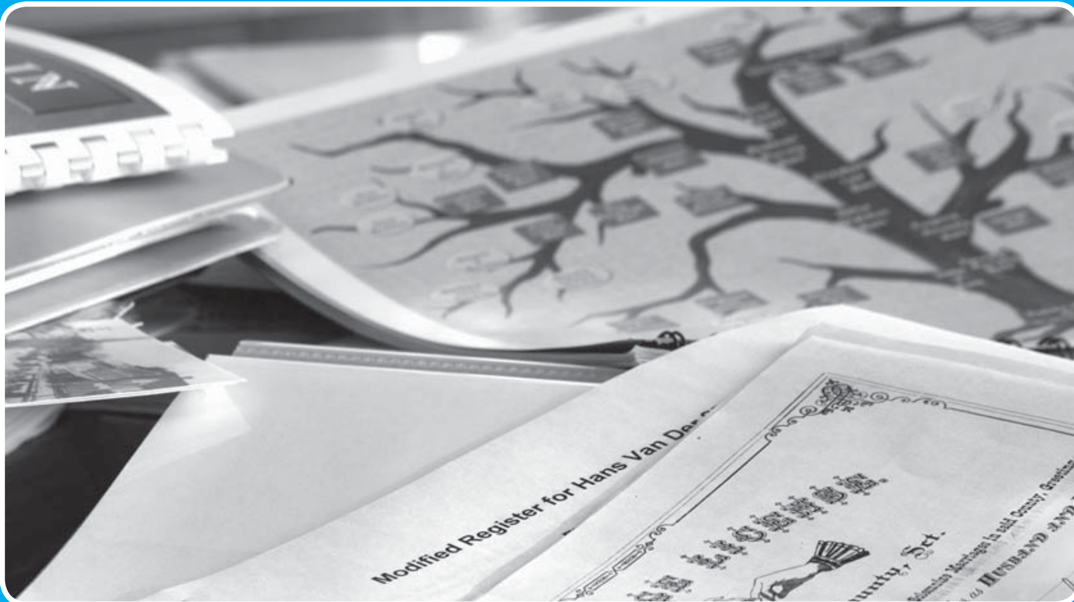
CHAPTER

1 Relations and Functions



Genealogy is the study of family histories. Genealogists investigate birth, marriage, and death records to learn about families' past generations. You will use exponential functions to calculate how many direct ancestors you had a certain number of generations ago.

Mathematical Representations

INTRODUCTION Mathematics is a human invention, developed as people encountered problems that they could not solve. For instance, when people first began to accumulate possessions, they needed to answer questions such as: How many? How many more? How many less?

People responded by developing the concepts of numbers and counting. Mathematics made a huge leap when people began using symbols to represent numbers. The first “numerals” were probably tally marks used to count weapons, livestock, or food.

As society grew more complex, people needed to answer questions such as: Who has more? How much does each person get? If there are 5 members in my family, 6 in your family, and 10 in another family, how can each person receive the same amount?

During this course, we will solve problems and work with many different representations of mathematical concepts, ideas, and processes to better understand our world. The following processes can help you solve problems.

Discuss to Understand

- Read the problem carefully.
- What is the context of the problem? Do you understand it?
- What is the question that you are being asked? Does it make sense?

Think for Yourself

- Do I need any additional information to answer the question?
- Is this problem similar to some other problem that I know?
- How can I represent the problem using a picture, a diagram, symbols, or some other representation?

Work with Your Partner

- How did you do the problem?
- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to one another?

Work with Your Group

- Show me your representation.
- This is the way I thought about the problem—how did you think about it?
- What else do we need to solve the problem?
- Does our reasoning and our answer make sense to one another?
- How can we explain our solution to one another? To the class?

Share with the Class

- Here is our solution and how we solved it.
- We could only get this far with our solution. How can we finish?
- Could we have used a different strategy to solve the problem?

1.1 Human Growth
Multiple Representations of Relations and Functions

Objectives
In this lesson you will:
- Represent relations and functions using tables, graphs, words, and algebraic equations.
- Determine the domain and range of a relation.
- Determine if relations are functions.
- Describe the graphs of relations and functions.
- Use graphs to make predictions.

Key Terms
- relation
- domain
- range
- function

Problem 1 Age and Height

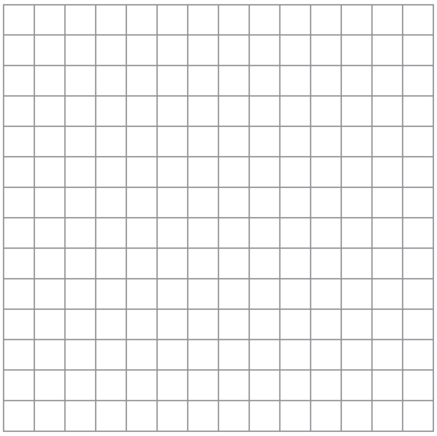
The following table shows values for the relation between age from 6 months to 13 years and the average height of boys.

Take Note

A **relation** is a mapping between a set of inputs and a set of outputs.

Age	Average Height (inches)
6 months	26
12 months	30
18 months	34
2 years	36
3 years	39
4 years	42
5 years	44
6 years	47
7 years	49
8 years	51
9 years	53
10 years	55
11 years	57
12 years	59
13 years	61

1. Create a scatter plot of the relation between age and average height on the grid shown.



2. Does it make sense to connect the points of the scatter plot? Why or why not?

3. Describe the shape of the graph.

4. What is the increase in average height for boys

   a. between 6 months and 18 months?

   b. between 1 year and 2 years?

   c. between 2 years and 3 years?

   d. between 7 years and 8 years?

5. During what one-year period does the average height for boys increase the most?

   a. How would you use the table to answer this question?

   b. How would you use the graph to answer this question?

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6. The set of inputs of a relation is the **domain.** What is the domain of the relation?

7. The set of outputs of a relation is the **range.** What is the range of the relation?

Take Note

A **function** is a relation that maps each member of the domain onto one and only one member of the range.

8. Is the relation a function? Explain.

9. Use the graph to answer each question.

   a. What is the average height of a 6½-year-old boy?

   b. What is the average height of a 14-year-old boy?

   c. At what age do boys have an average height of 54 inches?

10. The relation only includes ages up to 13 years.

a. Using the graph, what would you predict as the increase in average height of boys from the ages of 13 years to 21 years?

b. Using this predicted increase, what would be the average height at 21 years?

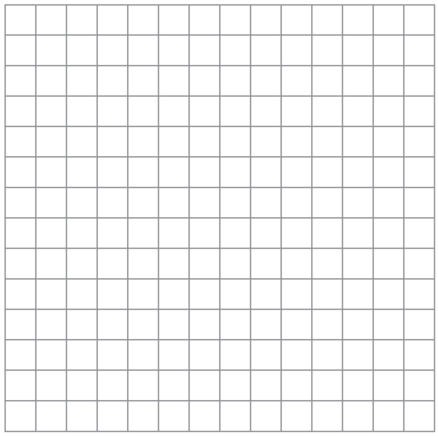
c. Does this prediction make sense? Why or why not?

Problem 2 Age and Weight

The following table shows values for the relation between age from 6 months to 13 years and average weight for boys.

Age	Average Weight (pounds)
6 months	16
12 months	23
18 months	24
2 years	31
3 years	35
4 years	40
5 years	45
6 years	49
7 years	55
8 years	61
9 years	69
10 years	75
11 years	85
12 years	89
13 years	99

1. Create a scatter plot of the relation between age and average weight on the grid shown.



2. Does it make sense to connect the points of the scatter plot? Why or why not?

3. Describe the shape of the graph.

4. During what one-year period does the average weight of boys increase the most?

   a. How would you use the table to answer this question?

   b. How would you use the graph to answer this question?

5. What are the domain and range of the relation?

6. Is the relation a function? Explain.

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7. Use the graph to answer each question.

a. What is the average weight of a 6-year, 6-month-old boy?

b. What is the average weight of a 14-year-old boy?

c. At what age does a boy have an average weight of 65 pounds?

8. The relation only includes ages up to 13 years.

a. Using the graph, what would you predict as the increase in average weight for boys from the ages of 13 years to 21 years?

b. Using this predicted average increase, what would be the average weight for boys at 21 years?

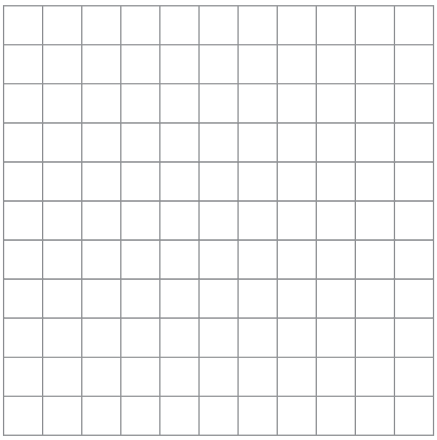
c. Does this prediction make sense? Why or why not?

Problem 3 Weight and Height

Dino's parents recorded his weight and height as he was growing up as shown in the table.

Dino's Age	Dino's Weight	Dino's Height
Years	Pounds	Inches
1	24	21
3	33	35
5	48	44
7	57	56
9	73	75
11	73	78
13	99	95

1. Create a scatter plot of the relation between Dino's weight and his height on the grid shown.



2. Does it make sense to connect the points of the scatter plot? Why or why not?

3. What are the domain and range of the relation?

4. Is the relation a function? Explain.

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Problem 4 U.S. Shirts

You have been hired at a custom T-shirt shop, U.S. Shirts. One of your jobs is to calculate the total cost of customers' orders. The shop charges $8 per shirt plus a one-time charge of $15 to set up the T-shirt design.

1. What is a one-time charge?

2. What is the total cost for an order of

   a. 3 shirts?

   b. 10 shirts?

   c. 100 shirts?

3. How many shirts can be ordered for each amount of money? What is the actual cost for an order with that number of shirts?

   a. $50

   b. $60

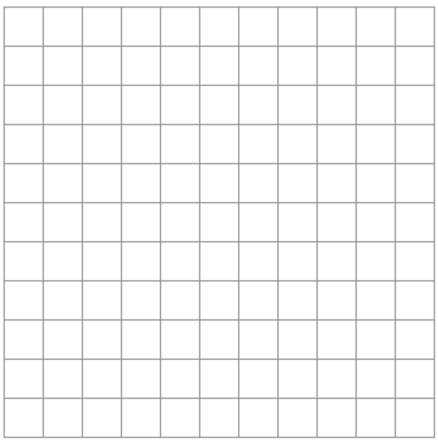
c. $220

4. Explain how to calculate the number of shirts that can be ordered for a given amount of money.

5. Enter the values you determined in Questions 2 and 3 in the following table.

Labels	Number of Shirts Ordered	Total Cost
Units	Shirts	Dollars

6. Create a scatter plot of the relation between the number of T-shirts ordered and the total cost.



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7. Does it make sense to connect the points of the scatter plot? Why or why not?

8. What are the domain and range of the relation?

9. Is the relation a function? Explain.

10. What are the variable quantities in this problem? Define a variable to represent each quantity.

Take Note

A relation involves two quantities. One quantity, the dependent quantity, depends on the other, the independent quantity.

11. Which variable quantity depends on the other variable quantity?

12. What is the independent quantity? What is the dependent quantity?

13. What are the constant quantities in this problem?

14. Use the variables from Question 10 to write an algebraic equation.

15. In this lesson, you represented problem situations using words, tables, graphs, and equations. What are the advantages and disadvantages of each representation?

	Advantages	Disadvantages
Words		
Tables		
Graphs		
Equations		

Be prepared to share your answers with the class.

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1.2 Down and Up
Linear and Absolute Value Functions

Objectives
In this lesson you will:
- Model problem situations with linear and absolute value functions.
- Represent linear and absolute value functions using words, tables, equations, and graphs.
- Interpret the graphs of linear and absolute value functions.

Key Terms
- linear function
- slope
- extreme points/extrema
- absolute value function
- line symmetry
- line of symmetry

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Problem 1 Water Tanks

Water is supplied to a house in the mountains using a well and a water tank. When full, the water tank contains 240 gallons of water. The occupants of the house use water at the average rate of 10 gallons per hour.

1. If the water tank begins full, how much water remains in the tank after

 a. 5 hours?

 b. 8 hours 30 minutes?

 c. 10 hours 15 minutes?

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2. If the water tank begins full, after how long will the tank

a. contain 100 gallons of water?

b. contain 75 gallons of water?

c. be one fourth full?

d. be completely empty?

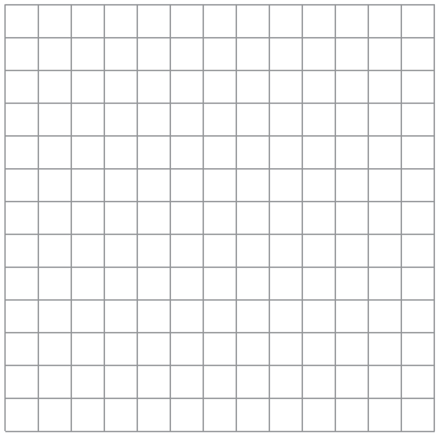
3. What are the variable quantities in this problem?

4. What is the independent quantity? What is the dependent quantity?

5. Use the values from Questions 1 and 2 to complete the following table.

	Independent Quantity	Dependent Quantity
Labels		
Units		

6. Create a scatter plot of the relation between the time and the amount of water in the tank.



7. Draw a line that represents all possible points. Why does it make sense in this problem situation to connect the points?

8. What are the domain and range of the function?

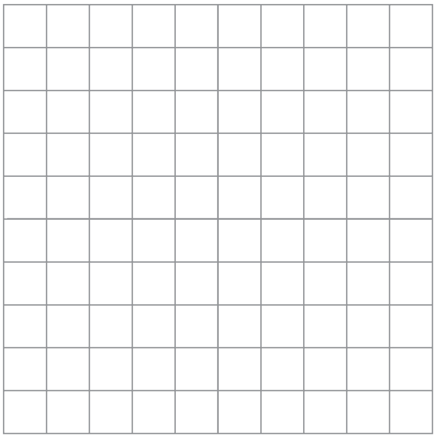
9. Define a variable to represent each variable quantity. Then use the variables to write an algebraic equation.

10. Identify each constant in the equation from Question 9. What does each constant represent in the problem situation?

This equation you wrote in Question 9 represents a **linear function** because its graph is a line. In a linear function, the dependent variable increases or decreases by a constant amount when the independent variable increases by one unit. This unit rate of change is called the **slope**.

11. What is the unit rate of change, or slope, of the function from Question 9?

12. Graph the linear function represented by the equation $y = 240 - 10x$.

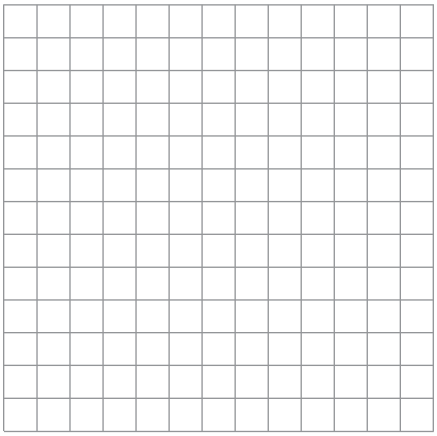


13. What are the domain and range of this linear function?

Problem 2 Water Pumps

Consider the water tank from Problem 1. The homeowner installs a water pump. The water pump starts automatically when the water tank is half full and fills the tank at a rate of 20 gallons per hour. When the water tank is full, the water pump automatically shuts off.

1. The water tank begins full. Create a scatter plot to represent the amount of water in the tank over a 24-hour period.



2. Connect the points of the scatter plot. Why does it make sense in this problem situation to connect the points?

3. Describe the shape of the graph.

4. What are the domain and range of this relation?

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5. Does the graph have a maximum point or points? If so, what is/are the maximum point(s)?

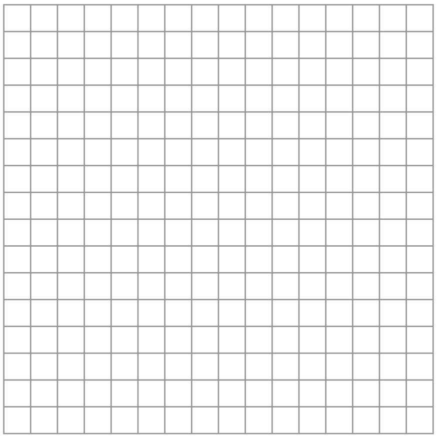
6. Does the graph have a minimum point or points? If so, what is/are the minimum point(s)?

The maximum and minimum points of a function are called **extreme points** or **extrema**.

7. Use a graphing calculator to graph the equation $y = 120 + |-10x + 120|$ for $0 \le x \le 24$. Sketch the graph on the grid.

Take Note

The absolute value symbol is | | and absolute value is defined as $|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a & \text{if } a < 0 \end{cases}$



8. How does the graph in Question 7 compare to the graph in Question 1?

The equation $y = 120 + |-10x + 120|$ represents an **absolute value function**. An absolute value function is a function that contains an absolute value expression. The basic absolute value function is $y = |x|$.

9. Draw the vertical line $x = 12$ on the graph in Question 7. What do you notice?

A graph in which a line can be drawn that divides the graph into two parts that are mirror images of one another displays a property called **line symmetry**. The line is called the **line of symmetry**. The reflection of one part of the graph about the line of symmetry is the same as the other part of the graph.

Problem 3

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1. Graph each absolute value function. Then identify the domain, range, extrema, and line of symmetry.

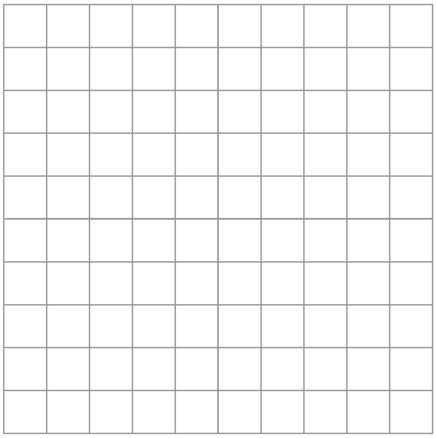
a. $y = |x|$

Domain:

Range:

Extrema:

Line of symmetry:



b. $y = -|2x|$

Domain:

Range:

Extrema:

Line of symmetry:

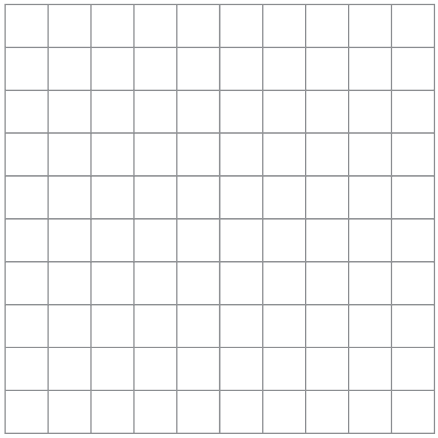
c. $y = 2 + |x + 3|$

Domain:

Range:

Extrema:

Line of symmetry:



Be prepared to share your answers with the class.

1.3 Let's Take a Little Trip with Me!

Every Graph Tells a Story

Objectives

In this lesson you will:

- Represent functions using words, tables, equations, and graphs.
- Determine intervals of increase and decrease for a function.
- Interpret the graphs of functions.

Key Terms

- interval of increase
- interval of decrease
- vertical motion
- quadratic function

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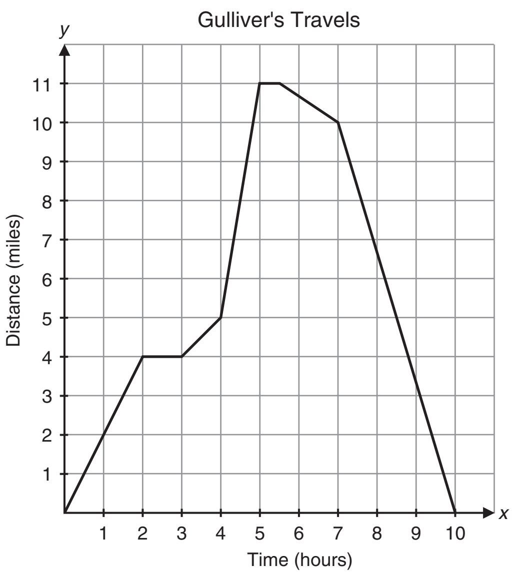
Problem 1 Gulliver's Travels

The following graph shows the relation between time and Gulliver's distance from home during 1 ten-hour period. Use the graph to answer the following questions.



1. Is the relation a function? Explain.

2. Identify any extreme points. What do the extreme points mean in this problem?

3. What is the domain and the range of the relation?

4. How far from home was Gulliver after

   a. two hours?

   b. two and one half hours?

   c. six hours?

   d. eight hours?

   e. ten hours?

5. After how many hours was Gulliver

   a. five miles from home?

   b. ten miles from home?

   c. four miles from home?

6. How far did Gulliver travel during the first two hours of the trip?

7. Consider Gulliver's average speed during the trip.

a. What was Gulliver's average speed during the first two hours?

b. What was Gulliver's average speed between seven hours and ten hours?

c. When was Gulliver moving the fastest? How is this shown on the graph?

d. What was Gulliver's average speed when he was moving the fastest?

e. When was Gulliver moving the slowest? How is this shown on the graph?

f. What was Gulliver's average speed when he was moving the slowest?

8. When was Gulliver moving away from his house? How is this shown on the graph?

9. When was Gulliver moving toward his house? How is this shown on the graph?

10. When was Gulliver standing still? How is this shown on the graph?

As you trace along a graph from left to right, **intervals of increase** are portions of the graph that are increasing. **Intervals of decrease** are portions of the graph that are decreasing.

11. Write a paragraph describing Gulliver's travels. Include each time interval when his speed changed.

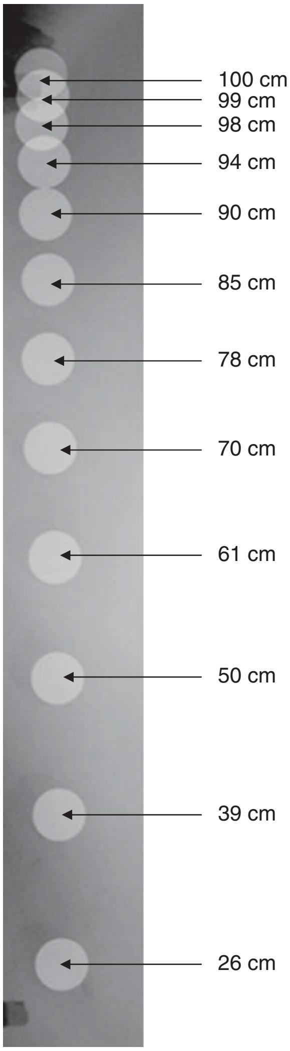
Problem 2 Don't Drop the Ball

If two objects of different weights are dropped from the same height, which one will land on the ground first?

This question was explored by the famous mathematician Galileo Galilei. He performed a series of experiments in which he dropped balls of different weights from the Leaning Tower of Pisa. He was able to prove that the weight of an object doesn't matter!

The motion of a dropped object is called **vertical motion** and can be modeled using an algebraic equation.

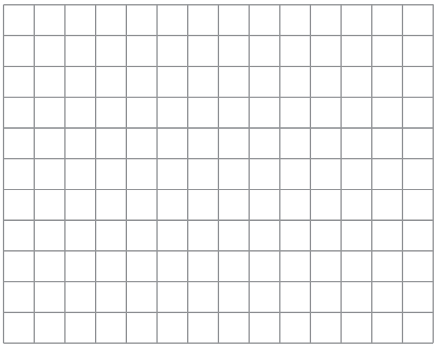
The following picture shows the height of a tennis ball that was dropped from an initial height of 100 centimeters. A strobe light was used to record the height of the ball every 0.025 seconds.



1. Complete the table using the picture.

Labels	**Time**	**Height**
Units	**Seconds**	**Centimeters**
	0	
	0.025	

2. Use the table to create a graph showing the relation between time and the height of the tennis ball.



3. Is the relation a function? Explain.

4. Describe the shape of the graph.

5. Does the shape of the graph describe the path of the ball? Explain.

6. Average speed is the ratio of the distance and the time. What was the ball's average speed in centimeters per second

   a. during the first 0.025 seconds?

   b. between 0.15 seconds and 0.175 seconds?

   c. between 0.25 seconds and 0.275 seconds?

7. What do you notice about the average speed of the ball over time? Based on your experience, does this make sense? Explain.

8. Perform the following steps using a graphing calculator.

   a. Enter the equation $y_1 = 100 - 980x^2$.

   b. Set the x-bounds from 0 to 0.325 with an interval of 0.025.

   c. Set the y-bounds from 0 to 100 with an interval of 10.

   d. Turn on the grid.

   e. Graph this function

   What do you notice about this graph and the graph from Question 2?

9. Perform the following steps using a graphing calculator.

   a. Keep the equation $y_1 = 100 - 980x^2$.

   b. Set up a table starting at 0 with intervals of 0.025.

   c. View the table.

   What do you notice about this table and the table from Question 1?

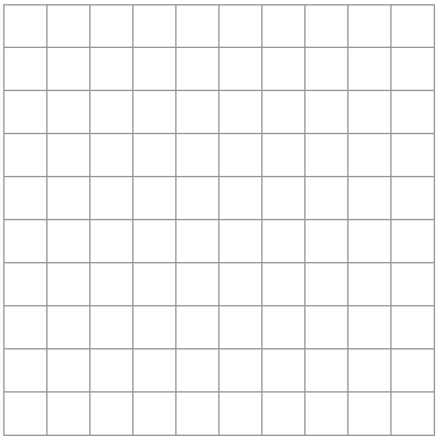
The relation between time and the height of a dropped object is an example of a **quadratic function**. A quadratic function is a function in which the independent variable is raised to a power of two.

Problem 3

For each quadratic function, complete the table and sketch a graph. Then, identify the domain and range, any extreme points and what type they are, intervals of increase and decrease, and the line of symmetry.

1. $y = x^2$

x	y
0	
1	
2	
−1	
−2	



Domain:

Range:

Extreme point:

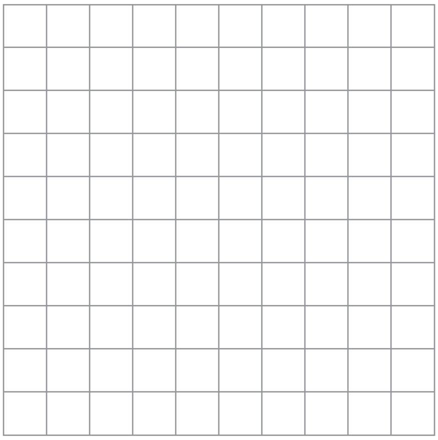
Interval of increase:

Interval of decrease:

Line of symmetry:

2. $y = -x^2$

x	y
0	
1	
2	
−1	
−2	



Domain:

Range:

Extreme point:

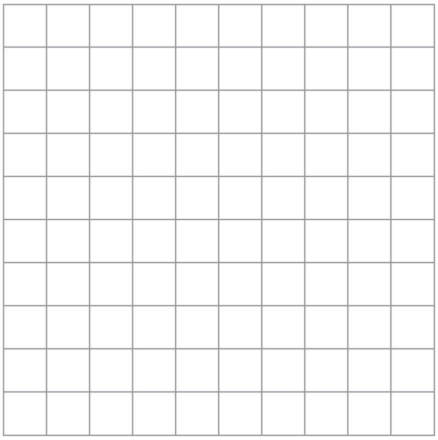
Interval of increase:

Interval of decrease:

Line of symmetry:

3. $y = 2x^2 - 5$

x	y
0	
1	
2	
−1	
−2	



Domain:

Range:

Extreme point:

Interval of increase:

Interval of decrease:

Line of symmetry:

Be prepared to share your answers with the class.

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1.4 Building a Better Box

Cubic and Indirect Variation Functions

Objectives

In this lesson you will:

- Represent cubic and indirect variation functions using words, tables, equations, and graphs.
- Interpret the graphs of cubic and indirect variation functions.

Key Terms

- cubic function
- indirect variation function

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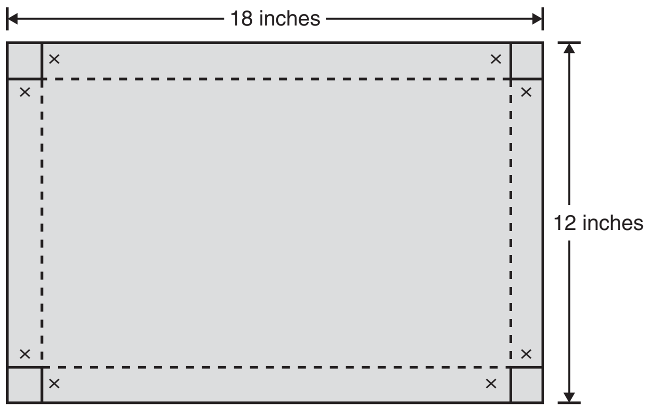
Problem 1

A company produces liners for planter boxes. To make the liners, a square is cut from each corner of a rectangular copper sheet. The sides are bent to form a box without a top. Cutting different sized squares from the corners results in different sized boxes.

Each rectangular copper sheet is 12 inches by 18 inches. In the diagram, the heavy lines indicate where the cuts are made and the dotted lines represent where the sides are bent.



1. What are the height, width, and length of the planter box that result if the length of each side of each corner square is

   a. one inch?

   b. two inches?

   c. four inches?

2. What is the largest size of corner square that can be cut to make a box?

3. Write a formula for the volume of the box.

4. What is the volume of the box that results if the length of each side of the corner square is

   a. one inch?

   b. two inches?

c. four inches?

5. Complete the table using your answers from Questions 1 and 4.

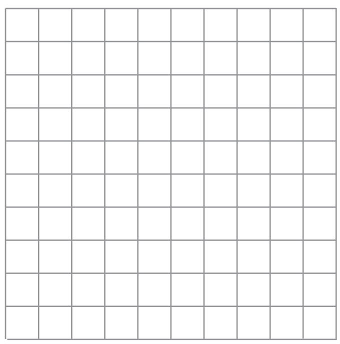
Labels	Side Length of Square	Height of Box	Width of Box	Length of Box	Volume of Box
Units	Inches	Inches	Inches	Inches	Cubic Inches
	0				
	1				
	2				
	3				
	4				
	5				
	6				

6. What is the volume of the box if each side of each corner square is 0 inches or 6 inches? Add these rows to the table in Question 5.

7. Use the table to create a scatter plot for the relation between the side length of the corner squares and the volume of the resulting box.

Take Note

Whenever points are included in a graph to represent endpoints and are not actually part of the graph, we use open points ° to represent them.



8. Draw a smooth curve connecting the points. Why does it make sense in this problem to connect the points?

9. Is the relation a function? Explain.

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10. What is the independent quantity? What is the dependent quantity?

11. Use the graph to estimate the largest possible volume. What size of squares must be cut to make a box with the maximum volume?

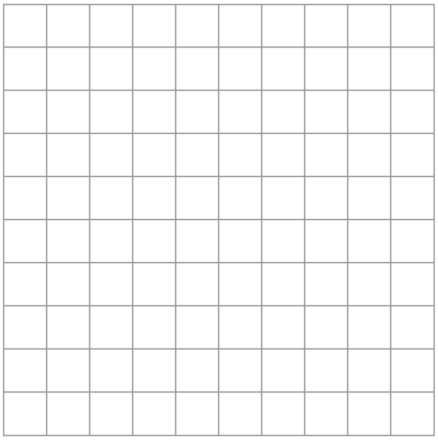
12. What are the domain and range of the relation?

13. If the side length of the squares is x inches, what are the height, width, and length of the resulting box?

14. If the side length of the square is x inches, what is the volume of the resulting box?

15. Graph the equation from Question 14 using a graphing calculator. Set the x-bounds of the graph from 0 to 6. What do you notice about this graph and the graph from Question 7?

16. Graph the equation from Question 14 again using a graphing calculator. This time, set the x-bounds of the graph from -10 to 10. Set the y-bounds of the graph from -300 to 300.



17. Graph the equation $y = 4x^3 - 60x^2 + 216x$ using a graphing calculator. Set the x-bounds of the graph from -10 to 10. Set the y-bounds of the graph from -300 to 300. What do you notice about this graph and the graph from Question 16?

The equations in Questions 14 and 17 are examples of cubic functions. A **cubic function** is a function in which the independent variable is raised to a power of three.

Problem 2

A bucket of paint can cover a flat surface that is 100 square feet with one coat.

1. You are painting a rectangular surface with one can of paint. What is the largest possible length if the width of the rectangular surface is

   a. one foot?

   b. two feet?

   c. five feet?

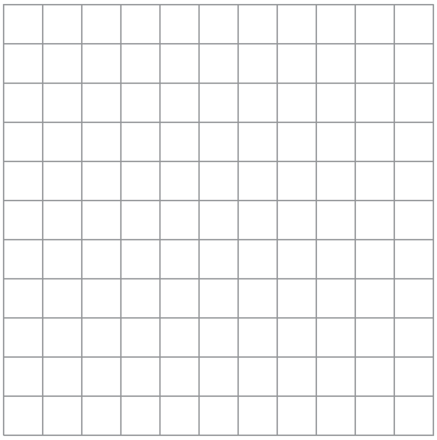
   d. ten feet?

2. Can the rectangular surface have a length that is more than 100 feet? Explain.

3. Complete the table using your answers from Question 1.

Labels	Width	Length
Units	Feet	Feet
	1	
	2	
	5	
	10	
		5
		2
		1

4. Use the table to create a scatter plot for the relation between width and length.



5. Draw a smooth curve connecting the points. Why does it make sense in this problem to connect the points?

6. Is this relation a function? Explain.

7. Is it possible for the width or length of the rectangle to be zero feet? Explain.

8. As the width of the rectangular surface increases, what happens to the length?

9. What are the domain and range of this function?

10. Define variables for the width and the length. Use the variables to write an equation.

11. Graph the equation from Question 10 using a graphing calculator. Set the x-bounds of the graph from 0 to 100. Set the y-bounds of the graph from 0 to 100. What do you notice about this graph and the graph from Question 4?

The equation in Question 10 is an example of an **indirect variation function.** An indirect variation function is a function where the value of y varies indirectly or in the opposite direction to x. When x increases, y decreases proportionally and when x decreases, y increases proportionally.

Be prepared to share your answers with the class.

1.5 How Far Can You See? How Many Ancestors?

Square Root and Exponential Functions

Objectives

In this lesson you will:

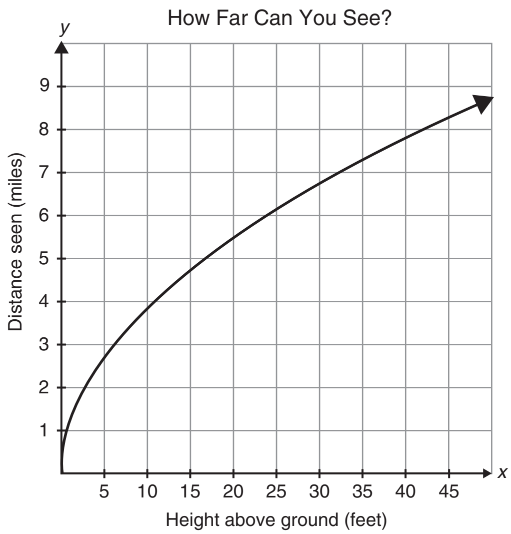
- Represent square root and exponential functions using words, tables, equations, and graphs.
- Interpret the graphs of square root and exponential functions.
- Calculate unit rates of change and average rates of change.

Key Terms

- square root function
- square root
- exponent
- exponential function
- average rate of change

Problem 1 How Far Can You See?

The following graph shows the relation between a person's height above the ground and the maximum distance a person can see across flat ground.



1. Describe the shape of the graph.

2. Based on the graph, what is the maximum distance that a person can see if they are at a height of

a. five feet?

b. 20 feet?

c. 35 feet?

3. Based on the graph, how high above ground would a person need to be in order to see an object that is at a distance of

a. six miles?

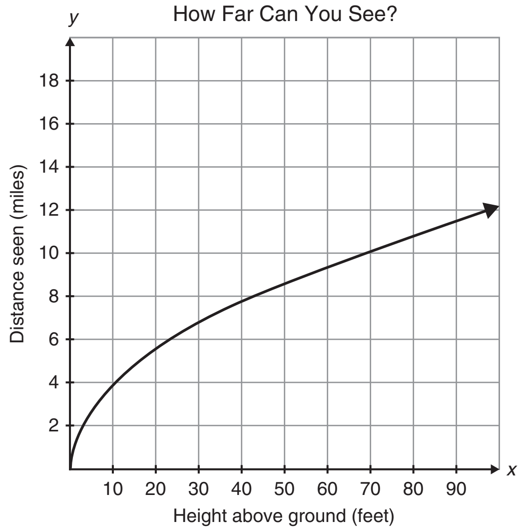
b. eight miles?

c. two miles?

4. Can you use the graph to predict the maximum distance a person could see from the top of a 100-foot building? Explain.

5. Can you use the graph to predict the maximum distance a person could see from an airplane at an altitude of 30,000 feet? Explain.

6. The following graph shows the same relation for heights up to 100 feet. Based on this graph, what is the maximum distance a person could see from the top of a 100-foot building? Compare this answer to you prediction in Question 4.



The equation that models this function is $y = \sqrt{1.5x}$, where x is the height above the ground in feet and y is the maximum distance that a person can see in miles. This equation is an example of a **square root function.** A square root function is a function in which the independent variable is contained within a **square root.**

7. Use the equation to answer Question 1. Compare the answers using the equation to the answers using the graph. What do you notice?

8. Use the equation to answer Question 5.

I

9. What are the advantages and disadvantages of using a graph to make predictions?

10. What are the domain and range of the function $y = \sqrt{1.5x}$?

11. Are the domain and range of the function the same as the domain and range of the problem situation? Explain.

Problem 2 How Many Ancestors?

Did you ever wonder how many relatives you have? You can begin to answer that question by looking at the ancestors you have at each generation. Your parents are one generation from you, your grandparents are two generations from you, and your great-grandparents are three generations from you.

1. How many ancestors do you have

   a. one generation ago?

   b. two generations ago?

   c. three generations ago?

   d. four generations ago?

2. What do you notice about the number of ancestors that you have at each generation?

3. How many ancestors do you have 10 generations ago?

> **Take Note**
>
> An **exponent** can be used to indicate repeated multiplication. For example,
>
> $$\overbrace{2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 \cdot 2}^{a \text{ times}} = 2^a$$

4. How many ancestors do you have 20 generations ago? How did you calculate this number?

5. Let x represent the number of generations ago and let y represent the number of ancestors at that generation. Use the variables to write an equation for the relation between the generation and the number of ancestors.

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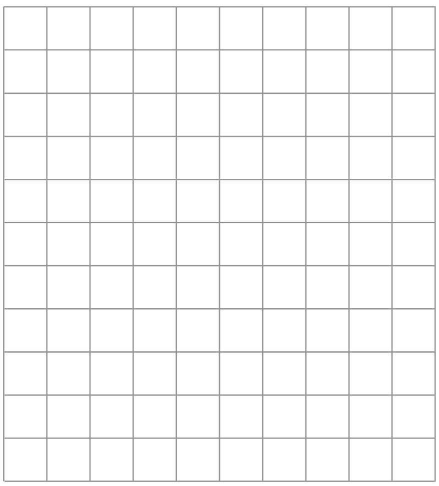
6. Is this relation a function? Explain.

7. What are the domain and range of the relation?

8. Create a scatter plot of the function for up to 10 generations ago.



9. Does it make sense to connect the points of the scatter plot? Why or why not?

10. Use the equation to calculate the number of ancestors you have 30 generations ago.

11. A generation is considered to be about 30 years. Using this approximation, during what year were the ancestors from 30 generations ago living? Explain.

12. In the year 1000 there were about 400 million people in the entire world. Is this consistent with your answer in Question 10?

The equation in Question 5 is an example of an **exponential function.** An exponential function is a function where the independent variable is an exponent. The graphs of exponential functions can increase or decrease very rapidly.

Problem 3

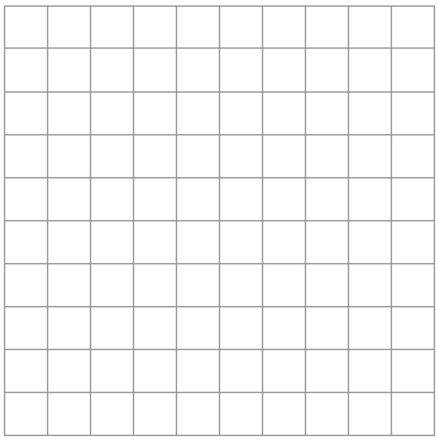
1. Complete the first two columns of each table. Then graph the linear function $y = 2x$ and the exponential function $y = 2^x$ on the same grid.

$y = 2x$

x	y	Unit Rate of Change
1		
2		
3		
4		
5		

$y = 2^x$

x	y	Unit Rate of Change
1		
2		
3		
4		
5		



2. Compare the graphs of the linear function $y = 2x$ and the exponential function $y = 2^x$.

3. The slope, or unit rate of change, is the ratio of the change in the dependent variable to a change in the independent variable of one unit. Complete the third column of each table in Question 1.

4. Describe the differences in the unit rates of change for the linear function $y = 2x$ and the exponential function $y = 2^x$.

The **average rate of change** is the ratio of the change in the dependent variable to the change in the independent variable, or $\frac{y_2 - y_1}{x_2 - x_1}$.

For example, the average rate of change for $y = 2x$ from 1 to 5 is $\frac{10 - 2}{5 - 1} = \frac{8}{4} = 2$.

5. What is the average rate of change for $y = 2x$ from 1 to 4? from 2 to 4?

6. What is the average rate of change for $y = 2^x$ from 1 to 5? from 2 to 4?

7. What can you conclude about the average rates of change and the unit rates of change for linear functions?

8. What can you conclude about the average rates of change and the unit rates of change for exponential functions?

Be prepared to share your answers with the class.