Skills Practice

Name _____ Date _____

Roots and Zeros
Calculating Roots of Quadratic Equations and Zeros of Quadratic Functions

Vocabulary

Write the term that best completes each statement.

1. The _____ states that if the product of two or more factors is equal to zero, then at least one of the factors must be equal to zero.
2. The *x*-intercepts of a function are also called the _____ of the function.
3. The solutions to a quadratic equation are called _____.

Problem Set

Factor and solve each quadratic equation, if possible.

1. $x^2 - 8x + 15 = 0$

   $x^2 - 8x + 15 = 0$

   $(x - 3)(x - 5) = 0$

   $x - 3 = 0$ or $x - 5 = 0$

   $x = 3$ or $x = 5$

2. $x^2 - 8x + 12 = 0$

3. $x^2 - 20x - 21 = 0$

4. $x^2 - 10x + 21 = 0$

5. $x^2 - 22x - 48 = 0$

6. $x^2 + 16x + 48 = 0$

7. $x^2 - 10x + 100 = 0$

8. $x^2 - 8x + 48 = 0$

10

9. $x^2 - 25x + 100 = 0$

10. $x^2 - 10x - 96 = 0$

Calculate the zero(s) of each quadratic function, if possible.

11. $f(x) = x^2 - 7x + 12$

$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x - 3 = 0$ or $x - 4 = 0$

$x = 3$ or $x = 4$

12. $f(x) = x^2 - 9x + 20$

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13. $f(x) = x^2 - 10x - 39$

14. $f(x) = x^2 + 12x - 45$

15. $f(x) = x^2 - 6x + 12$

16. $f(x) = x^2 + 20x + 36$

17. $f(x) = x^2 + 24x + 63$

18. $f(x) = x^2 + 12x + 45$

19. $f(x) = x^2 - 23x + 132$

20. $f(x) = x^2 - 50x - 104$

10

Skills Practice

Name _____ Date _____

Poly High
Factoring Polynomials

Vocabulary

Match each word with its corresponding definition.

1. polynomial equation

2. greatest common factor

3. factoring by grouping

a. a method of factoring that creates two groups of terms and factors with the greatest common factor of each term

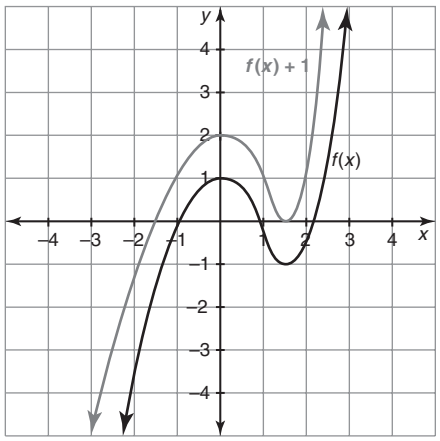
b. an equation that can be written in the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 = 0$

c. the largest factor that is common to all terms of a polynomial

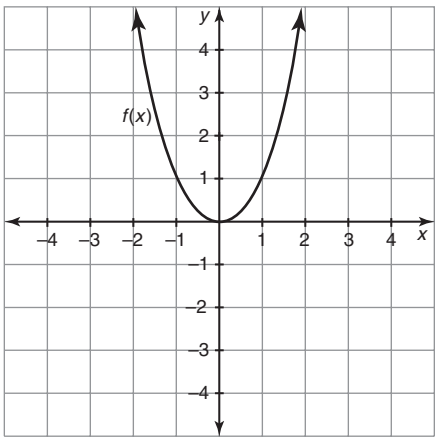
Problem Set

Given the graph of $f(x)$, sketch the graph of each transformed function.

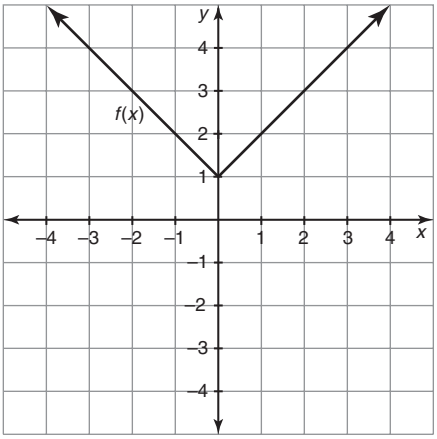
1. $f(x) + 1$



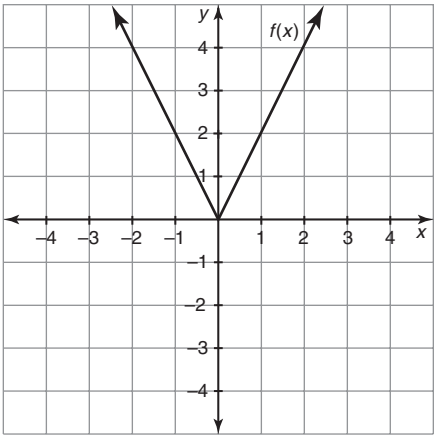
2. $f(x - 2)$



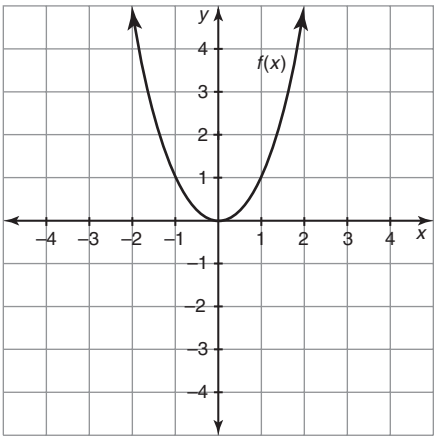
3. $-f(x)$



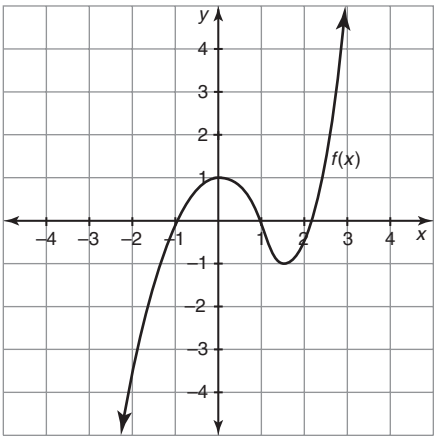
4. $2f(x)$



5. $\frac{1}{2}f(x)$



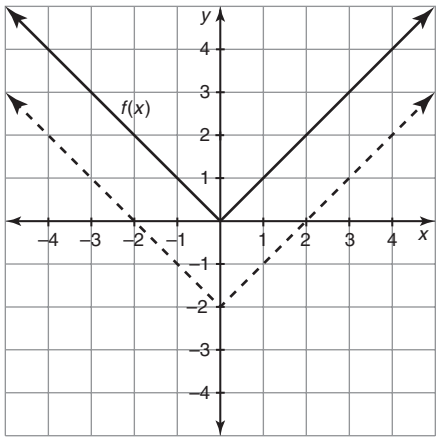
6. $f(-x)$



Name _____ Date _____

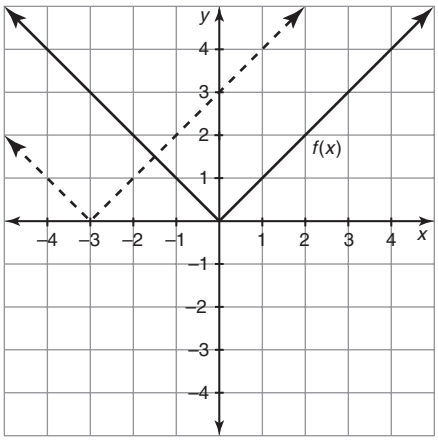
The graph of a function $f(x)$ is shown. Write a function in terms of $f(x)$ for each transformed graph.

7.



$f(x) - 2$

8.



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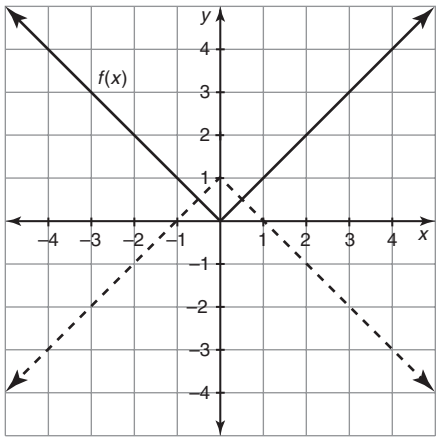

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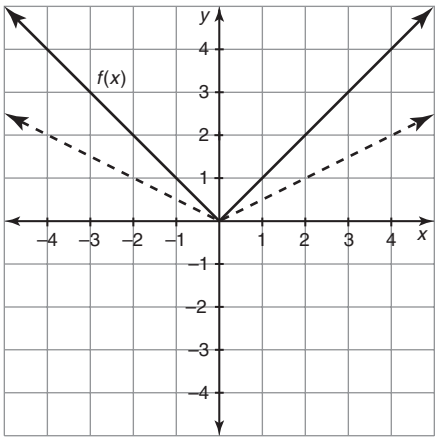

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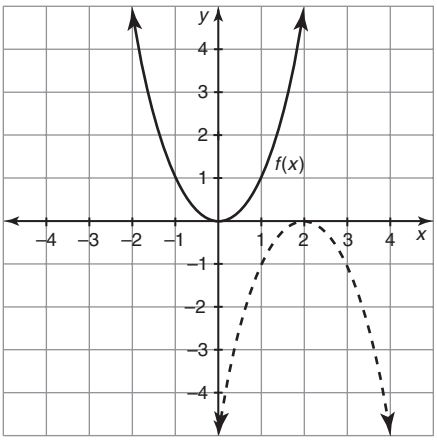

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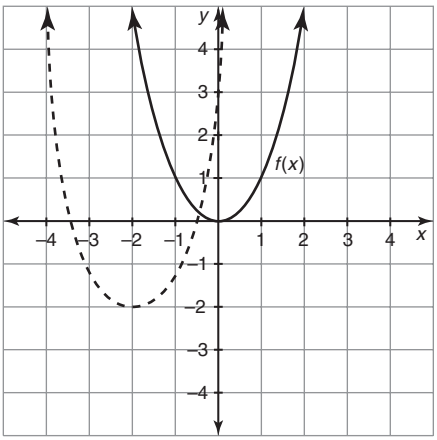

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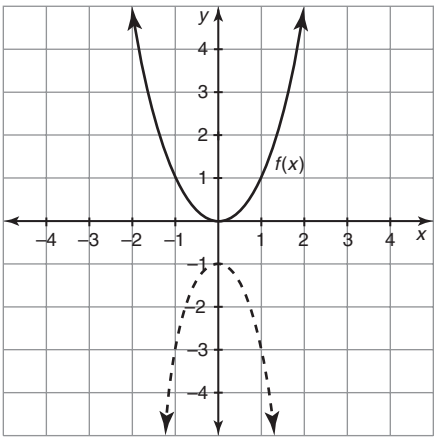

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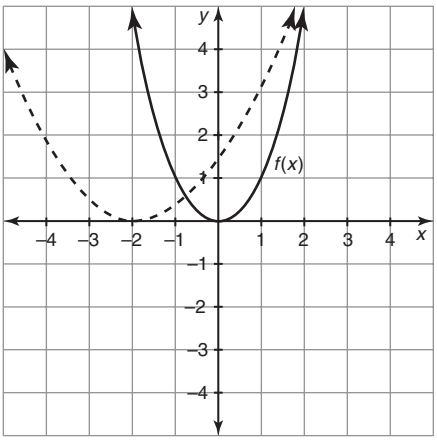

Name _____ Date _____

Factor and solve each polynomial equation.

15. $x^2 - 13x + 22 = 0$

$x^2 - 13x + 22 = 0$

$(x - 2)(x - 11) = 0$

$x - 2 = 0$ or $x - 11 = 0$

$x = 2$ or $x = 11$

16. $x^2 - 15x - 76 = 0$

17. $x^4 - 13x^2 + 36 = 0$

18. $x^4 - 20x^2 + 64 = 0$

19. $2x^3 - 16x^2 + 14x = 0$

20. $5x^3 - 15x^2 + 10x = 0$

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21. $3x^3 - 15x^2 - 72x = 0$

22. $6x^3 - 6x^2 - 12x = 0$

23. $x^3 + 4x^2 - 9x - 36 = 0$

24. $x^3 - 3x^2 + 16x + 48 = 0$

10

25. $2x^4 + 10x^3 - 8x^2 - 40x = 0$

26. $3x^4 + 6x^3 - 27x^2 - 54x = 0$

Skills Practice

Name _____ Date _____

Rational Thinking
Rational Equations and Functions

Vocabulary

Define each term in your own words.

1. rational equation

2. extraneous solution

Problem Set

Write a rational expression to model each situation.

1. Gina is 4 years older than her sister. Write an expression for the ratio of Gina's age to her sister's age.

   $\frac{x}{x-4}$

2. Leslie's uncle is 24 years older than her. Write an expression for the ratio of Leslie's age to her uncle's age.

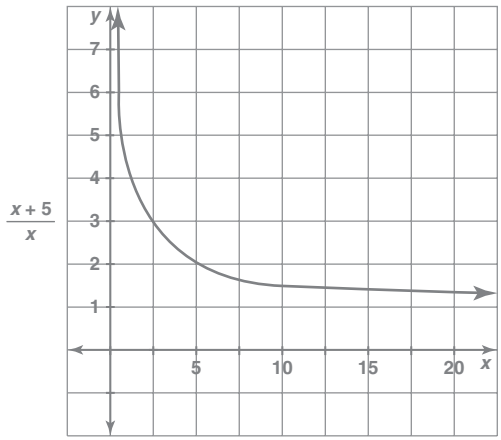
3. Mica weighs 10 pounds less than twice his nephew's weight. Write an expression for the ratio of Mica's nephew's weight to Mica's weight.

4. Jana weighs 15 pounds more than three times her daughter's weight. Write an expression for the ratio of Jana's daughter's weight to Jana's weight.

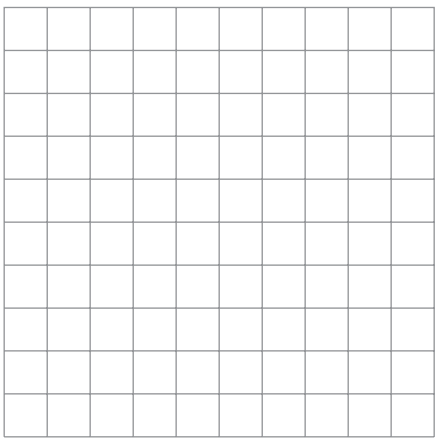
Write and graph a rational function to model each situation.

5. John was 5 years old when his brother was born. Let x represent John's brother's age. Write and graph a rational function that represents the ratio of John's age to his brother's age. Use bounds that make sense in terms of the problem situation.

$\frac{x + 5}{x}$

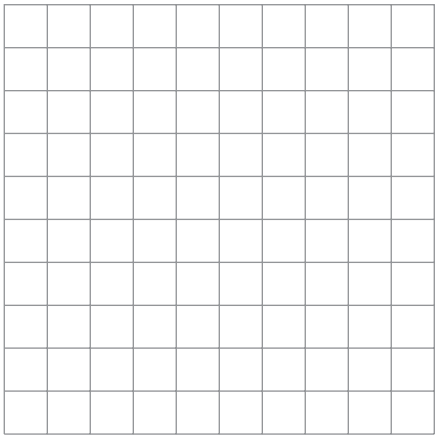


6. Liza was 25 years old when her daughter was born. Let x represent Liza's age. Write and graph a rational function that represents the ratio of Liza's age to her daughter's age. Use bounds that make sense in terms of the problem situation.

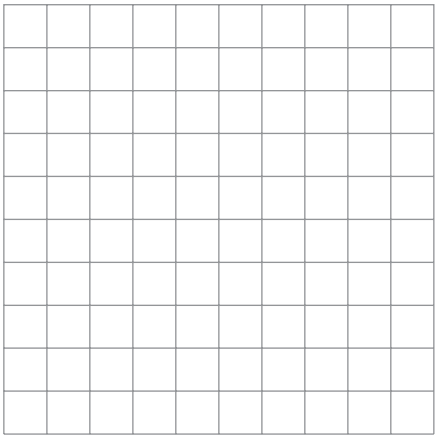


Name _____ Date _____

7. Jose's grandfather was 60 years old when Jose was born. Let x represent Jose's age. Write and graph a rational function that represents the ratio of Jose's age to his grandfather's age. Use bounds that make sense in terms of the problem situation.



8. Dante's sister was 3 years old when he was born. Let x represent Dante's age. Write and graph a rational function that represents the ratio of Dante's age to his sister's age. Use bounds that make sense in terms of the problem situation.



Solve each rational equation. Make sure to identify any extraneous solutions and list any restrictions to your solution set.

9. $\frac{-4}{x} = 13$

$x\left(\frac{-4}{x}\right) = x(13); \quad x \neq 0$

$-4 = 13x$

$x = -\frac{4}{13}$

Check:

$\frac{-4}{\left(\frac{-4}{13}\right)} = 13$

$-4\left(\frac{13}{-4}\right) = 13$

$13 = 13$

10. $\frac{-3}{x} = -5$

11. $\frac{3}{x + 4} = -3$

12. $\frac{7}{x - 5} = 2$

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13. $\frac{6}{x+1} = \frac{3}{x-2}$

14. $\frac{4}{x+4} = \frac{5}{x-5}$

15. $\frac{x-4}{x+4} = 5$

16. $\frac{x+6}{x+3} = 2$

17. $\frac{x^2 - 4x}{5} = x$

10

18. $\frac{x^2 - 7x}{3} = \frac{8x}{6}$

Name _____ Date _____

19. $\frac{2x^2 + 7x}{2} = \frac{3x^2 + 10x + 4}{3}$

20. $\frac{2x^2 - 3x}{4} = \frac{3x^2 - 7x + 5}{6}$

21. $\frac{1}{x + 4} - \frac{6}{2x + 8} = \frac{2}{x + 4}$

22. $\frac{8}{x - 2} + \frac{5}{3x - 6} = \frac{7}{x - 2}$

23. $\frac{3}{x-2} + \frac{1}{x+2} = \frac{4}{x^2-4}$

24. $\frac{3}{x-1} + \frac{5}{x+2} = \frac{17}{x^2+x-2}$

10

Skills Practice

Name _____ Date _____

Work, Mixture, and More
Applications of Rational Equations and Functions

Vocabulary

Compare and contrast the meanings of the terms.

1. work problems, mixture problems, and cost problems

Problem Set

Solve each work problem.

1. If Frank is working alone, he can finish a job in 30 minutes. If he is working with Britney, the two of them can complete the job in 20 minutes. How long would it take Britney to complete the job if she were working alone?

   Let t be the number of minutes it takes Britney to complete the job alone.
   Then Britney can complete $\frac{1}{t}$ of the job in 1 minute.

   Frank can complete $\frac{20}{30} = \frac{2}{3}$ of the job in 20 minutes.

   $$\frac{2}{3} + \frac{20}{t} = 1$$

   $$3t\left(\frac{2}{3} + \frac{20}{t}\right) = 1(3t)$$

   $$2t + 60 = 3t$$

   $$60 = t$$

   $$t = 60$$

   It would take Britney 60 minutes to complete the job if she were working alone.

2. If Jose is working alone, he can finish a job in 40 minutes. If he is working with Carmen, the two of them can complete the job in 15 minutes. How long would it take Carmen to complete the job if she were working alone?

10

3. If Jessica is working alone, she can wash a car in 60 minutes. If she is working with Jason, the two of them can complete the job in 36 minutes. How long would it take Jason to complete the job if he were working alone?

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4. If Elizabeth is working alone, she can clean the dishes in 30 minutes. If she is working with John, the two of them can complete the job in 24 minutes. How long would it take John to complete the job if he were working alone?

5. If Nicholas is working alone, he can mow the lawn in 90 minutes. If Don is working alone, he can mow the lawn in 72 minutes. How long would it take Nicholas and Don to mow the lawn if they both worked together?

6. If Serena is working alone, she can paint a room in 150 minutes. If Jonna is working alone, she can paint a room in 150 minutes. How long would it take Serena and Jonna to paint a room if they both worked together?

10

Solve each mixture problem.

7. A saline solution of 80 milliliters contains 10% salt. How much water should be added to produce a solution with 8% salt?

There are 8 milliliters of salt and 72 milliliters of water in the original solution.
Let x = the amount of water added to produce a solution with 8% salt.

$$\frac{8}{80 + x} = \frac{8}{100}$$
$$800 = 640 + 8x$$
$$160 = 8x$$
$$x = 20$$

To produce a solution with 8% salt, 20 milliliters of water should be added.

Name _____ Date _____

8. A saline solution of 150 milliliters contains 6% salt. How much water should be added to produce a solution with 4% salt?

9. A saline solution of 120 milliliters contains 15% salt. How much water should be added to produce a solution with 4% salt?

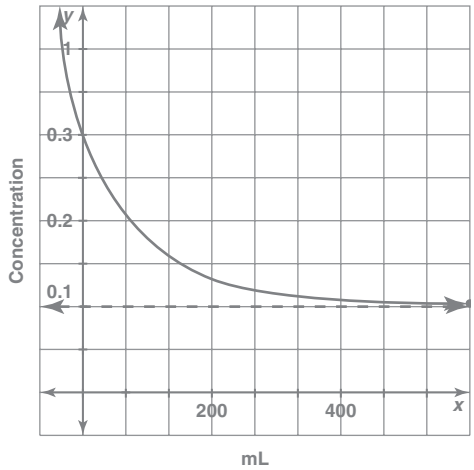
10. A saline solution of 300 milliliters contains 16% salt. How much water should be added to produce a solution with 3% salt?

Define a function $S(x)$ for the concentration of each resulting solution. Graph the function and identify the domain and range of the problem situation.

11. Forty milliliters of a 30% solution of hydrochloric acid will be mixed with x milliliters of a 10% solution to produce other solutions of other concentrations.

The 30% solution contains 12 milliliters of acid and 28 milliliters of water, and the 10% solution contains $0.1x$ milliliters of acid.

$$S(x) = \frac{12 + 0.1x}{40 + x}$$



Domain: all $x \geq 0$

Range: $0.1 < y \leq 0.3$

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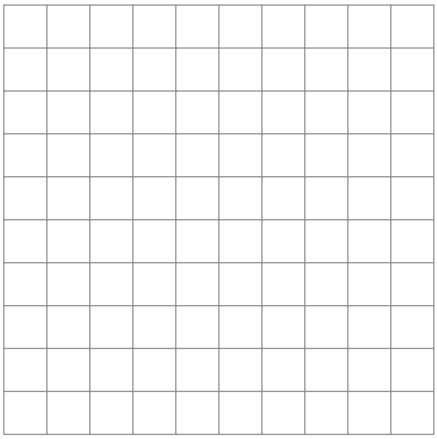
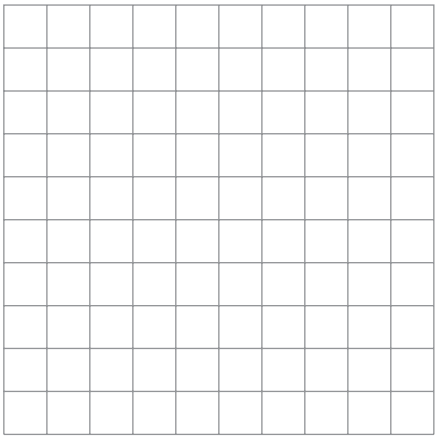
12. One hundred milliliters of a 27% solution of hydrochloric acid will be mixed with x milliliters of a 6% solution to produce other solutions of other concentrations.

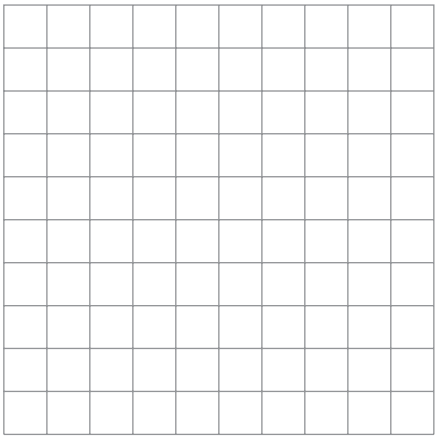


13. Sixty milliliters of a 25% solution of hydrochloric acid will be mixed with x milliliters of a 4% solution to produce other solutions of other concentrations.



Name _____ Date _____

14. Twenty-five milliliters of a 20% solution of hydrochloric acid will be mixed with x milliliters of a 2% solution to produce other solutions of other concentrations.



Solve each cost problem.

15. A new computer costs \$2000. Operating and repair costs are \$200 per year. What is the average yearly cost of ownership over the first two years? The first five years?

$\frac{2000 + 200 \cdot 2}{2} = \frac{2400}{2} = 1200$

The average yearly cost of ownership after 2 years would be \$1200.

$\frac{2000 + 200 \cdot 5}{5} = \frac{3000}{5} = 600$

The average yearly cost of ownership after 5 years would be \$600.

16. A new television costs $1200. Operating costs are $50 per year. What is the average yearly cost of ownership over the first 4 years? The first 10 years?

10

17. A new computer costs $1500. Operating and repair costs are $150 per year. After how many years will the average yearly cost of ownership be $650? $450?

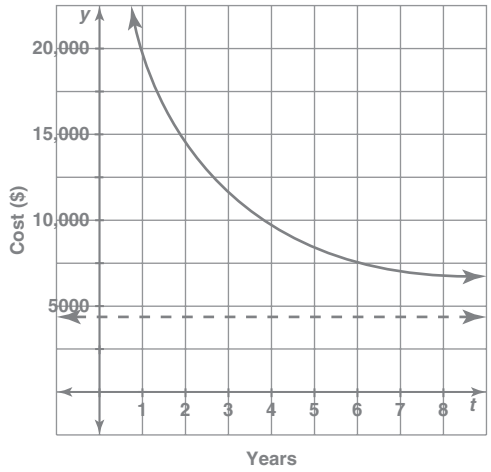
Name _____ Date _____

18. A new television costs $900. Operating costs are $60 per year. After how many years will the average yearly cost of ownership be $240? $160?

Define a function *C*(*t*) for the average cost of ownership as a function of time. Graph the function and identify the domain and range of the problem situation.

19. A new automobile costs $20,000 to purchase and the estimated cost of fuel, service, repairs, and insurance is $4500 per year.

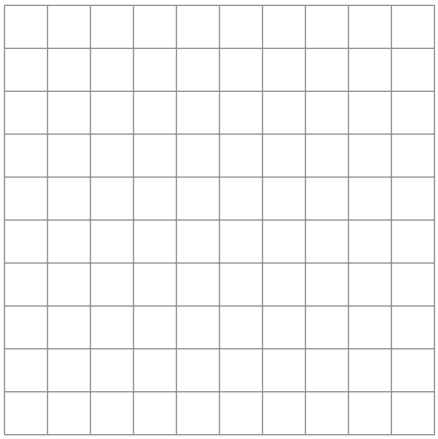
$$C(t) = \frac{20{,}000 + 4500t}{t}$$



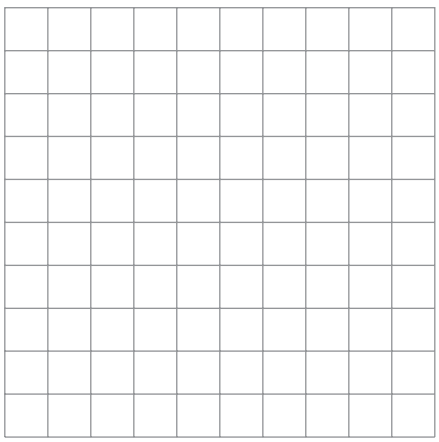
Domain: all $t > 0$

Range: $y > 4500$

20. A new automobile costs $40,000 to purchase and the estimated cost of fuel, service, repairs, and insurance is $7500 per year.

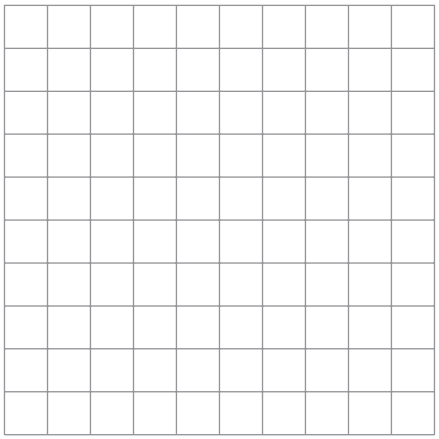


21. A boat costs $25,000 to purchase and the estimated cost of fuel, service, repairs, insurance, and docking is $8500 per year.



Name _____ Date _____

22. A horse costs $10,000 to purchase and the estimated cost of food, vet care, and stabling is $3500 per year.



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Skills Practice

Name _____ Date _____

Rad Man!
Radical Equations and Functions

Vocabulary

Match each word with its corresponding definition.

1. radicand

2. perfect square

3. radical symbol

a. the mathematical expression written as $\sqrt{\ }$

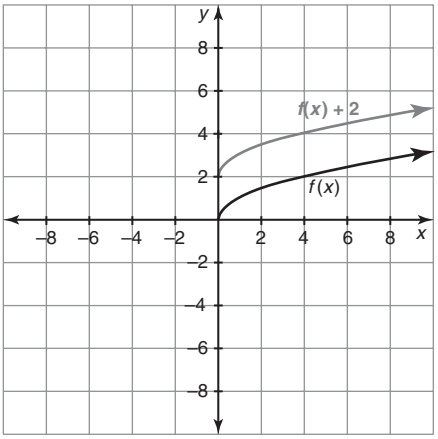
b. the expression under the radical symbol

c. a number that can be written as the square of an integer

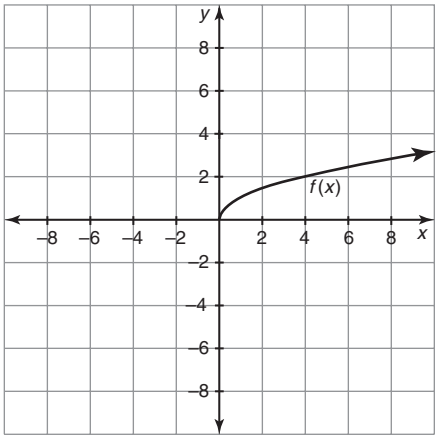
Problem Set

Given the graph of $f(x) = \sqrt{x}$, sketch the graph of each transformed function.

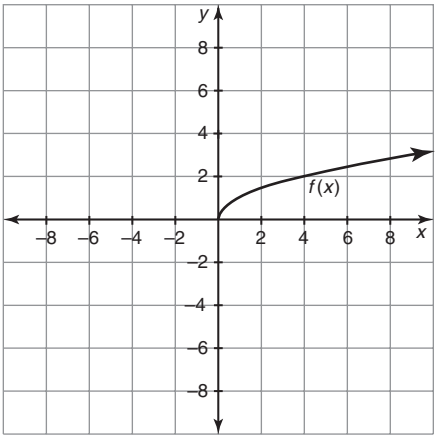
1. $f(x) + 2$



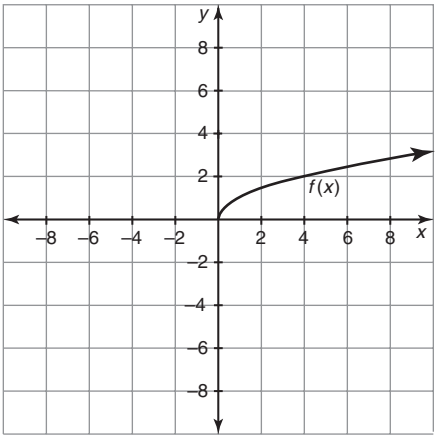
2. $f(x) - 3$



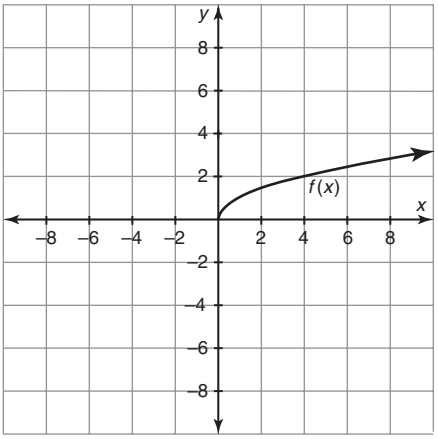
3. $f(x - 5)$



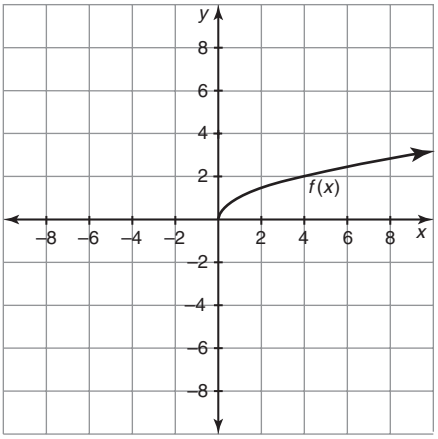
4. $f(x + 6)$



5. $f(x + 2) - 4$



6. $f(x - 3) + 6$



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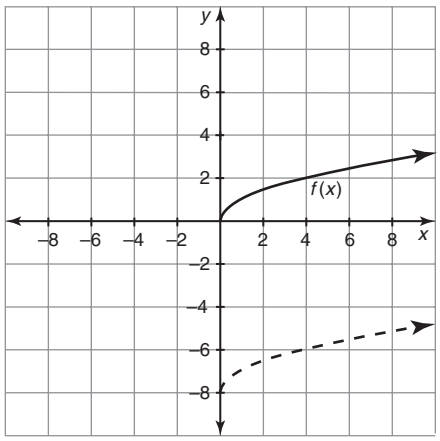
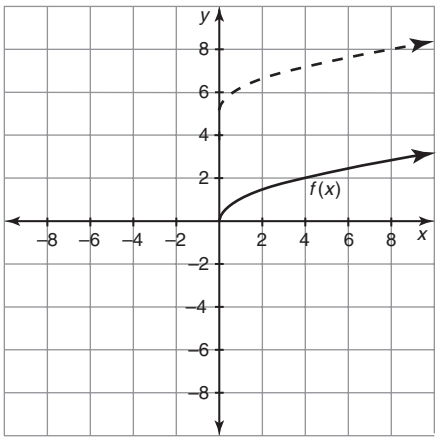
The graph of $f(x) = \sqrt{x}$ is shown. Write a function in terms of $f(x)$ for each transformed graph.

7.



$f(x) - 8$

8.



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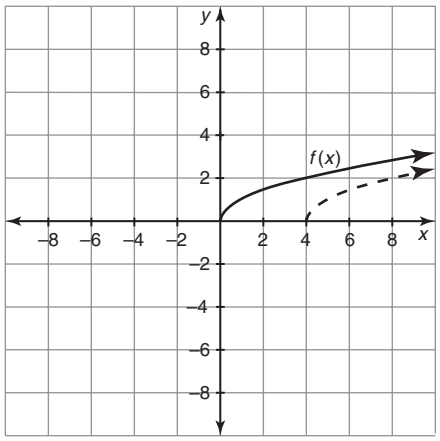

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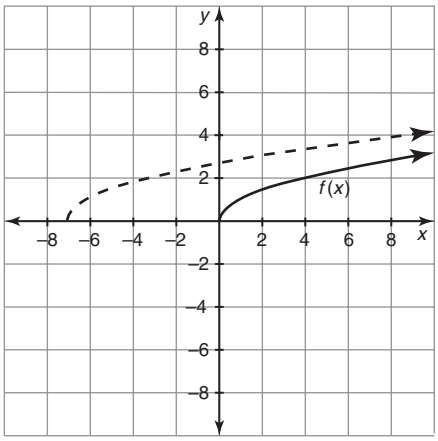

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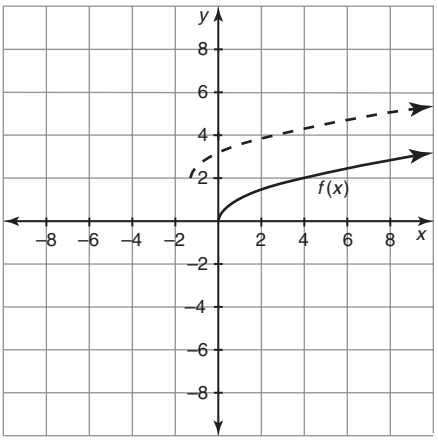
11.



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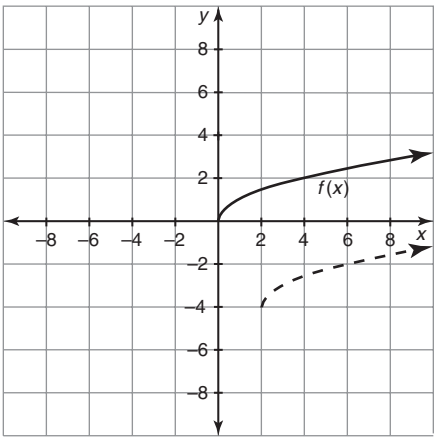

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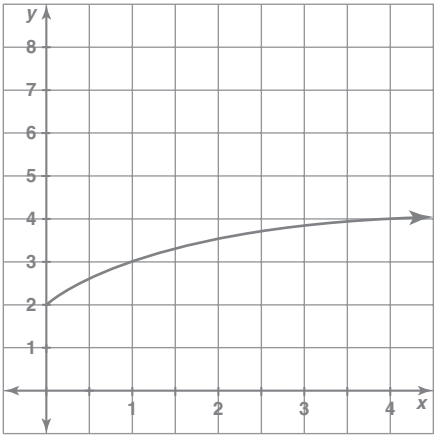
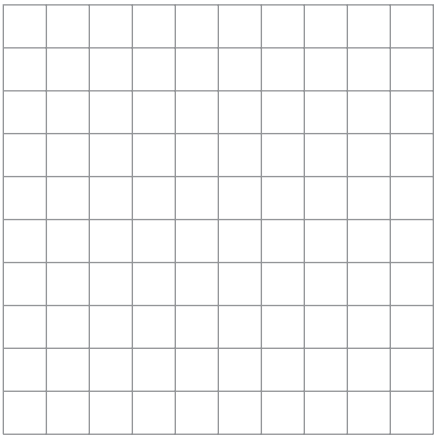
Sketch the graph of each transformed radical function.

13. $f(x) = \sqrt{x} + 2$



14. $f(x) = 3 - \sqrt{x}$



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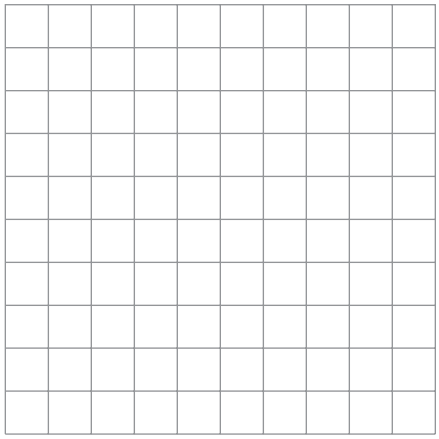
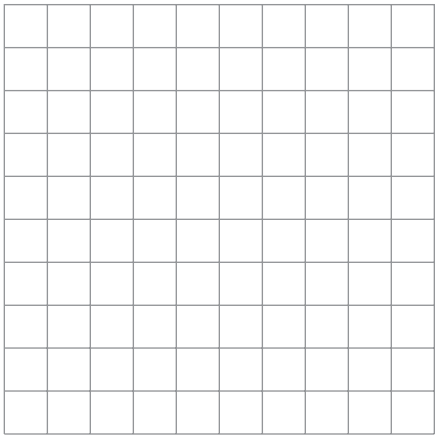
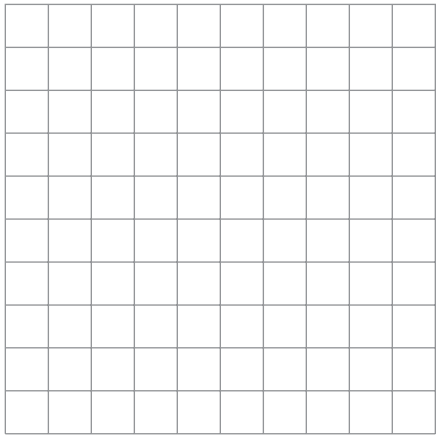
15. $f(x) = \sqrt{x - 3}$



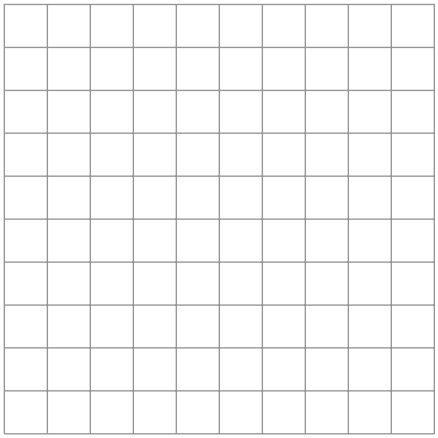
16. $f(x) = \sqrt{1 - x}$



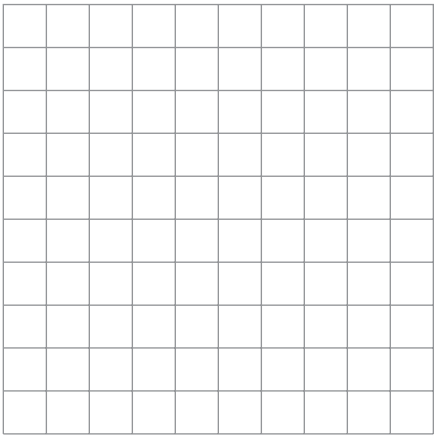
17. $f(x) = \sqrt{-x} + 4$



18. $f(x) = \sqrt{-x} - 1$



19. $f(x) = 1 - \sqrt{-x}$



20. $f(x) = -1 + \sqrt{x + 1}$

10

Solve each equation and check for extraneous solutions or roots.

21. $\sqrt{2x - 7} = 3$

$(\sqrt{2x - 7})^2 = 3^2$

$2x - 7 = 9$

$2x = 16$

$x = 8$

Check: $\sqrt{2(8) - 7} = 3$

22. $\sqrt{5x + 6} = 6$

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23. $\sqrt[3]{9x + 1} = 4$

24. $\sqrt[4]{5x + 1} = 3$

25. $\sqrt{x} - x = -20$

26. $\sqrt{x} + x = 12$

27. $x + \sqrt{2x} = 24$

28. $\sqrt{4x} - x = -15$

10

Skills Practice

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Connections
Algebraic and Graphical Connections

Vocabulary

Provide two examples of each term.

1. quadratic function

2. absolute value function

3. square root function

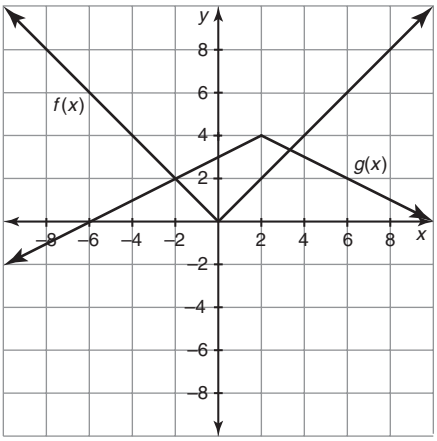
4. exponential function

5. rational function

Problem Set

Each grid shows two functions, $f(x)$ and $g(x)$. In each case $g(x)$ is a transformed version of $f(x)$. Describe the transformation graphically. Then write $g(x)$ algebraically.

1.



To form the graph of $g(x)$, the function $f(x) = |x|$ was:

shifted right 2 units

shifted up 4 units

reflected about the x-axis

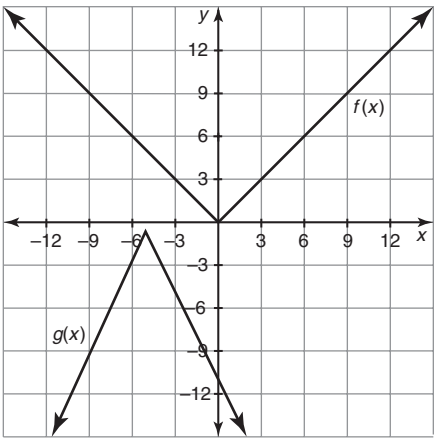
expanded by a factor of 2.

To express these transformations algebraically, we write:

$$g(x) = -\frac{1}{2}f(x - 2) + 4$$

$$= -\frac{1}{2}|x - 2| + 4$$

2.



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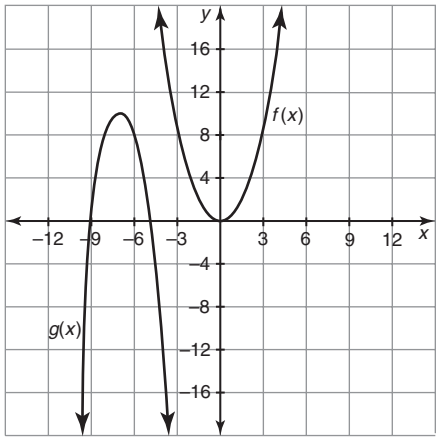
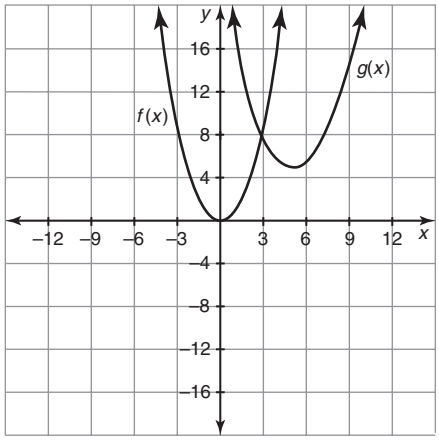
3.



4.



Determine the zeros of each function algebraically. Then graph the function. Label the *x*-intercepts and the *y*-intercepts.

5. $f(x) = x^3 + 2x^2 - x - 2$

$x^3 + 2x^2 - x - 2 = 0$

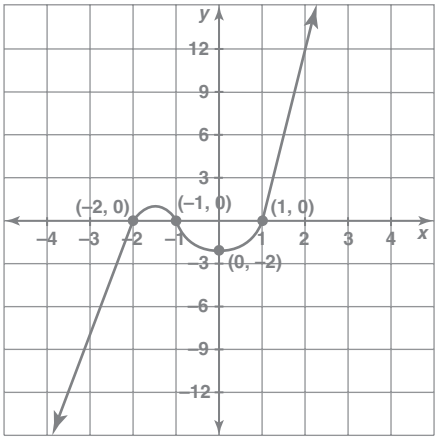
$x^2(x + 2) - (x + 2) = 0$

$(x^2 - 1)(x + 2) = 0$

$(x - 1)(x + 1)(x + 2) = 0$

$x - 1 = 0$ or $x + 1 = 0$ or $x + 2 = 0$

$x = 1$ or $x = -1$ or $x = -2$



The *x*-intercepts are at the roots of the equation: $x = -2, -1, 1$.

The *y*-intercept is $y = -2$.

6. $f(x) = x^3 - 4x$

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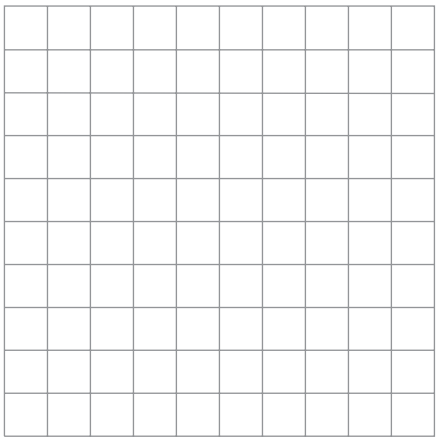
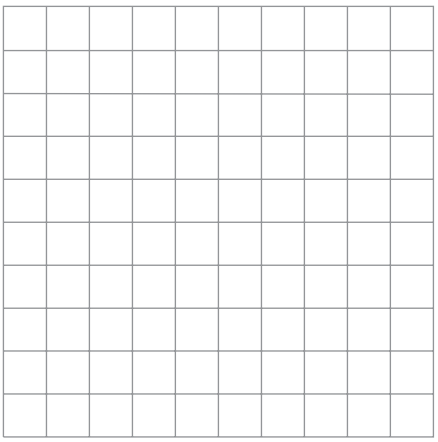
7. $f(x) = -x^3 + x^2 + 12x$



8. $f(x) = -x^3 - x^2 + 16x + 16$



Solve each equation algebraically.

9. $x^2 + 3x - 18 = 0$

$x^2 + 3x - 18 = 0$

$(x - 3)(x + 6) = 0$

$x - 3 = 0$ or $x + 6 = 0$

$x = 3$ or $x = -6$

10. $x^2 - 9x + 20 = 0$

11. $|x + 5| - 6 = 0$

12. $|x - 11| - 11 = 0$

10

13. $\sqrt{5x} - 15 = 0$

14. $\sqrt{8x} - 12 = 0$

15. $2^x = 128$

16. $3^x = 243$

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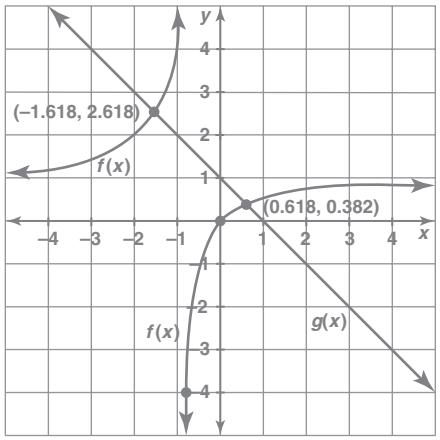
17. $\frac{7x + 8}{x} = x$

18. $\frac{7x - 12}{x} = x$

Solve each equation graphically.

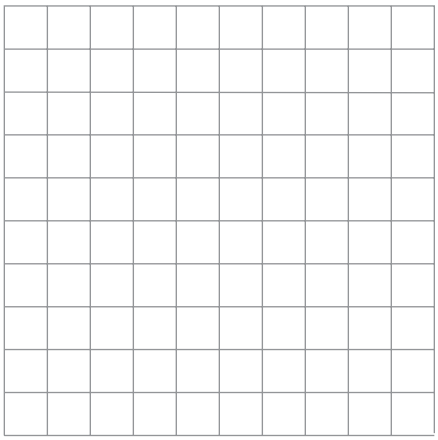
19. $\frac{x}{x + 1} = 1 - x$

$f(x) = \frac{x}{x + 1}$ $g(x) = 1 - x$

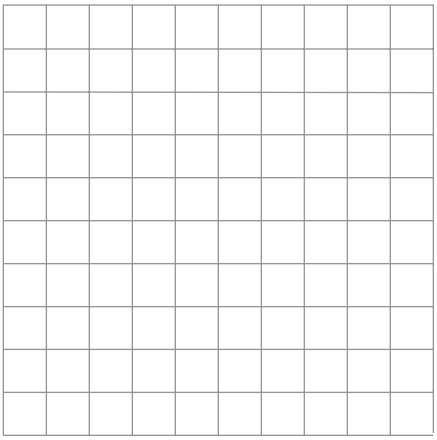


$x \cong -1.618$ or $x \cong 0.618$

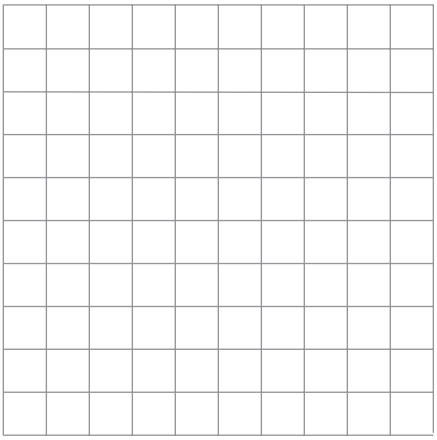
20. $4 - x^2 = 2^{-x}$



21. $|x + 2| = x^2 - 2$



22. $\frac{x - 1}{x} = 1 - x^2$



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