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A Little Dash of Logic
Two Methods of Logical Reasoning

Vocabulary

Define each term in your own words.

1. inductive reasoning

2. deductive reasoning

Problem Set

For each situation, identify the specific information, the general information, and the conclusion.

1. You read an article in the paper that says a high-fat diet increases a person's risk of heart disease. You know your father has a lot of fat in his diet, so you worry that he is at great risk of heart disease.

   Specific information: Your father has a lot of fat in his diet.

   General information: High-fat diets increase the risk of heart disease.

   Conclusion: Your father is at higher risk of heart disease.

2. You hear from your teacher that spending too much time in the sun without sunblock increases the risk of skin cancer. Your friend Susan spends as much time as she can outside working on her tan without sunscreen, so you tell her that she is increasing her risk of skin cancer when she is older.

3. Janice tells you that she has been to the mall three times in the past week, and every time there were a lot of people there. "It's always crowded at the mall," she says.

4. John returns from a trip out West and reports that it was over 100 degrees every day. "It's always hot out West," he says.

3

Determine the type of reasoning used in each situation. Then determine whether the conclusion is correct.

5. Jason sees a line of 10 school buses and notices that each is yellow. He concludes that all school buses must be yellow. What type of reasoning is this? Is his conclusion correct? Explain.

   It is inductive reasoning because he has observed specific examples of a phenomenon—the color of school buses—and come up with a general rule based on those specific examples.

   The conclusion is not necessarily true. It may be the case, for example, that all or most of the school buses in this school district are yellow, while another school district may have orange school buses.

6. Caitlyn has been told that every taxi in New York City is yellow. When she sees a red car in New York City, she concludes that it cannot be a taxi. What type of reasoning is this? Is her conclusion correct? Explain.

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7. Miriam has been told that lightning never strikes twice in the same place. During a lightning storm, she sees a tree struck by lightning and goes to stand next to it, convinced that it is the safest place to be. What type of reasoning is this? Is her conclusion correct? Explain.

8. Jose is shown the first six numbers of a series of numbers: 7, 11, 15, 19, 23, 27. He concludes that the general rule for the series of numbers is $a_n = 4n + 3$. What type of reasoning is this? Is his conclusion correct? Explain.

3

In each situation, identify the type of reasoning that each of the two people are using. Then compare and contrast the two types of reasoning.

9. When Madison babysat for the Johnsons for the first time, she was there 2-hours and was paid \$30. The next time she was there for 5-hours and was paid \$75. She decided that the Johnsons were paying her \$15 per hour. The third time she went, she stayed for 4-hours. She tells her friend Jennifer that she makes \$15 per hour babysitting. So, Jennifer predicted that Madison made \$60 for her 4-hour babysitting job.

Madison used inductive reasoning to conclude that the Johnsons were paying her at a rate of \$15 per hour. From that general rule, Jennifer used deductive reasoning to conclude that 4 hours of babysitting should result in a payment of \$60. The inductive reasoning looks at evidence and creates a general rule from the evidence. By contrast, the deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen in a particular instance.

10. When Holly was young, the only birds she ever saw were black crows. So, she told her little brother Walter that all birds are black. When Walter saw a bluebird for the first time, he was sure it had to be something other than a bird.

11. Tamika is flipping a coin and recording the results. She records the following results: heads, tails, heads, tails, heads, tails, heads. She tells her friend Javon that the coin alternates between heads and tails for each toss. Javon tells her that the next time the coin is flipped, it will definitely be tails.

12. John likes to watch the long coal trains moving past his house. Over the weeks of watching he notices that every train going east is filled with coal, but the trains heading west are all empty. He tells his friend Richard that all trains heading east have coal and all trains heading west are empty. When Richard hears a train coming from the west, he concludes that it will certainly be filled with coal.

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What's Your Conclusion? Hypotheses, Conclusions, Conditional Statements, Counterexamples, Direct and Indirect Arguments

Vocabulary

Write the term that best completes each statement.

1. A(n) ______________________ disproves a conditional statement by supplying a specific example for which that statement is false.
2. A logical statement that has the form "If *p*, then *q*" is called a(n) ______________________.
3. A(n) ______________________ is a form of proof that contains a conditional statement, a second statement formed by the hypothesis of the conditional statement, and a conclusion formed by the conclusion of the conditional statement.
4. The ______________________ is the first part of a conditional statement, or the *p* portion of the statement "If *p*, then *q*."
5. The ______________________ is the second part of a conditional statement, or the *q* portion of the statement "If *p*, then *q*."
6. A counterexample is an example of a form of proof called ______________________.
7. A(n) ______________________ is an argument that takes the form "If *q* is false, then *p* is false."

Problem Set

Write a conditional statement using each set of words. Explain why it is a conditional statement.

1. birthday and age

   If my age is 15 now, then I will be 16 on my next birthday.

   This is a conditional statement because it is in the form "If *p*, then *q*," where *p* is the statement "my age is 15 now," and *q* is the statement "I will be 16 on my next birthday."

2. rain and umbrellas

3. reading a notice and knowing information

4. studying and grades

3

For each conditional statement, draw a solid line beneath the hypothesis. Then draw a dotted line beneath the conclusion.

5. If it is sunny tomorrow, we will go to the beach.

6. If the groundhog sees its shadow, there will be six more weeks of winter.

7. If a and b are real numbers, then $a^2 + b^2$ is greater than or equal to 0.

8. If I am smiling, then I am happy.

For each conditional statement, write a direct argument that uses the conditional statement. Then write an indirect argument that uses the conditional statement.

9. If it is the weekend, I don't have to go to school.

   Direct argument:

   Today is Saturday.

   Therefore, I do not have to go to school.

   Indirect argument:

   I have to go to school today.

   Therefore, it cannot be the weekend.

10. If the last digit of a number is 0, that number is divisible by both 2 and 5.

11. If a banana is green, it is not ripe.

12. If a candidate for President receives at least 270 electoral votes, that candidate is elected President.

For each conditional statement, write a proof by contrapositive.

13. If N is an even number, then it is divisible by 2.

The number 3 is not divisible by 2.

Therefore, the number 3 is not an even number.

14. If a and b are both greater than 0, then ab is greater than 0.

15. If I get a good night's sleep, I am not tired in the morning.

16. If the temperature drops below 20 degrees Fahrenheit, our orange tree will not survive.

For each conditional statement, write a proof by counterexample.

17. If a is a real number, then $\sqrt{a^2} = a$.

Let a be -1. The number -1 is a real number, and $\sqrt{(-1)^2} = \sqrt{1} = 1$, which is not equal to -1.

So, the statement is false by counterexample.

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18. If a, b, c, and d are real numbers, then $(a + b)(c + d) = ac + bd$.

19. If a and b are negative integers, then $a - b$ is also a negative integer.

20. If a and b are irrational numbers, then ab is also an irrational number.

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You Can't Handle the Truth (Table)
Converses, Inverses, Contrapositives, Biconditionals, Truth Tables, Postulates, and Theorems

Vocabulary

Explain how each set of terms is related by identifying similarities and differences.

1. inverse, converse, and contrapositive

2. truth value and truth table

3. postulate and theorem

4. propositional form and propositional variables

5. logically equivalent and biconditional statement

Problem Set

Complete the truth table for each conditional statement. Then explain what each row means in the truth table.

1. "If I can play the violin, then I can join the orchestra."

p	q	$p \to q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

Row 1: If p is true, then I can play the violin. If q is true, then I can join the orchestra. It is true that if I can play the violin, I can join the orchestra, so the truth value of the conditional statement is true.

Row 2: If p is true, then I can play the violin. If q is false, then I cannot join the orchestra. It is false that if I can play the violin, I cannot join the orchestra, so the truth value of the conditional statement is false.

Row 3: If p is false, then I cannot play the violin. If q is true, then I can join the orchestra. It could be true that if I cannot play the violin, I can join the orchestra, so the truth value of the conditional statement in this case is true.

Row 4: If p is false, then I cannot play the violin. If q is false, then I cannot join the orchestra. It could be true that if I cannot play the violin, I cannot join the orchestra, so the truth value of the conditional statement in this case is true.

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2. "If $n = 2$, then $n^2 = 4$."

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

3. "If a plant is an oak, then the plant is a tree."

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

3

4. "If your mode of transportation is a motorcycle, then your mode of transportation has two wheels."

p	*q*	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

3

Write the converse of each conditional statement.

5. If today is Tuesday, then Janis has a piano lesson after school.

   If Janis has a piano lesson after school, then today is Tuesday.

6. If that animal is a dog, then it has four legs.

7. If he believed that the sky is green, then he would be crazy.

8. If one book costs $10, then five books cost $50.

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Write the inverse of each conditional statement.

9. If you go to the grocery store on Saturday, then there will be very long lines.

If you do not go to the grocery store on Saturday, then there will not be very long lines.

10. If Krista gets an A on her history test, then she is allowed to spend the weekend with her friend.

11. If the bus does not arrive on time, then Milo will be late for work.

12. If there is a chance of rain this weekend, then Liza will cancel her camping trip.

3

Write the contrapositive of each conditional statement.

13. If a triangle is an equilateral triangle, then all of its sides are equal.

If the sides of a triangle are not all equal, then the triangle is not an equilateral triangle.

14. If it is dark outside, then it is nighttime.

15. If there are more than 30 students in this classroom, then it is too crowded.

16. If the next animal you see is a kangaroo, then you are in Australia.

For each conditional statement, write the converse of that statement. If possible, write a true biconditional statement. If not possible, explain why.

17. If N is divisible by 10, then the last digit in N is 0.

If the last digit in *N* is 0, then *N* is divisible by 10. True.

Biconditional statement: *N* is divisible by 10 if and only if the last digit in *N* is 0.

18. If two triangles are congruent, then the triangles have equal angles.

19. If the last digit in N is 5, then N is divisible by 5.

3

20. If x is greater than 0, then x^3 is greater than 0.

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Proofs Aren't Just for Geometry
Introduction to Direct and Indirect Proof with the Properties of Numbers

Vocabulary

Write the term that best completes each statement.

1. For addition, the ____________________ states that $a + 0 = a$.
2. For multiplication, the ____________________ states that $ab = ba$.
3. A(n) ____________________ proves a statement by first assuming that the conclusion of the statement is false and then showing that such an assumption leads to a contradiction.
4. For addition, the ____________________ states that $a + (b + c) = (a + b) + c$.
5. For multiplication, the ____________________ states that $a \cdot \frac{1}{a} = 1$ if $a \neq 0$.
6. The ____________________ states that $a(b + c) = ab + ac$.

Problem Set

Identify whether the commutative law, associative law, identity law, inverse law of addition, or inverse law of multiplication explains why each statement is true.

1. $(5 + 3) + 4 = 5 + (3 + 4)$

   Associative law of addition

2. $172.3 + (-172.3) = 0$

3. $107 \cdot \frac{1}{107} = 1$

4. $12 \cdot 23 = 23 \cdot 12$

5. $13{,}416.7 \cdot 1 = 13{,}416.7$

6. $37 + 92 = 92 + 37$

7. $13(24 \cdot 117) = (13 \cdot 24)117$

8. $16\frac{3}{5} + 0 = 16\frac{3}{5}$

9. $65 \cdot 987 = 987 \cdot 65$

10. $344 \cdot 1 = 344$

11. $555 + 333 = 333 + 555$

12. $(45 \cdot 906)11 = 45(906 \cdot 11)$

13. $65.6 + 0 = 65.6$

14. $(177 + 32) + 1714 = 177 + (32 + 1714)$

15. $4\frac{2}{3} + \left(-4\frac{2}{3}\right) = 0$

16. $32 \cdot \frac{1}{32} = 1$

Use the distributive law to calculate each value.

17. $12\ (6 + 10)$

$12\ (6 + 10) = 12(6) + 12\ (10) = 72 + 120 = 192$

3

18. $(13 + 22) \cdot 4$

19. $4\ (x + y)$

20. $13\ (a - b)$

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Proof or disprove each statement.

21. If $a(b + c) = b(a + c) + ac$, then either $b = 0$ or $c = 0$ (or both).

$a(b + c) = b(a + c) + ac$	
$ab + ac = ba + bc + ac$	Distributive law
$ab + ac = ab + bc + ac$	Commutative law of addition
$ab + ac - ac = ab + bc + ac - ac$	Subtraction law of equality
$ab = ab + bc$	Inverse law of addition
$ab - ab = ab + bc - ab$	Subtraction law of equality
$0 = bc + ab - ab$	Additive inverse and commutative law
$0 = bc$	Additive inverse
$b = 0$ or $c = 0$ (or both)	If a product is equal to zero, at least 1 factor in the product is equal to zero.

22. If $ab + bc + ac = a(b + c)$, then either $b = 0$ or $c = 0$ (or both).

3

23. If $(x + a)(x + b) = x^2 + ab$ for all x, then either $a = 0$ or $b = 0$.

3

24. If $(a + b)c = c(a - b)$, then either $b = 0$ or $c = 0$.

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25. If a and b are real numbers, then $a(b + 2) = ab + 2$.

26. If a, b, and c are real numbers, then $\frac{a}{b + c} = \frac{a}{b} + \frac{a}{c}$.

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