## Skills Practice

Name $\qquad$ Date $\qquad$

## Quilting and Tessellations Introduction to Quadrilaterals

## Vocabulary

## Write the term that best completes each statement.

1. A quadrilateral with all congruent sides and all right angles is called $a(n)$ $\qquad$ .
2. $A(n)$ $\qquad$ is a parallelogram whose four sides have the same length.
3. $A(n)$ $\qquad$ uses circles to show how elements among sets of numbers or objects are related.
4. A polygon that has four sides is $a(n)$ $\qquad$ .
5. A quadrilateral with two pairs of parallel sides is called $a(n)$ $\qquad$ .
6. $A(n)$ $\qquad$ of a plane is a collection of polygons that are arranged so that they cover the plane with no gaps.
7. $A(n)$ $\qquad$ is a quadrilateral with exactly one pair of parallel sides.
8. A parallelogram with four right angles is a(n) $\qquad$ .
9. $A(n)$ $\qquad$ is a four-sided figure with two pairs of adjacent sides of equal length, with opposite sides not equal in length.

## Problem Set

Identify all of the terms from the following list that apply to each figure: quadrilateral, parallelogram, rectangle, square, trapezoid, rhombus, kite.
1.

2.

rhombus
parallelogram
quadrilateral
3.

4.

5.

6.


Name the type of quadrilateral that best describes each figure. Explain your answer.
8.


Rectangle. The quadrilateral has two pairs of parallel sides and four right angles, but the four sides are not all congruent.
$\qquad$
$\qquad$

10.

11.

12.


List all possible names for each quadrilateral based on its vertices.
13.


ABDC ACDB
BDCA BACD
DCAB DBAC
CABD CDBA
14.

15.

16.


## Name the indicated parts of each quadrilateral.

17. Name the parallel sides.

$A D$ and $B C$
18. Name the congruent sides.

19. Name the right angles.

© 2009 Carnegie Learning, Inc.

## Draw a Venn diagram for each description.

21. Suppose that a part of a Venn diagram has two circles. One circle represents all types of quadrilaterals with four congruent sides. The other circle represents all types of quadrilaterals with four congruent angles. Draw this part of the Venn diagram and label it with the appropriate types of quadrilaterals.

22. Suppose that a part of a Venn diagram has two circles. One circle represents all types of quadrilaterals with two pairs of congruent sides (adjacent or opposite). The other circle represents all types of quadrilaterals with at least one pair of parallel sides. Draw this part of the Venn diagram and label it with the appropriate types of quadrilaterals.
23. Suppose that a part of a Venn diagram has two circles. One circle represents all types of quadrilaterals with two pairs of parallel sides. The other circle represents all types of quadrilaterals with four congruent sides. Draw this part of the Venn diagram and label it with the appropriate types of quadrilaterals.
24. Suppose that a part of a Venn diagram has two circles. One circle represents all types of quadrilaterals with four right angles. The other circle represents all types of quadrilaterals with two pairs of parallel sides. Draw this part of the Venn diagram and label it with the appropriate types of quadrilaterals.

## Skills Practice

Name $\qquad$ Date $\qquad$

## When Trapezoids Are Kites Kites and Trapezoids

## Vocabulary

Identify all instances of each term in the figure.


1. isosceles trapezoid
2. base angles of a trapezoid

## Problem Set

Use the given figure to answer each question.

1. The figure shown is a kite with $\angle D A B \cong \angle D C B$. Which of the kite's sides are congruent?

2. The figure shown is a kite with $\overline{F G} \cong \overline{F E}$. Which of the kite's angles are congruent?


[^0]3. Given that IJLK is a kite, what kinds of triangles are formed by diagonal IL?

5. Given that $P Q R S$ is a kite, which angles are congruent?

4. Given that $L M N O$ is a kite, what is the relationship between the triangles formed by diagonal $\overline{M O}$ ?

6. Given that TUVW is a kite, which angles are congruent?


## Write a paragraph proof to prove each statement.

7. Given that $A B E F$ and $B C D E$ are both kites, prove that $\angle F A B \cong \angle D C B$.


You are given that $A B E F$ and $B C D E$ are both kites. This fact means that each has two pairs of adjacent sides that are congruent. By visual inspection, $\overline{A B} \cong \overline{A F}, \overline{B E} \cong \overline{F E}, \overline{B C} \cong \overline{D C}$, and $\overline{B E} \cong \overline{D E}$. By the Transitive Property of Congruence, $\overline{F E} \cong \overline{B E} \cong \overline{D E}$.

You are also given that $\overline{A B} \cong \overline{C B}$. By the Transitive Property of Congruence, $\overline{A F} \cong \overline{A B} \cong \overline{C B} \cong \overline{C D}$.

Because each pair of corresponding sides is congruent, $A B E F$ and CBED are congruent.

By the definition of congruence, corresponding angles $F A B$ and $D C B$ are congruent. So, $\angle F A B \cong \angle D C B$.
8. Given that $G H K L$ and $I H K J$ are both kites, prove that $\angle L G H \cong \angle J I H$.

9. Given that $A B F G$ and $C B E D$ are both kites, prove that $\triangle A B G \cong \triangle E B D$.

10. Given that $H I M N$ and $J I L K$ are both kites, prove that $\Delta N H I \cong \Delta K J$.

$\qquad$

## Use the given figure to answer each question.

11. The figure shown is an isosceles trapezoid with $\overline{A B} \| \overline{C D}$. Which sides are congruent?

$A C$ and $B D$ are congruent.
12. The figure shown is an isosceles trapezoid with $\overline{I J} \cong \overline{K L}$. Given that $I J K L$ is an isosceles trapezoid, what are the bases?
13. The figure shown is an isosceles trapezoid with $\overline{E H} \cong \overline{F G}$. Which sides are parallel?

14. The figure shown is an isosceles trapezoid with $\overline{M P} \cong \overline{N O}$. Given that MPON is an isosceles trapezoid, what are the pairs of base angles?
15. Given that QRVS is an isosceles trapezoid, which angles are congruent?

16. Given that $W X Z Y$ is an isosceles trapezoid, which angles are congruent?


## Write a paragraph proof to prove each statement.

17. Given that $A B C D$ is an isosceles trapezoid, prove that $\triangle A C D \cong \triangle B D C$.


You are given that $A B C D$ is an isosceles trapezoid. This fact means that $\overline{A D} \cong \overline{B C}$, and $\angle A D C$ and $\angle B C D$ are congruent.

Also, by the Reflexive Property of Congruence, $\overline{D C} \cong \overline{C D}$.
By the SAS Congruence Theorem, $\triangle C D A \cong \triangle D C B$.
$\angle A C D$ and $\angle B D C$ are corresponding angles. By the definition of congruent figures, $\angle A C D \cong \angle D B C$.

Because all three pairs of corresponding sides are congruent, $\triangle A C D \cong \triangle B D C$.
$\qquad$
$\qquad$
18. Given that $E F H G$ is an isosceles trapezoid, prove that $\angle G E H \cong \angle H F G$.

19. Given that $A B C F$ and $F E D C$ are isosceles trapezoids, prove that $\angle A F C \cong \angle E F C$.

20. Given that $G H K L$ and $J K H I$ are isosceles trapezoids, prove that $\angle G \cong \angle J$.


## Skills Practice

Name $\qquad$ Date $\qquad$

## Binocular Stand Design Parallelograms and Rhombi

## Vocabulary

## Match each definition to its corresponding term.

1. two angles of a polygon that do not share a common side
a. opposite sides
b. consecutive sides
2. two angles of a polygon that share a common side
c. consecutive angles
3. two sides of a polygon that do not intersect
d. opposite angles
4. two sides of a polygon that share a common vertex

## Problem Set

Identify the indicated parts of the given parallelogram.

1. Name the pairs of consecutive sides of the parallelogram.

$A B$ and $B D$
$B D$ and $D C$
$D C$ and CA
$C A$ and $A B$
2. Name the pairs of opposite sides of the parallelogram.

3. Name the pairs of opposite angles of the parallelogram.

4. Name the pairs of consecutive angles of the parallelogram.


## Write a paragraph proof to prove each statement.

5. Given that $\overline{A B} \| \overline{C D}$ and $\overline{A C} \| \overline{B D}$, use the ASA Congruence Theorem to prove that $\angle B \cong \angle C$.


Sides $A B$ and $C D$ are parallel segments that are cut by a transversal. By the Alternate Internal Angles Theorem, corresponding angles CDA and BAD are congruent.

Sides $A C$ and $B D$ are parallel segments that are cut by a transversal. By the Alternate Internal Angles Theorem, corresponding angles CAD and BDA are congruent.

By the Reflexive Property of Equality, $\overline{A D} \cong \overline{D A}$.
Because corresponding angles CAD and BDA are congruent and corresponding angles $C D A$ and $B A D$ are congruent (and the included sides are congruent), by the ASA Congruence Theorem, $\triangle A C D \cong \triangle D B A$.

By the definition of congruence, corresponding angles $B$ and $C$ are congruent.
So, $\angle B \cong \angle C$.
6. Given that $\overline{H G} \| \overline{E F}$ and $\overline{H G} \| \overline{G F}$, use the ASA Congruence Theorem to prove that $\overline{H G} \cong \overline{E F}$.

7. Given that $\overline{I K} \| \overline{L J}$ and $\overline{I K} \cong \overline{L J}$, use the AAS Congruence Theorem to prove that $\Delta I M K \cong \Delta L M J$.

8. Given that $N O \| Q P$ and $N O \cong \overline{Q P}$, use the AAS Congruence Theorem to prove that $\triangle N O M \cong \triangle Q P M$.


Name Date $\qquad$

Use what you know about rhombi to answer each question.
9. What is the relationship between consecutive angles of a rhombus?

Consecutive angles of a rhombus are supplementary.
10. What is the relationship between opposite angles of a rhombus?
11. What is the relationship between consecutive sides of a rhombus?
12. Explain the difference between parallelograms and rhombi in terms of opposite and consecutive sides.

## Use the given information to complete each two-column proof.

13. If $\overline{A C}$ bisects $\angle D A B$ and $\angle D C B$, then $\angle D \cong \angle B$.


| Statement | Reason |
| :--- | :--- |
| 1. $\overline{A C}$ bisects $\angle D A B$ and $\angle D C B$. | 1. Given |
| 2. $\angle D A C \cong \angle B A C$ | 2. Definition of angle bisector |
| 3. $\angle D C A \cong \angle B C A$ | 3. Definition of angle bisector |
| 4. $\overline{A C} \cong \overline{A C}$ | 4. Reflexive Property of Congruence |
| 5. $\triangle A D C \cong \triangle A B C$ | 5. ASA Congruence Theorem |
| 6. $\angle D \cong \angle B$ | 6. Definition of congruence |

14. If $\overline{E G}$ bisects $\angle F E H$ and $\angle F G H$, then $\overline{E F} \cong \overline{E H}$.


| Statement | Reason |
| :--- | :--- |
| 1. $\overline{E G}$ bisects $\angle F E H$ and $\angle F G H$. | 1. |
| 2. $\angle F E G \cong$ | 2. Definition of angle bisector |
| 3. $\angle F G E \cong$ | 3. Definition of angle bisector |
| 4. $\overline{E G} \cong \overline{E G}$ | 4. |
| 5. $\Delta F E G \cong$ | 5. ASA Congruence Theorem |
| 6. $\overline{E F} \cong \overline{E H}$ | 6. Definition of |

15. If $\overline{I K}$ bisects $\angle J I L$ and $\overline{I L} \cong \overline{I J}$, then $\angle I M J \cong \angle I M L$.


| Statement | Reason |
| :--- | :--- |
| 1. | 1. Given |
| 2. $\angle L I M \cong$ | 2. Definition of angle bisector |
| 3. $\overline{I M} \cong \overline{I M}$ | 3. |
| 4. | 4. Given |
| 5. $\triangle J I M \cong \triangle L I M$ | 5. |
| 6. $\angle I M J \cong \angle I M L$ | 6. Definition of |

$\qquad$
16. If $\overline{O N}$ bisects $\angle M O P$ and $\overline{M O} \cong \overline{P O}$, then $\overline{M Q} \cong \overline{P Q}$.


| Statement | Reason |
| :--- | :--- |
| 1. $\overline{O N}$ bisects $\angle M O P$. | 1. |
| 2. $\cong \angle P O Q$ | 2. Definition of angle bisector |
| 3. $\overline{O Q} \cong \overline{O Q}$ | 3. |
| 4. | 4. Given |
| 5. $\triangle M O Q \cong \triangle P O Q$ | 5. |
| 6. | 6. Definition of congruence |

## Skills Practice

Name $\qquad$ Date $\qquad$

## Positive Reinforcement Rectangles and Squares

## Vocabulary

Identify similarities and differences between the terms.

1. square and rectangle

## Problem Set

## Explain why each statement is true.

1. A rectangle is always a parallelogram.

A rectangle must have two pairs of parallel sides, so a rectangle is always a parallelogram.
2. A parallelogram is sometimes a rectangle.
3. A rectangle is sometimes a square.
4. A square is always a rectangle.
5. The diagonals of a square are perpendicular.
6. The diagonals of a rectangle are sometimes perpendicular.
7. A rectangle is sometimes a rhombus.
8. A square is always a rhombus.
9. A rhombus is sometimes a rectangle.
10. A rhombus is sometimes a square.
$\qquad$

## Given the lengths of the sides of a rectangle, calculate the length of each diagonal. Simplify radicals, but do not evaluate.

11. A rectangular construction scaffold with diagonal support beams is 8 feet high and 10 feet wide.

What is the length of each diagonal?
$A D^{2}=A C^{2}+C D^{2}$
$A D^{2}=8^{2}+10^{2}$
$A D^{2}=64+100$
$A D^{2}=164$
$A D=\sqrt{164}=2 \sqrt{41}$


The length of diagonal $A D$ is $2 \sqrt{41}$ feet.
$B C=A D=2 \sqrt{41}$
The length of diagonal $B C$ is $2 \sqrt{41}$ feet.
12. A fence has rectangular sections that are each 4 feet tall and 8 feet long.

Each section has a diagonal support beam.
What is the length of each diagonal?

13. A community garden has a rectangular frame for sugar snap peas. The frame is 9 feet high and 6 feet wide, and it has two diagonals to strengthen it.

What is the length of each diagonal?

14. The sides of a shelving unit are metal rectangles with two diagonals for support. Each rectangle is 12 inches wide and 40 inches high.

What is the length of each diagonal?

$\qquad$

## Given the length of a side of a rectangle and the length of a diagonal, calculate the length of another side. Simplify radicals, but do not evaluate.

15. Given that $A B D C$ is a rectangle, find $C D$.
$A D^{2}=A C^{2}+C D^{2}$
$C D^{2}=A D^{2}-A C^{2}$
$C D^{2}=10^{2}-5^{2}$
$C D^{2}=100-25$
$C D^{2}=75$

$C D=\sqrt{75}=5 \sqrt{3}$
$C D$ is $5 \sqrt{3}$ centimeters.
16. Given that $E F G H$ is a rectangle, find $F G$.

17. Given that $I J K L$ is a rectangle, find $I L$.

18. Given that $M N O P$ is a rectangle, find $M N$.

19. Given that QRTS is a rectangle, find QS.

20. Given that $U V W X$ is a rectangle, find $X W$.

$\qquad$
$\qquad$
21. Given that $A B C D$ is a rectangle, find $A D$.

22. Given that $E F G H$ is a rectangle, find $G H$.

23. A square garden is divided into quarters by diagonal paths. If each diagonal is 50 meters long, how long is each side of the garden?

$$
\begin{aligned}
A C^{2} & =A D^{2}+D C^{2}=2\left(A D^{2}\right) \\
50^{2} & =2\left(A D^{2}\right) \\
A D^{2} & =\frac{2500}{2}=1250 \\
A D & =\sqrt{1250}=\sqrt{625 \cdot 2}=25 \sqrt{2} \approx 35.4
\end{aligned}
$$

The length of each side of the garden is approximately
 35.4 meters.
24. A square porch has diagonal support beams underneath it. If each diagonal beam is 12 feet long, what is the length of each side of the porch?

25. A heavy picture frame in the shape of a square has a diagonal support across the back. If each side of the frame is 24 inches, what is the length of the diagonal?

26. A square shelving unit has diagonal supports across the back. If each side of the frame is 60 inches, what is the length of each diagonal?


## Skills Practice

Name $\qquad$ Date $\qquad$

## Stained Glass

Sum of the Interior Angle Measures in a Polygon

## Vocabulary

Draw a diagram to illustrate each term. Explain how your diagram illustrates the term.

1. interior angle
2. convex polygon
3. regular polygon

## Problem Set

## Calculate the sum of the interior angle measures of the polygon. Show all your work.

1. Draw all of the diagonals that connect to vertex $A$. What is the sum of the internal angles of quadrilateral $A B D C$ ?


The diagonal divides the figure into two triangles. The sum of the interior angles of each triangle is $180^{\circ}$, so multiply $180^{\circ}$ by 2 to find the sum of the interior angles of the quadrilateral:
$180^{\circ} \times 2=360^{\circ}$
The sum of the interior angles is $360^{\circ}$.
2. Draw all of the diagonals that connect to vertex $E$. What is the sum of the interior angles of polygon EFGHI?

$\qquad$
$\qquad$
3. Draw all of the diagonals that connect to vertex J . What is the sum of the interior angles of polygon JKMONL?

4. Draw all of the diagonals that connect to vertex $P$. What is the sum of the interior angles of polygon PQRSTUV?


## Calculate the sum of the interior angle measures of the polygon.

5. If a convex polygon has 5 sides, what is the sum of its interior angle measures?

The sum is equal to $(n-2) \cdot 180^{\circ}$ :
$(5-2) \cdot 180^{\circ}=3 \cdot 180^{\circ}=540^{\circ}$
The sum of the interior angles of the polygon is $540^{\circ}$.
6. If a convex polygon has 6 sides, what is the sum of its interior angle measures?
7. If a convex polygon has 8 sides, what is the sum of its interior angle measures?
8. If a convex polygon has 9 sides, what is the sum of its interior angle measures?
9. If a convex polygon has 12 sides, what is the sum of its interior angle measures?
10. If a convex polygon has 13 sides, what is the sum of its interior angle measures?
11. If a convex polygon has 16 sides, what is the sum of its interior angle measures?
$\qquad$
$\qquad$
12. If a convex polygon has 17 sides, what is the sum of its interior angle measures?

## Determine the measure of each interior angle of each regular polygon.

13. What is the measure of each interior angle of the regular polygon?


The sum of the interior angles is equal to $(n-2) \cdot 180^{\circ}$ :
$(8-2) \cdot 180^{\circ}=6 \cdot 180^{\circ}=1080^{\circ}$
Because the figure is a regular polygon, the measure
of each interior angle can be found by dividing by $n$ :
$1080^{\circ} \div 8=135^{\circ}$
The measure of each interior angle is $135^{\circ}$.
14. What is the measure of each interior angle of the regular polygon?

15. What is the measure of each interior angle of the regular polygon?

16. What is the measure of each interior angle of the regular polygon?


## Use the given information to determine the number of sides of each regular polygon.

17. The measure of each angle of a regular polygon is $108^{\circ}$. How many sides does the polygon have?

$$
\begin{aligned}
n\left(108^{\circ}\right) & =(n-2)\left(180^{\circ}\right) \\
n\left(108^{\circ}\right) & =n\left(180^{\circ}\right)-2\left(180^{\circ}\right) \\
n\left(72^{\circ}\right) & =360^{\circ} \\
360^{\circ} \div 72^{\circ} & =5
\end{aligned}
$$

The regular polygon has 5 sides. It is a pentagon.
18. The measure of each angle of a regular polygon is $120^{\circ}$. How many sides does the polygon have?
19. The measure of each angle of a regular polygon is $144^{\circ}$. How many sides does the polygon have?
20. The measure of each angle of a regular polygon is $156^{\circ}$. How many sides does the polygon have?
21. The measure of each angle of a regular polygon is $160^{\circ}$. How many sides does the polygon have?
22. The measure of each angle of a regular polygon is $162^{\circ}$. How many sides does the polygon have?

## Skills Practice

Name $\qquad$ Date $\qquad$

## Pinwheels

Sum of the Exterior Angle Measures in a Polygon

## Vocabulary

Define each term in your own words.

1. exterior angle
2. regular polygon

## Problem Set

Extend each vertex of the polygon to create one exterior angle at each vertex.
1.

2.

3.

4.


## Given the measure of an interior angle of a polygon, calculate the measure of the adjacent exterior angle. Explain how you found your answer.

5. What is the measure of an exterior angle if it is adjacent to an interior angle of a polygon that measures $90^{\circ}$ ?

Interior and exterior angles are supplementary. So subtract $90^{\circ}$, the measure of the interior angle, from $180^{\circ}$ :
$180^{\circ}-90^{\circ}=90^{\circ}$
6. What is the measure of an exterior angle if it is adjacent to an interior angle of a polygon that measures $120^{\circ}$ ?
7. What is the measure of an exterior angle if it is adjacent to an interior angle of a polygon that measures $108^{\circ}$ ?
8. What is the measure of an exterior angle if it is adjacent to an interior angle of a polygon that measures $135^{\circ}$ ?
9. What is the measure of an exterior angle if it is adjacent to an interior angle of a polygon that measures $115^{\circ}$ ?
10. What is the measure of an exterior angle if it is adjacent to an interior angle of a polygon that measures $124^{\circ}$ ?

Name
Date

For each regular polygon, calculate the measure of each of its external angles. Explain how you found your answer.
11. What is the measure of each external angle of a square?

Find the sum of the internal angle measures:
$(4-2) \cdot 180^{\circ}=2 \cdot 180^{\circ}=360^{\circ}$
Then divide $360^{\circ}$ by 4 to find the measure of each internal angle:
$360^{\circ} \div 4=90^{\circ}$
Then subtract the measure of an internal angle from $180^{\circ}$ to find the measure of an external angle:
$180^{\circ}-90^{\circ}=90^{\circ}$
Each external angle of a square measures $90^{\circ}$.
12. What is the measure of each external angle of a regular pentagon?
13. What is the measure of each external angle of a regular hexagon?
14. What is the measure of each external angle of a regular octagon?

## For each regular polygon, calculate the sum of the measures of its external angles. Show all your work.

15. What is the sum of the external angle measures of a regular pentagon?

Sum of the internal angle measures:
$(5-2) \cdot 180^{\circ}=3 \cdot 180^{\circ}=540^{\circ}$
Internal angle measure $=540^{\circ} \div 5=108^{\circ}$
External angle measure $=180^{\circ}-108^{\circ}=72^{\circ}$
Sum of the external angle measures $=72^{\circ} \cdot 5=360^{\circ}$
The sum of the external angle measures of a regular pentagon is $360^{\circ}$.
16. What is the sum of the external angle measures of a regular hexagon?
$\qquad$
$\qquad$
17. What is the sum of the external angle measures of a regular octagon?
18. What is the sum of the external angle measures of a square?

For each polygon, calculate the sum of the measures of its external angles. Show all your work.
19. What is the sum of the external angle measures of the polygon?


External angle measures:
$180^{\circ}-120^{\circ}=60^{\circ}$
$180^{\circ}-60^{\circ}=120^{\circ}$
$180^{\circ}-120^{\circ}=60^{\circ}$
$180^{\circ}-60^{\circ}=120^{\circ}$
Sum of the external angle measures $=120^{\circ}+60^{\circ}+120^{\circ}+60^{\circ}=360^{\circ}$
The sum of the external angle measures of the polygon is $360^{\circ}$.
20. What is the sum of the external angle measures of the polygon?

21. What is the sum of the external angle measures of the polygon?

$\qquad$
22. What is the sum of the external angle measures of the polygon?

23. What is the sum of the external angle measures of the polygon?

24. What is the sum of the external angle measures of the polygon?



[^0]:    $A B$ and $C B$ are congruent.
    $A D$ and $C D$ are congruent.

