

Name _____

Date _____

Properties of Triangles Angle Relationships in a Triangle

Vocabulary

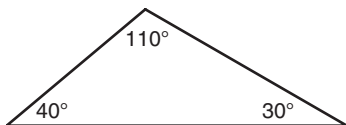
Match each word with its definition.

- | | |
|--------------------------------------|--|
| 1. acute triangle | a. an angle formed by one side of a triangle and an extension of another side |
| 2. obtuse triangle | b. The measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles. |
| 3. right triangle | c. a triangle with three equal angles |
| 4. equiangular triangle | d. a triangle with three acute angles |
| 5. exterior angle | e. a triangle that has one right angle |
| 6. remote interior angles | f. a triangle with one obtuse angle |
| 7. Exterior Angle Inequality Theorem | g. the two angles of a triangle that are not supplementary to a given exterior angle |

Problem Set

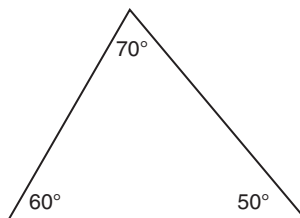
Classify each triangle as acute, obtuse, right, or equiangular.

1.

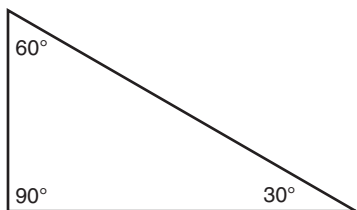


obtuse

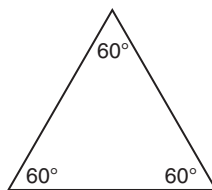
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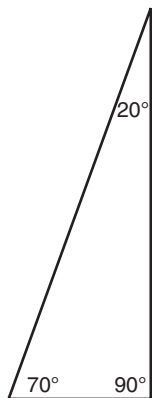
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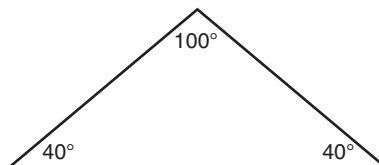
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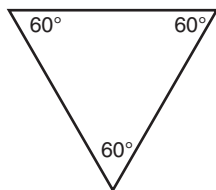


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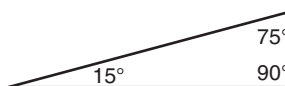


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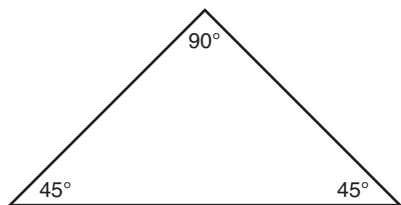
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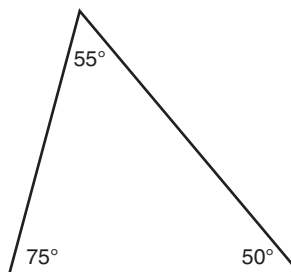
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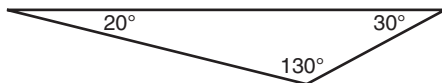
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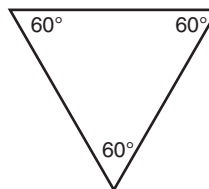
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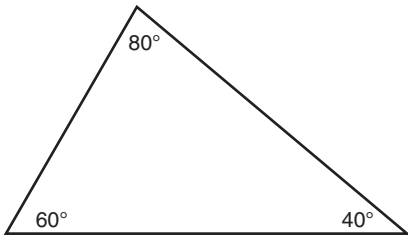
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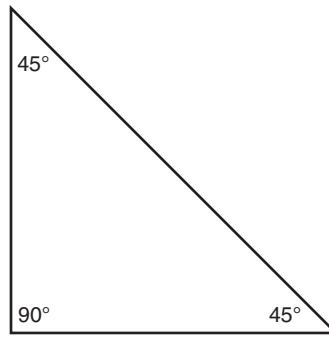
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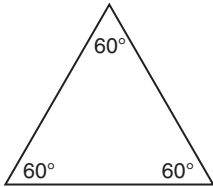
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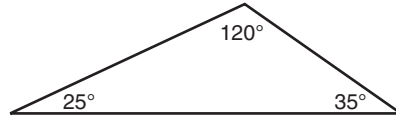
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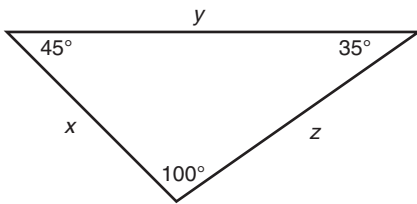


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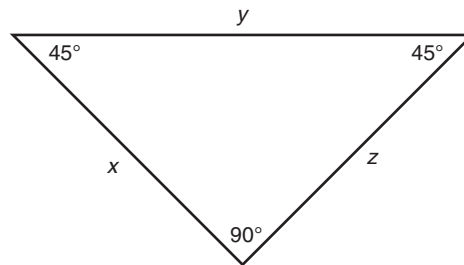
Identify the shortest and longest side of each triangle. Identify any sides that have the same length.

17.



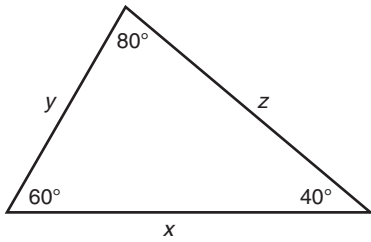
x is the shortest,
 y is the longest

18.

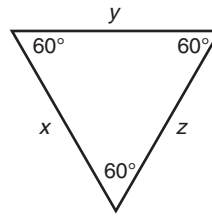


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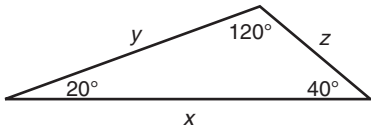
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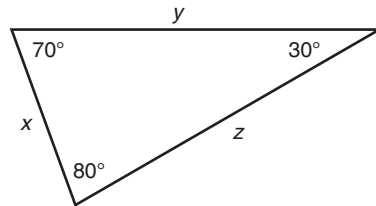
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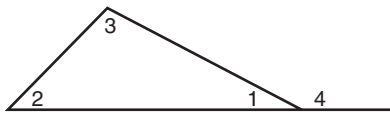


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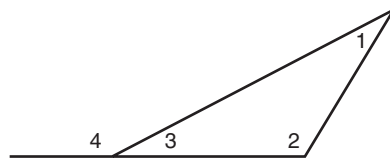
Identify the remote interior angles of each exterior angle.

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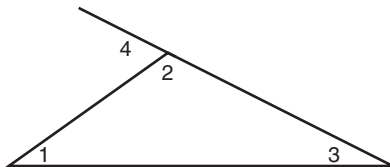


$\angle 2, \angle 3$

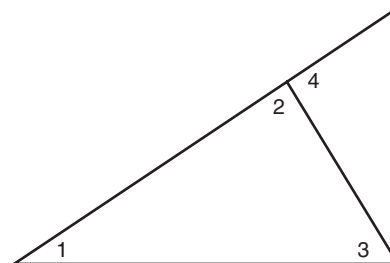
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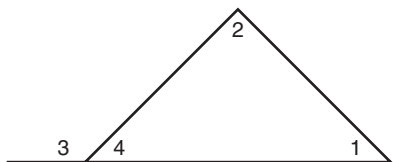


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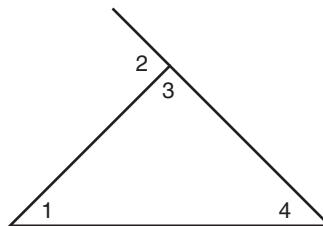
Identify the exterior angle of each triangle.

27.

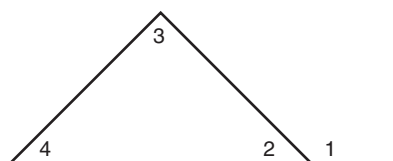


$\angle 3$

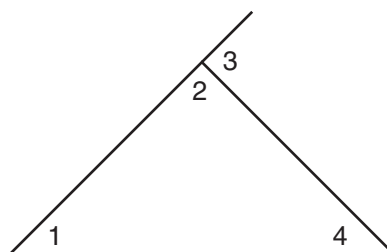
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29.

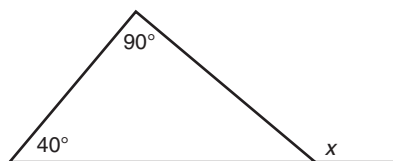


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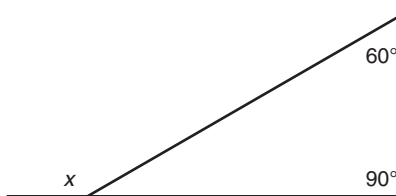
Solve for x in each triangle.

31.



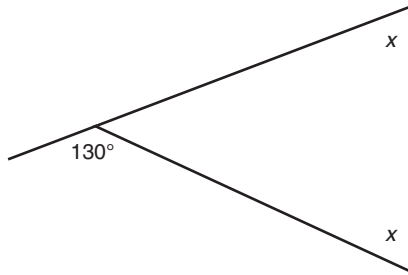
$x = 130^\circ$

32.

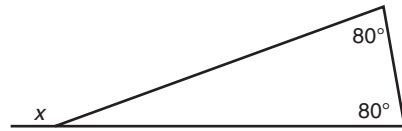


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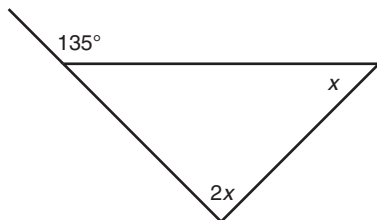
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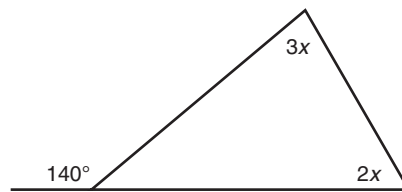
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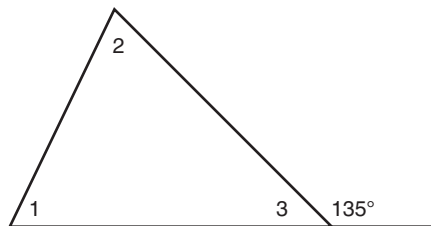


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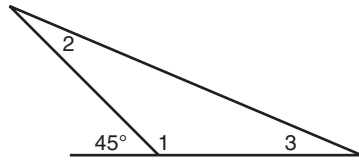
Use the Exterior Angle Inequality Theorem to describe the angles of each triangle.

37.

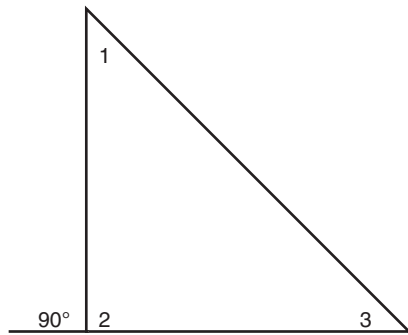


An exterior angle is equal to the sum of the remote interior angles. In this case, $\angle 1 + \angle 2 = 135^\circ$. Because $\angle 1$ and $\angle 2$ must be positive, and because their sum is 135° , then both $\angle 1$ and $\angle 2$ must be less than 135° . In other words, the measure of the exterior angle, 135° , is greater than the measure of $\angle 1$ and greater than the measure of $\angle 2$.

38.

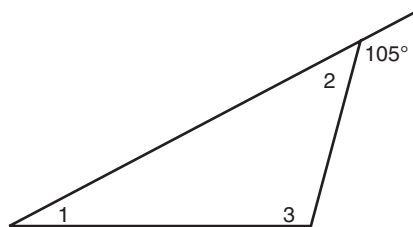


39.



5

40.



Name _____ Date _____

Properties of Triangles Side Relationships of a Triangle

Vocabulary

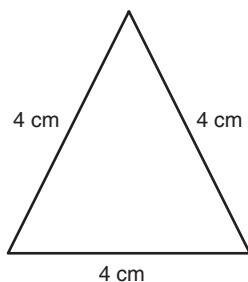
Define each term in your own words.

1. scalene triangle
2. isosceles triangle
3. equilateral triangle

Problem Set

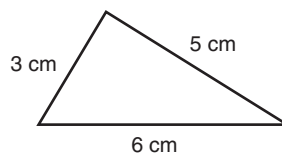
Identify each triangle as scalene, isosceles, or equilateral.

1.

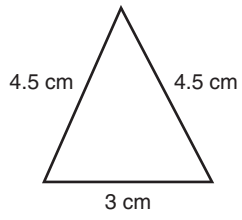


equilateral

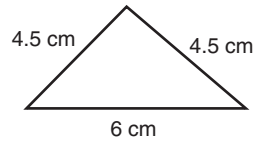
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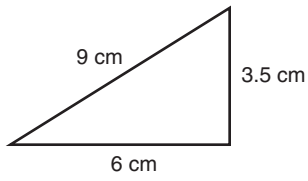
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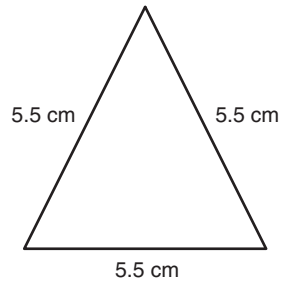
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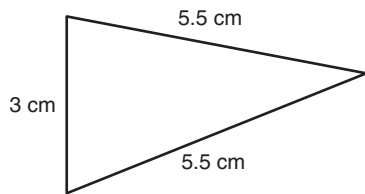
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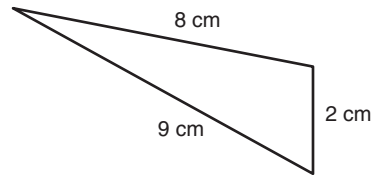
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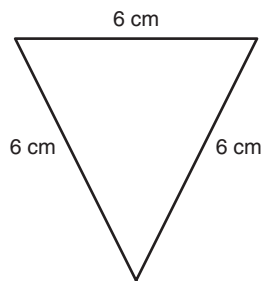


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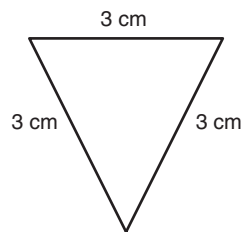


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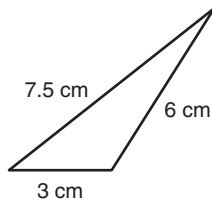
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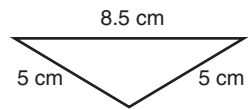
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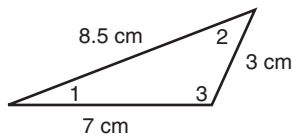


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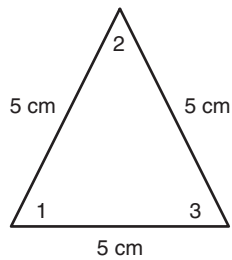


Identify the smallest and largest angle in each triangle. Identify any angles that have the same measure.

13.

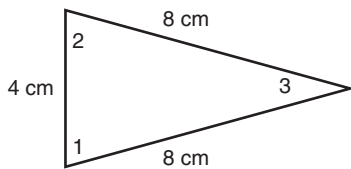


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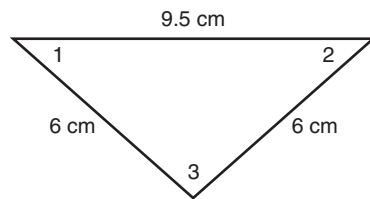


$\angle 1$ is the smallest, $\angle 3$ is the largest

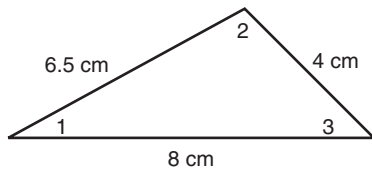
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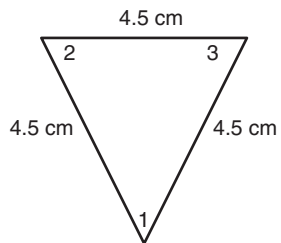
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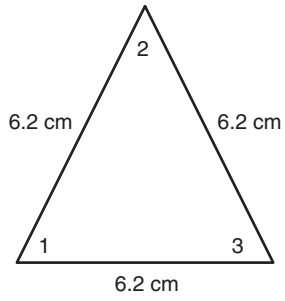
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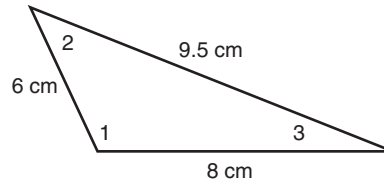
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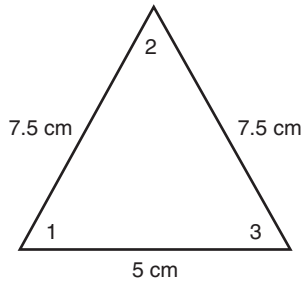
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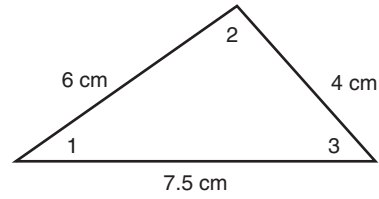
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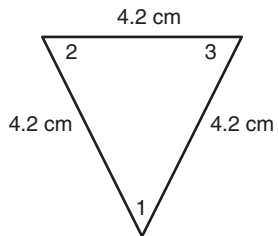


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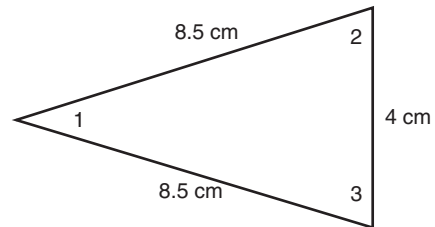


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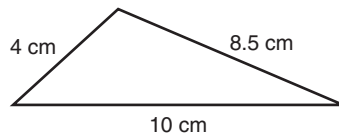


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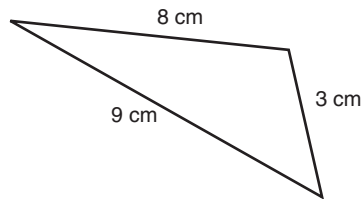
Explain why the Triangle Inequality Theorem is true for each triangle.

25.

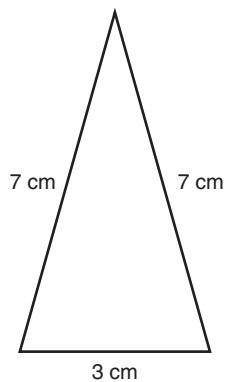


The measure of the longest side, 10 centimeters, is less than the sum of the measures of the two other sides: $4\text{ cm} + 8.5\text{ cm} = 12.5\text{ cm}$.

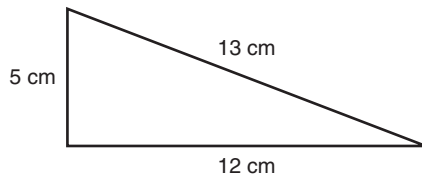
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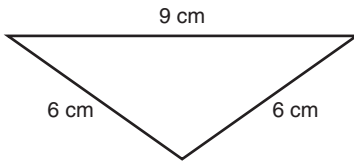
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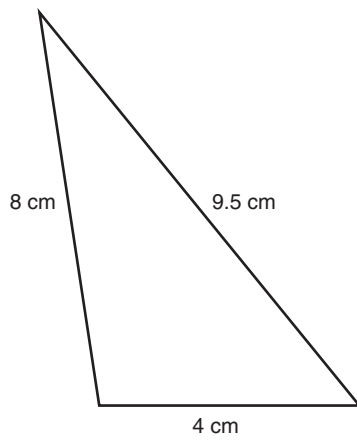
28.



29.



30.



5

Name _____ Date _____

Determine whether each set of segment lengths can form a triangle.

31. 4 cm, 6 cm, 9 cm

Yes

32. 4 in., 6 in., 10 in.

33. 12 ft, 12 ft, 25 ft

34. 2 m, 20 m, 21 m

35. 17 yd, 18 yd, 32 yd

36. 12 mm, 13 mm, 30 mm

37. 1 in., 11 in., 12 in.

38. 14 cm, 14 cm, 20 cm

Name _____ Date _____

Properties of Triangles Points of Concurrency

Vocabulary

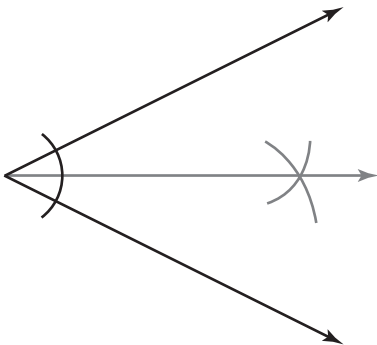
Write the term that best completes each statement.

1. A(n) _____ is a line that divides a segment into two smaller segments of equal length.
2. The _____ of a triangle is the point at which the three medians intersect.
3. Three or more lines that intersect at a common point are called _____.
4. A(n) _____ of a triangle is a line segment that connects a vertex to the midpoint of the side opposite the vertex.
5. The _____ of a triangle is the point at which the three perpendicular bisectors intersect.
6. The point at which three or more lines intersect is called the _____.
7. A(n) _____ is a segment bisector that is also perpendicular to the line segment.
8. The _____ of a triangle is the point at which the three angle bisectors intersect.
9. To divide an angle into two smaller angles of equal measure is to _____.
10. A perpendicular line segment that is drawn from a vertex to the opposite side is called an _____.
11. A(n) _____ is a line that divides an angle into two smaller angles of equal measure.
12. The _____ of a triangle is the point at which the three altitudes intersect.
13. To divide a segment into two smaller segments of equal length is to _____.

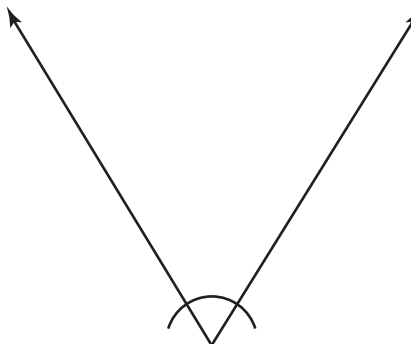
Problem Set

Use a compass and straightedge to construct an angle bisector of each angle.

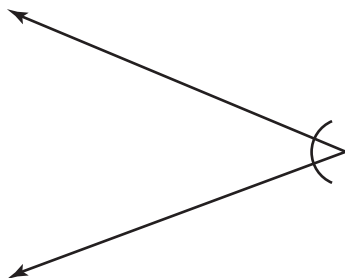
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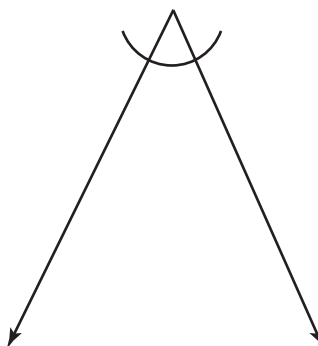
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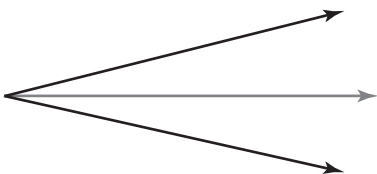
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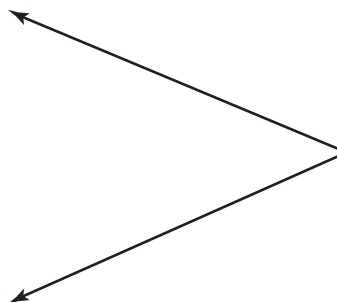
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Trace each angle onto patty paper and construct an angle bisector on the patty paper.

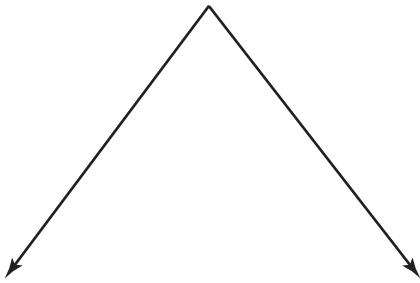
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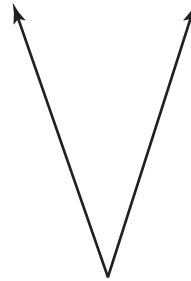
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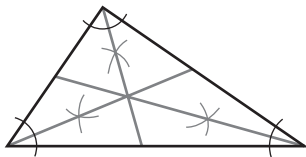


8.

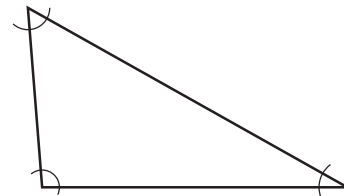


Use a compass and straightedge to construct angle bisectors for each angle of the triangle to construct the incenter of the triangle. Then, using patty paper, trace the triangle and construct the incenter on the patty paper to see if you get the same result.

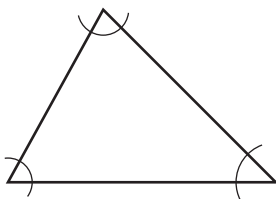
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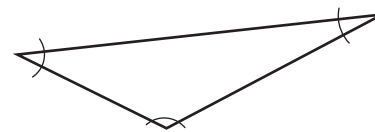
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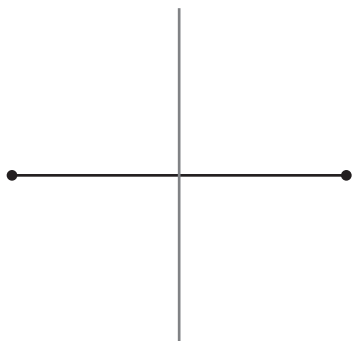


12.



Use a compass and straightedge to construct the perpendicular bisector of each line segment. Then, using patty paper, trace the line segment and construct a perpendicular bisector with the patty paper to see if you get the same result.

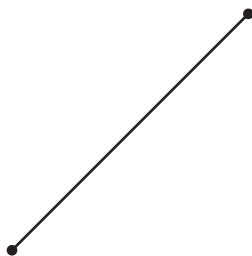
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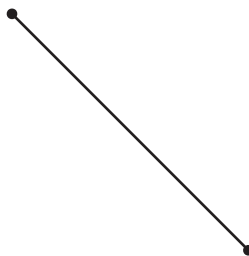
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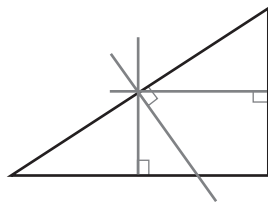


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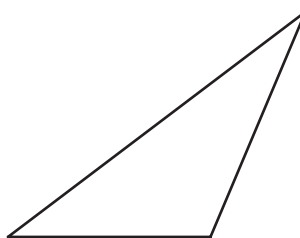


Use a compass and straightedge to construct the perpendicular bisector of each side of the triangle and then construct the circumcenter of the triangle.

17.



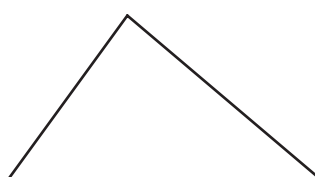
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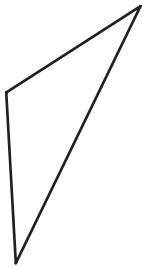


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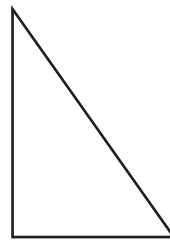


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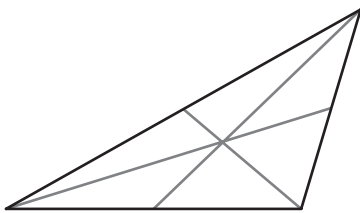


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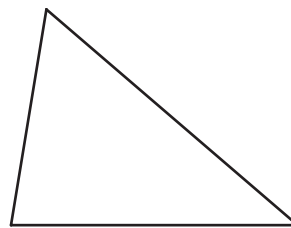


Use a compass and straightedge to construct the three medians of the triangle and construct the centroid. Then, using patty paper, trace the triangle and construct the centroid with the patty paper to see if you get the same result.

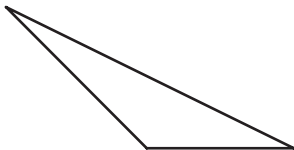
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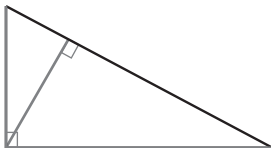


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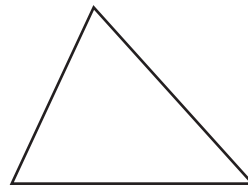


Use a compass and straightedge to construct the three altitudes of the triangle and construct the orthocenter. Then, using patty paper, trace the triangle and construct the orthocenter with the patty paper to see if you get the same result.

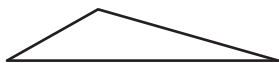
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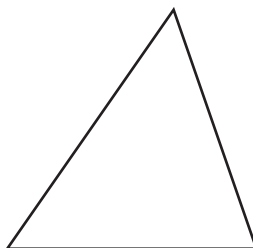
28.



29.



30.



31.



Compare the parts of the given types of triangles.

32. Compare the placement of the incenter and circumcenter for acute, obtuse, and right triangles.

For acute triangles, both the incenter and the circumcenter lie on the inside. For right triangles, the incenter lies on the inside of the triangle, while the circumcenter lies on the hypotenuse. For obtuse triangles, the incenter lies on the inside, while the circumcenter is outside the triangle.

33. Compare the placement of the incenter and centroid for acute, obtuse, and right triangles.

Name _____ Date _____

- 34.** Compare the placement of the incenter and orthocenter for acute, obtuse, and right triangles.
- 35.** Compare the placement of the circumcenter and centroid for acute, obtuse, and right triangles.
- 36.** Compare the placement of the circumcenter and orthocenter for acute, obtuse, and right triangles.
- 37.** Compare the placement of the centroid and orthocenter for acute, obtuse, and right triangles.

Name _____

Date _____

Properties of Triangles Direct and Indirect Proof

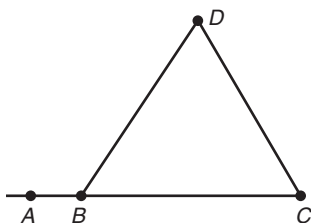
Vocabulary

Write the term that best completes each statement.

1. A(n) _____ is a way of writing a proof such that each step is listed in one column and the reason for each step is listed in the other column.
2. The _____ says an exterior angle of a triangle is greater than either of the two remote interior angles of the triangle.
3. According to the _____, if $a = b + c$ and $c > 0$, then $a > b$.
4. The _____ says that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.
5. The _____ means assuming the opposite of the conclusion.

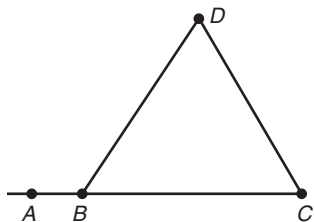
Problem Set

For the triangle shown, use a direct proof to prove each statement.



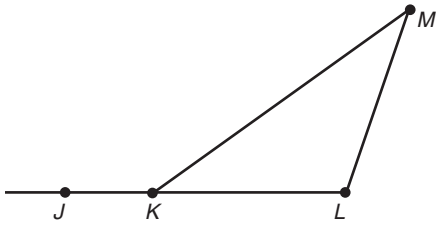
1. $m\angle ABD > m\angle C$

Statements	Reasons
1. Angle ABD is an exterior angle of triangle BCD .	1. Given
2. $\angle CBD + \angle C + \angle D = 180^\circ$	2. Triangle Sum Theorem
3. $\angle ABD$ and $\angle CBD$ are a linear pair	3. Linear Pair Postulate
4. $m\angle ABD + m\angle CBD = 180^\circ$	4. Definition of a linear pair
5. $m\angle C + m\angle D = m\angle ABD$	5. Subtraction Property of Equality
6. $m\angle D > 0$	6. Definition of angle measure
7. $m\angle ABD > m\angle C$	7. Inequality Property



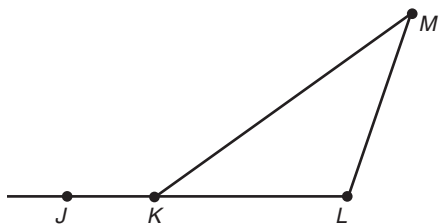
2. $m\angle ABD > m\angle D$

Statements	Reasons
1. Angle ABD is an exterior angle of triangle BCD .	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.



3. $m\angle JKM > m\angle L$

Statements	Reasons
1. Angle JKM is an exterior angle of triangle KLM .	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

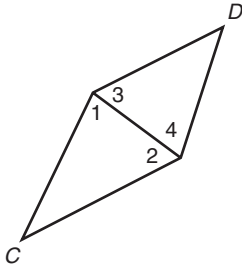


4. $m\angle JKM > m\angle M$

Statements	Reasons
1. Angle JKM is an exterior angle of triangle KLM .	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Complete each proof using the indicated method.

5. Use a direct proof to prove the statement.



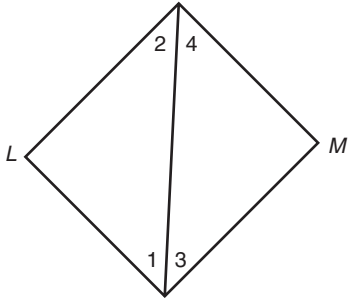
Given: $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$

Prove: $m\angle C = m\angle D$

Statements	Reasons
1. $m\angle 1 = m\angle 4$	1. Given
2. $m\angle 2 = m\angle 3$	2. Given
3. $m\angle 1 + m\angle 2 + m\angle C = 180^\circ$	3. Triangle Sum Theorem
4. $m\angle 3 + m\angle 4 + m\angle D = 180^\circ$	4. Triangle Sum Theorem
5. $m\angle 1 + m\angle 2 + m\angle C =$ $m\angle 3 + m\angle 4 + m\angle D$	5. Substitution using equations from steps 3 and 4
6. $m\angle 1 + m\angle 2 + m\angle C =$ $m\angle 1 + m\angle 2 + m\angle D$	6. Substitution using equations from steps 1, 2, and 5
7. $m\angle C = m\angle D$	7. Subtraction Property of Equality

5

6. Use a direct proof to prove the statement.

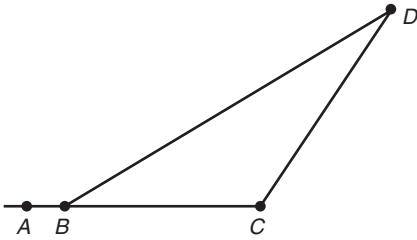


Given: $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$

Prove: $m\angle L = m\angle M$

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

7. Use an indirect proof with proof by contradiction to prove the statement.

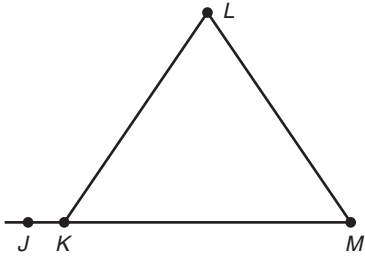


Given: $\angle ABD$ is an exterior angle of $\triangle BCD$

Prove: $m\angle ABD > m\angle C$

Statements	Reasons
1. Angle ABD is an exterior angle of triangle BCD .	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.

8. Use an indirect proof with proof by contradiction to prove the statement.

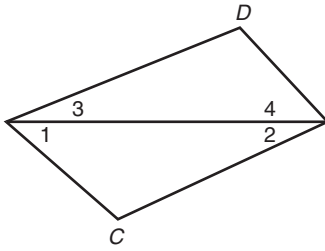


Given: $\angle JKL$ is an exterior angle of $\triangle KLM$.

Prove: $m\angle JKL > m\angle M$

Statements	Reasons
1. Angle JKL is an exterior angle of triangle KLM .	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.

9. Use an indirect proof with proof by contradiction to prove the statement.

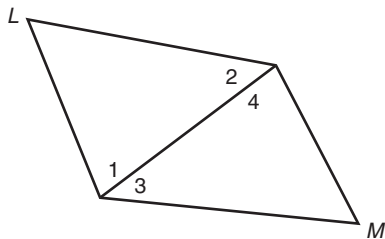


Given: $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$

Prove: $m\angle C = m\angle D$

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

10. Use an indirect proof with proof by contradiction to prove the statement.



Given: $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$

Prove: $m\angle L = m\angle M$

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

Name _____ Date _____

Computer Graphics Proving Triangles Congruent: SSS and SAS

Vocabulary

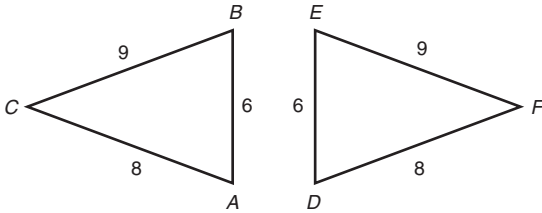
Match each term with its definition.

- | | |
|---------------------------------------|---|
| 1. Side-Side-Side Congruence Theorem | a. If two pairs of corresponding sides of triangles are congruent, and the included angles are congruent, the triangles themselves are congruent. |
| 2. Side-Angle-Side Congruence Theorem | b. a proof written in paragraph form |
| 3. paragraph proof | c. If the corresponding sides of two triangles are congruent, then the triangles themselves are congruent. |

Problem Set

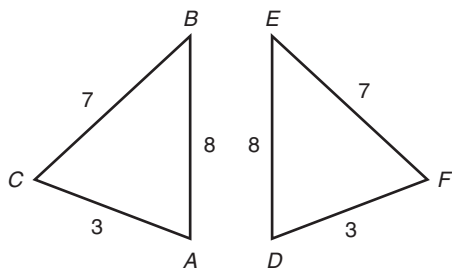
Complete the two-column proof and use the Side-Side-Side Similarity Postulate to prove that the two triangles are congruent.

1.



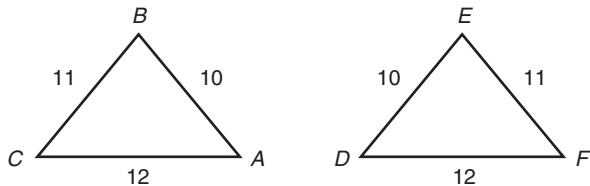
Statements	Reasons
1. $AB = DE = 6, BC = EF = 9, CA = FD = 8$	1. Given
2. $\frac{AB}{DE} = 1, \frac{BC}{EF} = 1, \frac{CA}{FD} = 1$	2. Division Property of Equality
3. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$	3. Transitive Property of Equality
4. $\triangle ABC \sim \triangle DEF$	4. SSS Similarity Postulate
5. $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$	5. Definition of similar triangles
6. $\triangle ABC \cong \triangle DEF$	6. Definition of congruence

2.



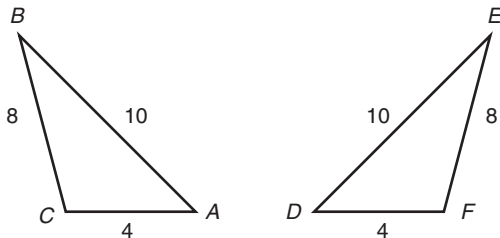
Statements	Reasons
1. $AB = DE = 8, BC = EF = 7, CA = FD = 3$	1.
2. $\frac{AB}{DE} = 1, \frac{BC}{EF} = 1, \frac{CA}{FD} = 1$	2.
3.	3. Transitive Property of Equality
4. $\triangle ABC \sim \triangle DEF$	4.
5.	5. Definition of similar triangles
6.	6. Definition of congruence

3.



Statements	Reasons
1.	1. Given
2.	2. Division Property of Equality
3. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$	3.
4.	4. SSS Similarity Postulate
5. $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$	5.
6.	6. Definition of congruence

4.

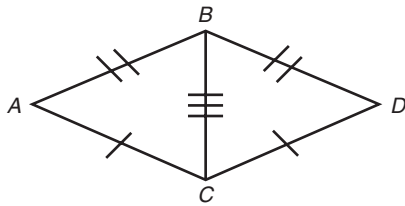


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Statements	Reasons
1. $AB = DE = 10, BC = EF = 8, CA = FD = 4$	1.
2.	2. Division Property of Equality
3. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$	3.
4.	4. SSS Similarity Postulate
5. $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$	5.
6.	6. Definition of congruence

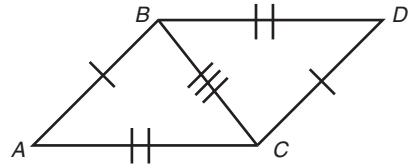
Use either the Side-Side-Side Congruence Theorem or the Side-Angle-Side Congruence Theorem to show that each pair of triangles is congruent.

5.

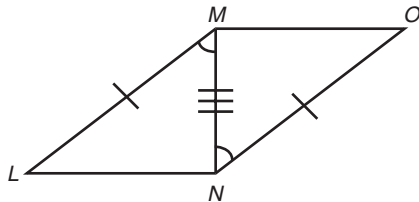


From the figure, $\overline{AB} \cong \overline{DB}$, $\overline{AC} \cong \overline{DC}$, $\overline{BC} \cong \overline{BC}$. Thus $\triangle ABC \cong \triangle DBC$ by the SSS Congruence Theorem.

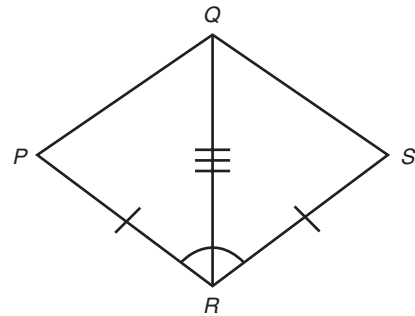
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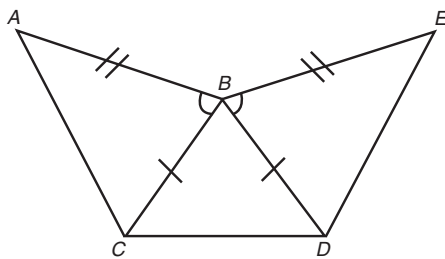
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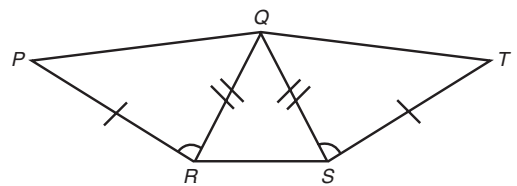
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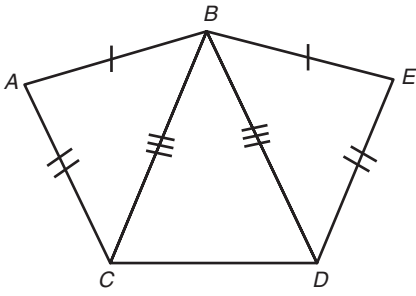
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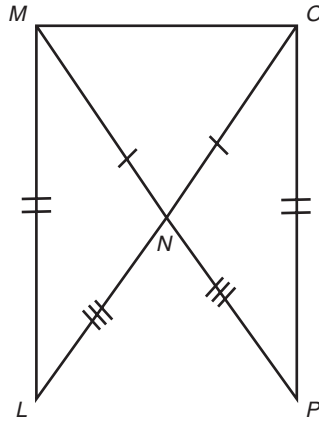
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11.



12.



Skills Practice

Skills Practice for Lesson 5.6

Name _____ Date _____

Wind Triangles Proving Triangles Congruent: ASA and AAS

Vocabulary

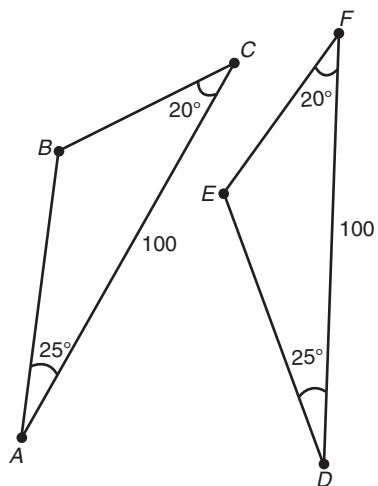
Explain the similarities and differences between the two terms.

1. Angle-Side-Angle Congruence Postulate and Angle-Angle-Side Congruence Theorem

Problem Set

Use the ASA Postulate to show that each pair of triangles is congruent.

1.

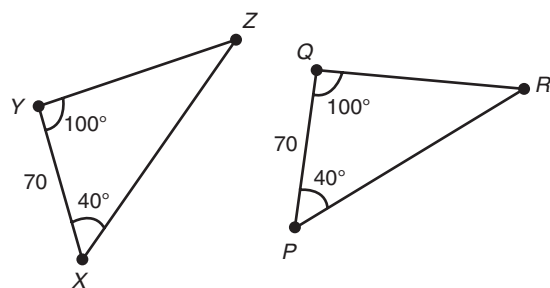


$$\angle A = 25^\circ, \angle C = 20^\circ, \overline{AC} = 100$$

$$\angle D = 25^\circ, \angle F = 20^\circ, \overline{DF} = 100$$

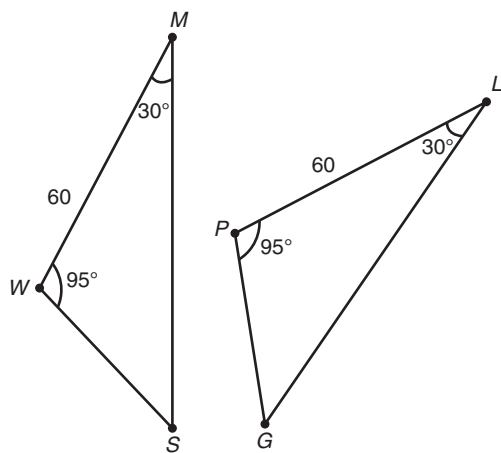
Therefore $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DF}$, and by the ASA Postulate,
 $\triangle ABC \cong \triangle DEF$.

2.

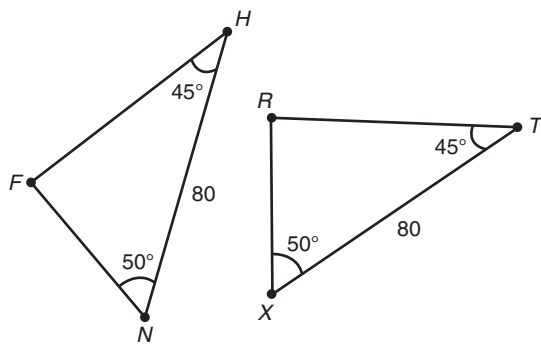


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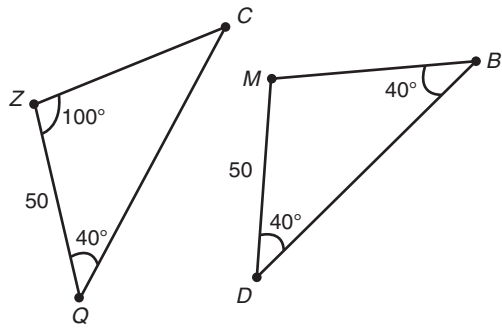
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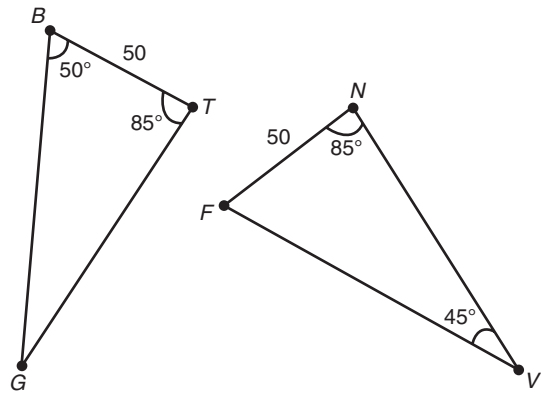
4.



5.

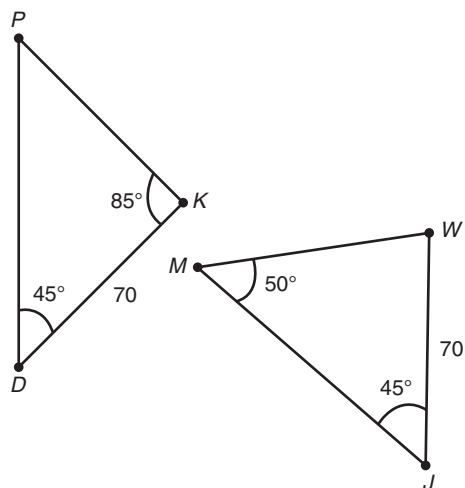


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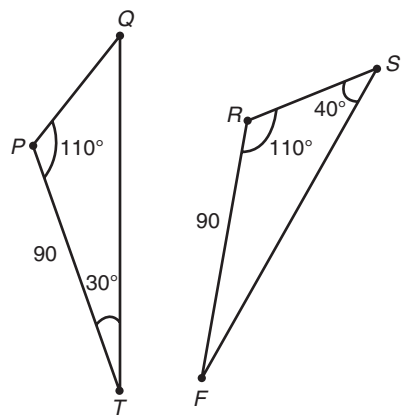


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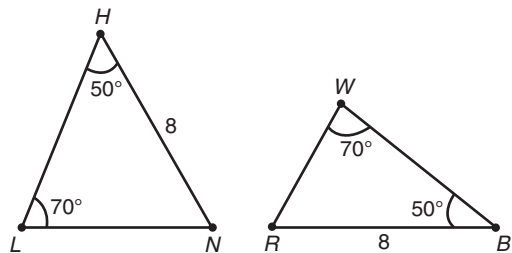


8.



Use the AAS Theorem to show that each pair of triangles is congruent.

9.

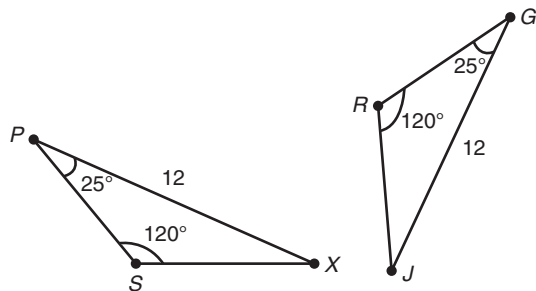


$$\angle L = 70^\circ, \angle H = 50^\circ, \overline{HN} = 8$$

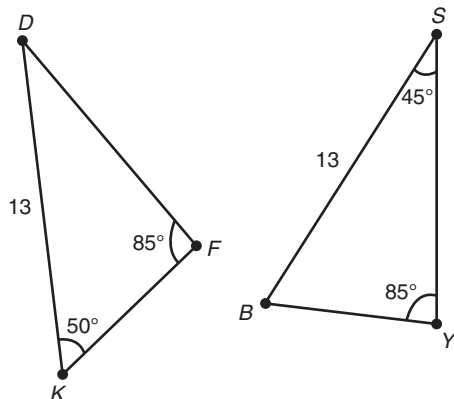
$$\angle W = 70^\circ, \angle B = 50^\circ, \overline{BR} = 8$$

Therefore $\angle L \cong \angle W$, $\angle H \cong \angle B$, $\overline{HN} \cong \overline{BR}$, and by the AAS Theorem,
 $\triangle HLN \cong \triangle WBR$.

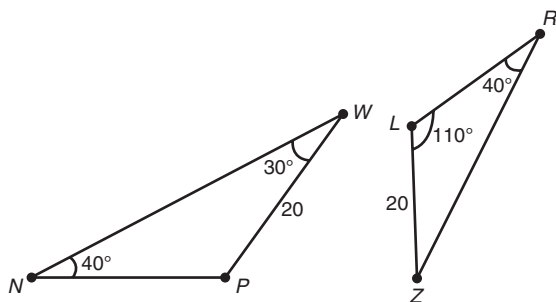
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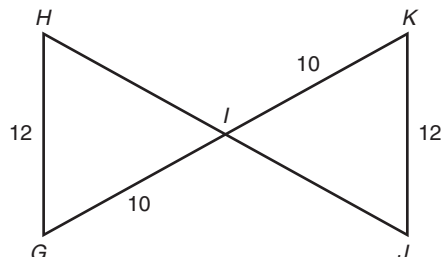


12.



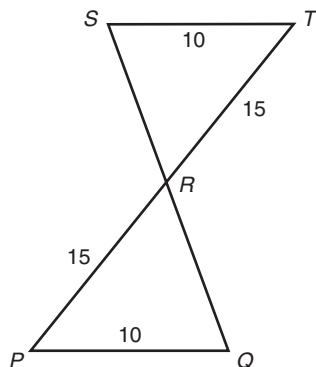
Determine whether there is enough information to tell whether the two triangles are congruent. If there is enough information, determine whether they are congruent.

13.



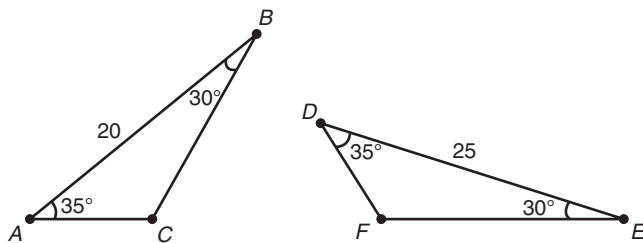
No, there is not enough information. We can see that $\angle HIG \cong \angle JIK$, and we know that $\overline{GI} = \overline{KI} = 10$ and $\overline{HG} = \overline{JK} = 12$, so two pairs of sides are congruent and one pair of angles is congruent. But these are not the included angles, and the relationship between the remaining angles cannot be determined, so none of the congruence postulates or theorems can be used.

14.

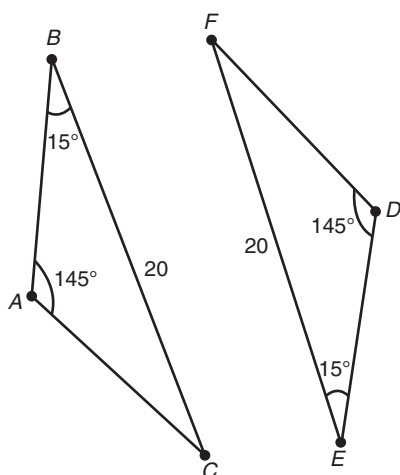


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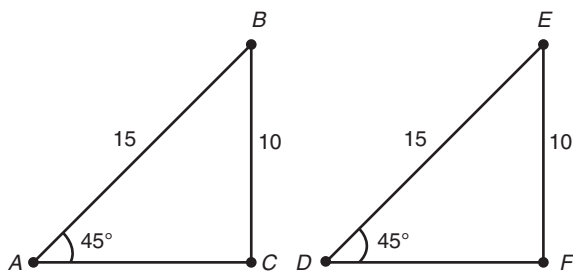
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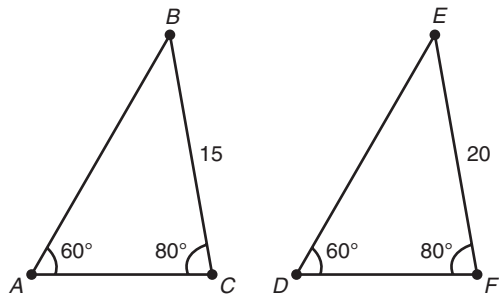
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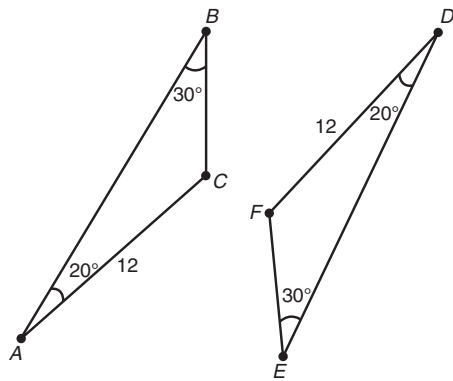
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18.

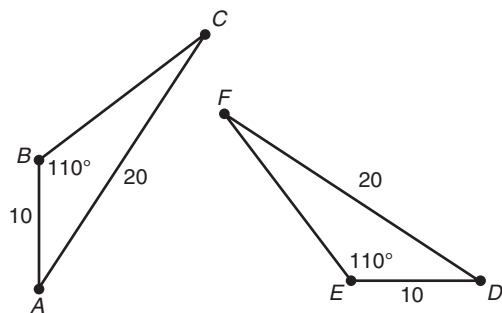


19.



5

20.



Name _____ Date _____

Planting Grape Vines Proving Triangles Congruent: HL

Vocabulary

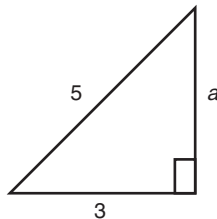
Define each term in your own words.

1. hypotenuse
2. Hypotenuse-Leg Congruence Theorem

Problem Set

Show that the two triangles in each figure are congruent.

1.



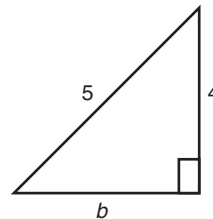
In the first triangle, you can apply the Pythagorean Theorem to get a value for a :

$$3^2 + a^2 = 5^2$$

$$9 + a^2 = 25$$

$$a^2 = 16$$

$$a = 4$$



Similarly, in the second triangle, you can apply the Pythagorean Theorem to get a value for b :

$$b^2 + 4^2 = 5^2$$

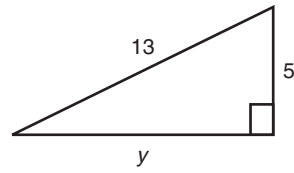
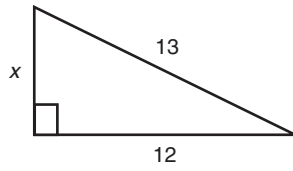
$$b^2 + 16 = 25$$

$$b^2 = 9$$

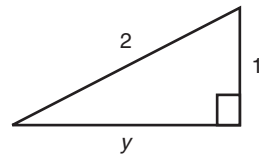
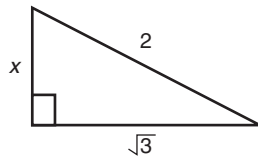
$$b = 3$$

Thus the two right triangles have both legs congruent, and by the SAS, SSS, or HL Theorems, the two triangles are congruent.

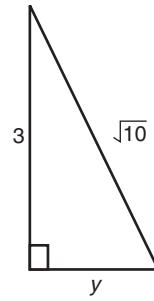
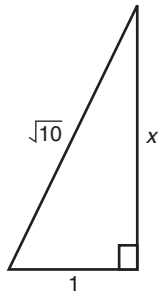
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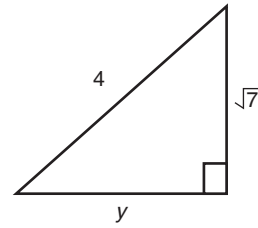
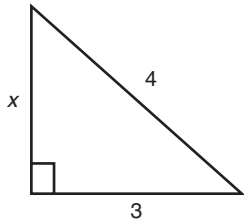
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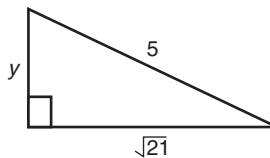
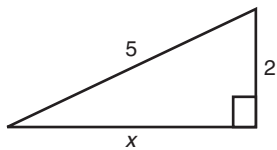
4.



5.

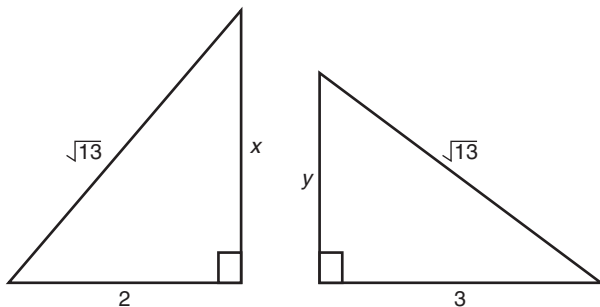


6.



Use the Hypotenuse Leg Congruence Theorem to show that the two triangles in each figure are congruent.

7.



In the first triangle, you can apply the Pythagorean Theorem to get a value for x :

$$2^2 + x^2 = (\sqrt{13})^2$$

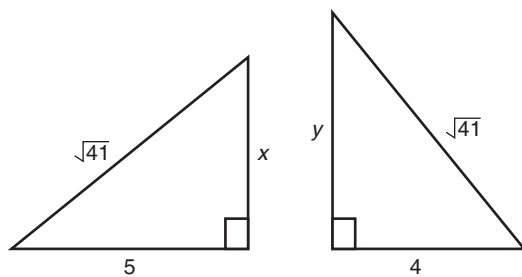
$$4 + x^2 = 13$$

$$x^2 = 9$$

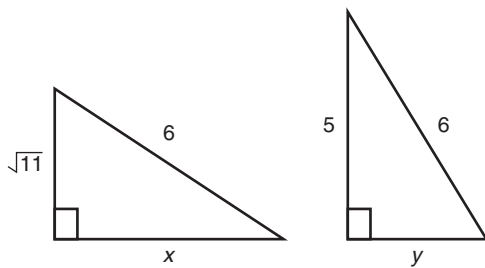
$$x = 3$$

Now the hypotenuse and one leg of the first right triangle are congruent to the hypotenuse and one leg in the other right triangle. Thus by the Hypotenuse-Leg Congruence Theorem, the two triangles are congruent.

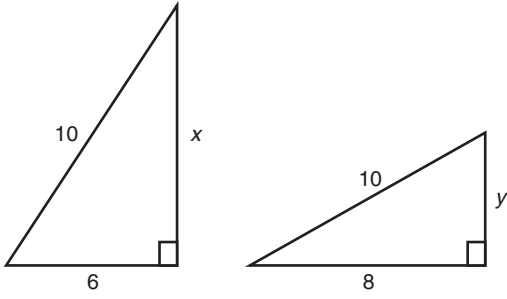
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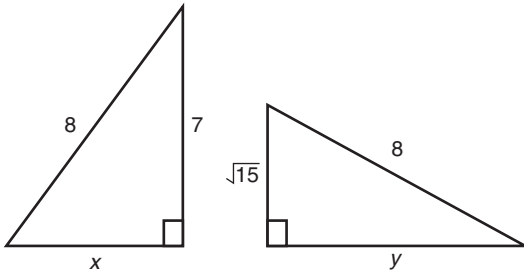
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10.

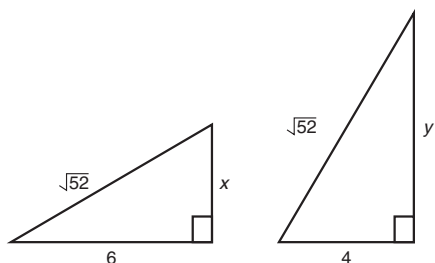


11.



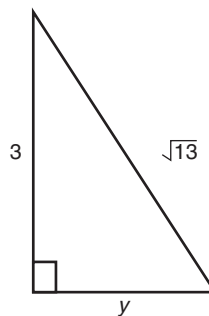
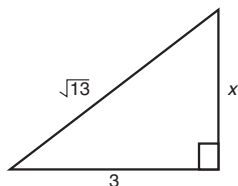
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12.



Show that the two triangles in each figure are congruent.

13.



In the first triangle, you can apply the Pythagorean Theorem to get a value for x :

$$x^2 + 3^2 = (\sqrt{13})^2$$

$$x^2 + 9 = 13$$

$$x^2 = 4$$

$$x = 2$$

You can also apply the Pythagorean Theorem to the second triangle to solve for y :

$$y^2 + 3^2 = (\sqrt{13})^2$$

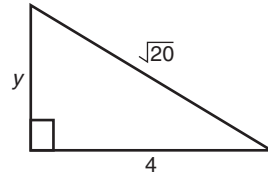
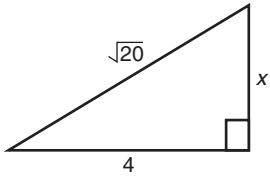
$$y^2 + 9 = 13$$

$$y^2 = 4$$

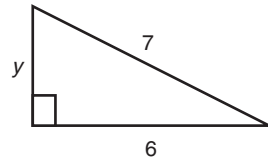
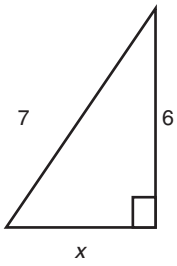
$$y = 2$$

Therefore, both right triangles have legs of length 2 and 3 and a hypotenuse of length $\sqrt{13}$. By the SAS or SSS Theorems, the two triangles are congruent.

14.



15.



16.

