Assignment

Name _____   Date _____

A Little Dash of Logic
Two Methods of Logical Reasoning

Joseph reads a journal article that states that yogurt with live cultures greatly helps digestion and prevents problems associated with lactose intolerance. He notices that his mother has problems with digestion and is lactose intolerant. He suggests that she try eating yogurt, because he thinks it may help her feel better.

1. What is the specific information in this situation?

2. What is the general information in this situation?

3. What is the conclusion in this situation?

4. Did Joseph use inductive or deductive reasoning to make his conclusion? Explain.

5. Is Joseph's conclusion correct? Explain.

Sadie reads an article that gives statistics about American families. She learns that the average American family has 2 parents and 2.3 children. She concludes that her neighbors, who are average Americans, probably have 2.3 children.

6. What is the specific information in this situation?

7. What is the general information in this situation?

8. What is the conclusion in this situation?

9. Did Sadie use inductive or deductive reasoning to make her conclusion? Explain.

10. Is Sadie's conclusion correct? Explain.

Nick watches baseball games on television. He noticed that the last five times that the Wolverines played against the Spartans, the Spartans won. Nick concludes that the Spartans always win against the Wolverines.

11. What is the specific information in this situation?

12. What is the general information in this situation?

13. What is the conclusion in this situation?

14. Did Nick use inductive or deductive reasoning to reach his conclusion? Explain.

15. Is Nick's conclusion correct? Explain.

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Lena has noticed that sometimes her face swells after she eats. She thinks she may be allergic to a type of food, so she takes careful notes over the next month, writing down all the ingredients of each meal and noting when her face swells. Reviewing her notes, she finds that the only common ingredient in the meals she ate previous to her face swelling was coconut. Lena concludes that she is allergic to coconut.

16. What is the specific information in this situation?

17. What is the general information in this situation?

18. What is the conclusion in this situation?

19. Did Lena use inductive or deductive reasoning to reach her conclusion? Explain.

20. Is Lena's conclusion correct? Explain.

Chaun is looking through records at a record store with her friend Ronaldo. She comes across a record she has not heard by a band she enjoys. Ronaldo knows that Chaun has five records at home by this band and that she likes all of them. He concludes that she will probably like any record made by this band. He tells Chaun so. She buys the record, saying to herself, "I will probably like this record, because I like records made by this band."

21. What conclusion did Ronaldo make? Why?

22. What type of reasoning did Ronaldo use? Explain.

23. What conclusion did Chaun make? Why?

24. What type of reasoning did Chaun use? Explain.

25. Is Ronaldo's conclusion definitely true? Is Chaun's conclusion definitely true? Explain.

The first five numbers in a sequence are 7, 21, 63, 189, and 567.

26. What is the next number in the sequence? How did you calculate the next number?

27. What type or types of reasoning did you use to find the next number? Explain the reasoning you used and the order of your conclusions.

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Assignment

Name _____ Date _____

What's Your Conclusion?
Hypotheses, Conclusions, Conditional Statements, Counterexamples, Direct and Indirect Arguments

Read each pair of statements. Then write a valid conclusion.

1. Statement: If you hear thunder, there must be lightning.

   Statement: You hear thunder.

   Conclusion:

2. Statement: If Jon is hungry, he wants food.

   Statement: Jon is hungry.

   Conclusion:

3. Statement: We will not be able to go sledding if it does not snow.

   Statement: It has not snowed.

   Conclusion:

4. Statement: A number is positive if it is greater than zero.

   Statement: The number -3 is less than zero.

   Conclusion:

5. Statement: If Trisha's cat Alexander stops meowing, he must have been fed.

   Statement: Alexander has not been fed.

   Conclusion:

Read each statement and conclusion. Then write the additional statement required to reach the conclusion.

6. Statement: If Matt soundproofs his basement, he can play drums at home.

   Statement:

   Conclusion: Therefore, Matt can play drums at home.

7. Statement: If Jason leaves his dinner in the oven too long, it gets burned.

Statement:

Conclusion: Therefore, he did not leave it in the oven too long.

8. Statement: You are at the North Pole if walking in any direction takes you south.

Statement:

Conclusion: Therefore, you are at the North Pole.

9. Statement: Julie will get tired if she dances all night.

Statement:

Conclusion: Therefore, Julie did not dance all night.

10. Statement:

Statement: Dutch is making yogurt.

Conclusion: Therefore, he needs milk.

11. Statement:

Statement: Alina got home early.

Conclusion: Therefore, she had time to fix her bicycle.

12. Statement:

Statement: It is not April.

Conclusion: Therefore, Jonas and Gabriel are not the same age.

Use Questions 1 through 12 to answer the following questions.

13. In Questions 1 through 12, the first statement is a conditional statement. For each conditional statement in Questions 1 through 12, underline the hypothesis with a solid line and underline the conclusion with a dotted line.

14. List the questions that used direct argument to reach their conclusions.

15. List the questions that used proof by contrapositive to reach their conclusions.

Melania noticed that $3^2 = 9 > 3$, $10^2 = 100 > 10$, and $5^2 = 25 > 5$. She proposes that the square of any number is greater than the number.

16. Did Melania use inductive or deductive reasoning to reach her conclusion? Explain.

17. Is Melania correct? If she is incorrect, give a counterexample.

P.J. noticed that $(-3)^2 = 9$, $(-3)^4 = 81$, and $(-3)^0 = 1$. Thinking, "each power of -3 is positive," he concluded that $(-3)^3$ must also be positive.

18. Did P.J. use inductive reasoning, deductive reasoning, or both to reach his conclusion? Explain.

19. Is P.J. correct? Explain. What type of argument did you use to decide? Explain.

In Questions 20 through 23, write a statement or statements that fit the given criteria.

20. Write a conditional statement. Underline the hypothesis with a solid line and the conclusion with a dotted line.

21. Use your conditional statement from Question 20 to write a direct argument.

22. Use your conditional statement from Question 20 to write an indirect argument.

23. Write a false assertion. Then provide a counterexample to show that it is false.

Assignment

Name _____ Date _____

You Can't Handle the Truth (Table)
Converses, Inverses, Contrapositives, Biconditionals, Truth Tables, Postulates, and Theorems

Consider the conditional statement "If I complete my homework, then I receive extra credit in class." Use this statement to answer Questions 1 through 10.

1. What is the hypothesis, p? What is the conclusion, q?

   Hypothesis, p:

   Conclusion, q:

2. Assume that p is true and q is true. What does that mean?

3. Could this statement be true? What is the truth value of the conditional statement when p is true and q is true?

4. Assume that p is true and q is false. What does that mean?

5. Could this statement be true? What is the truth value of the conditional statement when p is true and q is false?

6. Assume that p is false and q is true. What does that mean?

7. Could this statement be true? What is the truth value of the conditional statement when p is true and q is true?

8. Assume that p is false and q is false. What does that mean?

9. Could this statement be true? What is the truth value of the conditional statement when p is true and q is true?

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10. Summarize the information from Questions 1 through 10 by completing the truth table below.

p	q	p ↦ q

For each conditional statement, (a) identify the hypothesis, (b) identify the conclusion, (c) determine whether the conditional statement is true, (d) write the converse of the conditional statement, and (e) determine whether the converse is true.

11. If an organism can perform photosynthesis, then the organism is a plant.

    a. Hypothesis, *p*:

    b. Conclusion, *q*:

    c. Is the conditional statement true? Explain.

    d. Converse:

    e. Is the converse true? Explain.

12. If Agioso is in his room, then he is in his house.

    a. Hypothesis, *p*:

    b. Conclusion, *q*:

    c. Is the conditional statement true? Explain.

    d. Converse:

    e. Is the converse true? Explain.

13. If the Sun is down, then it is daytime.

    a. Hypothesis, p: .

    b. Conclusion, q:

    c. Is the conditional statement true? Explain.

    d. Converse:

    e. Is the converse true? Explain.

14. If Molly is a teenager, she is 13 years old.

    a. Hypothesis, p:

    b. Conclusion, q:

    c. Is the conditional statement true? Explain.

    d. Converse:

    e. Is the converse true? Explain.

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For each conditional statement, (a) determine whether the conditional statement is true, (b) identify the negation of the hypothesis, (c) identify the negation of the conclusion, (d) write the inverse of the conditional statement, and (e) determine whether the inverse is true.

15. If an organism can perform photosynthesis, then the organism is a plant.

    a. Is the conditional statement true?

    b. Not p:

    c. Not q:

    d. Inverse:

    e. Is the inverse true? Explain.

16. If Agioso is in his room, then he is in his house.

    a. Is the conditional statement true?

    b. Not *p*:

    c. Not *q*:

    d. Inverse:

    e. Is the inverse true? Explain.

17. If the Sun is down, then it is daytime.

    a. Is the conditional statement true?

    b. Not *p*:

    c. Not *q*:

    d. Inverse:

    e. Is the inverse true? Explain.

18. If Molly is a teenager, she is 13 years old.

    a. Is the conditional statement true?

    b. Not *p*:

    c. Not *q*:

    d. Inverse:

    e. Is the inverse true? Explain.

For each conditional statement, (a) determine whether the conditional statement is true, (b) write the contrapositive of the conditional statement, and (c) determine whether the contrapositive is true.

19. If an organism can perform photosynthesis, then the organism is a plant.

    a. Is the conditional statement true?

    b. Contrapositive:

    c. Is the contrapositive true? Explain.

20. If Agioso is in his room, then he is in his house.

    a. Is the conditional statement true?

    b. Contrapositive:

    c. Is the contrapositive true? Explain.

21. If the Sun is down, then it is daytime.

    a. Is the conditional statement true?

    b. Contrapositive:

    c. Is the contrapositive true? Explain.

22. If Molly is a teenager, she is 13 years old.

    a. Is the conditional statement true?

    b. Contrapositive:

    c. Is the contrapositive true? Explain.

23. Summarize the information from Questions 11 through 22 in by completing the sentences.

    a. If a conditional statement is true, then its converse

    b. If a conditional statement is false, then its converse

    c. If a conditional statement is true, then its inverse

    d. If a conditional statement is false, then its inverse

    e. If a conditional statement is true, then its contrapositive

    f. If a conditional statement is false, then its contrapositive

    g. A conditional statement and its _____ are logically equivalent.

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24. Which conditional statement from Questions 11 through 22 can be rewritten as a true biconditional statement? Rewrite it as a biconditional statement.

a. Conditional statement:

b. Biconditional statement:

25. Describe what a *postulate* is and what a *theorem* is.

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Assignment

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Proofs Aren't Just for Geometry
Introduction to Direct and Indirect Proof with the Properties of Numbers

For each conditional statement in Questions 1 through 4, determine whether the statement is true or false. If it is true, write a direct and an indirect proof of the statement. For each proof, write the steps in one column and the reasons in another. If the statement is false, prove it is false by counterexample.

1. Conditional statement: If $ab + ac = b + c$, then $a = 1$.

2. Conditional statement: If a and b are real numbers, then $10a + b = 10(a + b)$.

3. Conditional statement: If a and b are real numbers, then $\dfrac{3(2a + 2b)}{6} = a + b$.

4. Conditional statement: If $ab + ac = \dfrac{b + c}{\frac{1}{a}}$, then $b + c = 0$.

5. Identify the error in the proof. Correct the statement if possible.

If $2a + 2b = 4a$, then $b = 2$.

Steps	Reasons
$2(a + b) = 4a$	Distributive law
$\dfrac{2(a + b)}{2} = \dfrac{4a}{2}$	Algebraic equations remain true if you perform the same operation on both sides.
$a + b = 2a$	Multiplicative inverse
$a - a + b = 2a - a$	Algebraic equations remain true if you perform the same operation on both sides.
$b = 2$	Additive inverse

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