CHAPTER

2 Algebraic Functions

About 1.4 billion cotton T-shirts, with a retail value of about $20 billion, are sold annually in North America. Several thousand T-shirt retailers do business on the Internet alone. You will use arithmetic sequences to calculate the cost of ordering T-shirts in bulk.

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2.1 Functional Function: *F* of *x* it is!

Functional Notation

Objectives

In this lesson you will:

- Write functions using functional notation.
- Evaluate functions using functional notation.
- Identify independent and dependent values using tables, graphs, and equations.

Key Terms

- functional notation
- evaluating a function

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Functional notation is a way of representing functions algebraically. Function notation makes it easier to recognize the independent and dependent variables in an equation. The function $f(x)$ is read as "f of x" and indicates that x is the independent variable.

Consider the equation $c = 8s + 15$, where the independent variable s represents the number of shirts ordered and the dependent variable c represents the cost of the order. The equation can be written using functional notation as $f(s) = 8s + 15$. The cost, defined by f, is a function of s, the number of shirts ordered.

The process of calculating the value of a function for a specific value of the independent variable is called **evaluating a function.** For example, the cost of ordering 4 shirts can be calculated by evaluating the function at $s = 4$.

This is written as $f(4)$ and read as "f of 4."

To evaluate, substitute 4 for s in the rule $f(s) = 8s + 15$.

$f(4) = 8(4) + 15 = 32 + 15 = 47$

$f(4) = 47$

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Problem 1 Functions as Equations

1. The function $f(s) = 8s + 15$ represents the cost of ordering s shirts. What are the domain and range of the function?

2. Use the equation $f(s) = 8s + 15$ to evaluate the function at each value. Explain what each means in terms of the problem.

   a. $f(7)$

   b. $f(100)$

   c. $f(11)$

   d. $f(0)$

   e. $f(2.5)$

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3. Evaluating the function f at 4 is written as $f(4) = 47$, and read as "the value of the function f at 4 is 47" or "f of 4 is 47." How would each evaluation in Question 1 be read?

4. Calculate the value of x that makes each equation true. Explain what each means in terms of the problem.

   a. $f(x) = 55$

   b. $f(x) = 175$

   c. $f(x) = 151$

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Problem 2 Functions as Tables

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The function $h(a)$ represents the average height of boys that are a years old.

Boy's Age	Average Height in Inches
6 months	26
12 months	30
18 months	34
2 years	36
3 years	39
4 years	42
5 years	44
6 years	47
7 years	49
8 years	51
9 years	53
10 years	55
11 years	57
12 years	59
13 years	61

1. Use the table to evaluate the function at each value. Explain what each means in terms of the problem.

a. $h(7)$

b. $h(1.5)$

c. $h(11)$

d. $h(12.5)$

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2. Calculate the value of a that makes each equation true. Explain what each means in terms of the problem.

a. $h(a) = 61$

b. $h(a) = 36$

c. $h(a) = 53$

d. $h(a) = 45$

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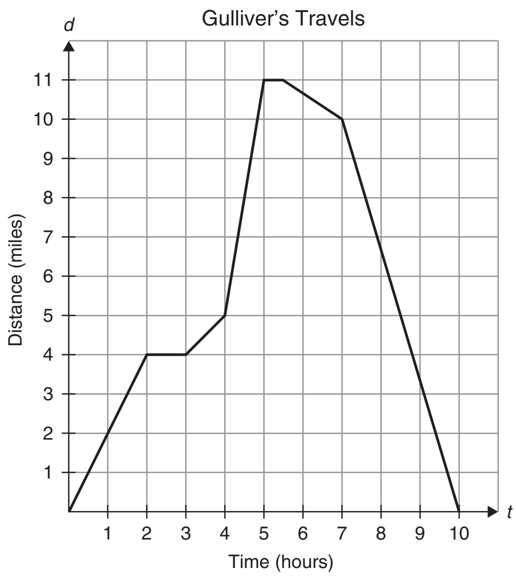
Problem 3 Functions as Graphs

The function $d(t)$ represents Gulliver's distance from home after t hours.



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1. Use the graph to evaluate the function at each value. Explain what each means in terms of the problem.

   a. $d(2)$

   b. $d(5)$

   c. $d(2.9)$

   d. $d(10)$

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2. Calculate the value of t that makes each equation true. Explain what each means in terms of the problem.

   a. $d(t) = 2$

   b. $d(t) = 5$

   c. $d(t) = 4$

   d. $d(t) = 0$

Be prepared to share your solutions and methods.

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2.2 Numbers in a Row!
Introduction to Sequences

Objectives
In this lesson you will:
- Define a mathematical sequence as a function with domain of the counting numbers.
- Describe sequences using words, numbers, diagrams, and figures.
- Define numerical sequences using an explicit formula for the *n*th or general term.
- Define numerical sequences using recursive formulas.

Key Terms
- mathematical sequence
- term
- finite sequence
- infinite sequence
- explicit or general term formula
- recursive formula

A **mathematical sequence**, or just a **sequence**, is a number pattern or a list of numbers. Each number of the sequence is called a **term**.

The ability to recognize patterns, especially in numbers, is very important. Sometimes, the pattern is easy to notice.

1, 3, 5, 7, 9, ______, ______, . . .

Sometimes, the pattern is more difficult to recognize.

0, 1, 5, 14, 30, 55, ______, _______, . . .

You can use a pattern to calculate a particular term of a sequence. As with recognizing a pattern, sometimes calculating a term is easy.

$$4, 8, 12, 16, \ldots, \underbrace{\qquad\quad}_{\text{10th term}}, \ldots$$

However, sometimes it may be difficult to calculate a term.

$$1, 1, 2, 3, 5, 8, 13, \ldots, \underbrace{\qquad\quad}_{\text{10th term}}, \ldots$$

Sequences can be described by a list of numbers, diagrams, or other figures.

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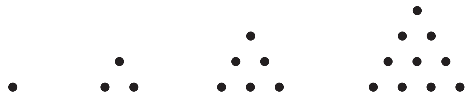
Problem 1 Representing Sequences

1. For each sequence, describe the pattern. Then determine the next two figures and the next two corresponding terms of the sequence.

a.

Figures:



Number of points: 1 3 6 10

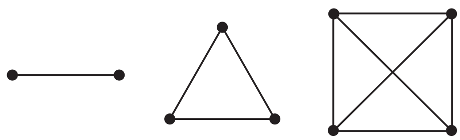
b. Each figure is made from toothpicks.

Figures:

Number of toothpicks: 4 7 10

c.

Figures:



Number of segments 1 3 6

2. For each sequence, describe the pattern. Then determine the next two terms and the tenth term.

a. 1, 4, 9, 16, ___, ___, . . . , ____, . . .
10th term

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b. 2, 4, 6, 8, ___, ___, . . . , $\underbrace{\qquad}_{\text{10th term}}$ . . .

c. 3, 7, 11, 15, ___, ___, . . . , $\underbrace{\qquad}_{\text{10th term}}$ . . .

d. $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$, ___, ___, . . . , $\underbrace{\qquad}_{\text{10th term}}$ . . .

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Sequences are often represented as $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$, where the subscript represents the position of the term in the sequence. For example, a_{100} is the one hundredth term of a sequence.

A **finite sequence** is a sequence with a finite number of terms. For example, the sequence 2, 4, 6, 8, 10, 12 is a finite sequence. An **infinite sequence** is a sequence with an infinite number of terms. For example, the sequence 2, 4, 6, 8, 10, 12, . . . is an infinite sequence.

3. For each sequence, describe the pattern. Then write a formula to calculate the *n*th term.

a. 10, 20, 30, 40, . . . , $\underbrace{\qquad}_{n\text{th term}}$ . . .

b. 2, 4, 8, 16, . . . , $\underbrace{\qquad}_{n\text{th term}}$ . . .

c. 23, 31, 39, 47, . . . , $\underbrace{\qquad}_{n\text{th term}}$ . . .

d. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, . . . , $\underbrace{\qquad}_{n\text{th term}}$ . . .

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Problem 2 Sequences as Functions

1. Use the sequence from Problem 1 Question 2(a) to complete the table.

Term Number	Value of Term
1	1
2	4
3	
4	
5	
10	
n	

a. Describe how to calculate the value of each term from the term number.

b. Write a function $f(n)$ to calculate the nth term of the sequence.

c. What are the domain and range of $f(n)$?

d. Is $f(1.5)$ defined? Explain.

2. Use the sequence from Problem 1 Question 2(b) to complete the table.

Term Number	Value of Term
1	2
2	
3	
4	
5	
10	
n	

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a. Describe how to calculate the value of each term from the term number.

b. Write a function $g(n)$ to calculate the nth term of the sequence.

c. What are the domain and range of $g(n)$?

d. Calculate each term of the sequence.

a_{20}

a_{25}

a_{50}

a_n

2

3. Use the sequence from Problem 1 Question 2(c) to answer each question.

a. Describe how to calculate the value of each term from the term number.

b. Write a function $h(n)$ to calculate the nth term of the sequence.

c. What are the domain and range of $h(n)$?

d. Calculate each term of the sequence.

a_{20}

a_{25}

a_{50}

a_n

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4. What is the domain of a function used to model a sequence?

5. What is the range of a function used to model a sequence?

Problem 3 Explicit Formulas

In Problems 2 and 3 you wrote formulas to calculate a_n, the value of a term of a sequence, using n, the term number. This formula, called a **general term formula** or **explicit formula**, defines all terms of a sequence in terms of the term number.

1. Use each explicit formula to generate the first four terms and the tenth term of the sequence.

   a. $a_n = 3n + 5$

   b. $a_n = n^2 + n$

   c. $a_n = \frac{2}{n}$

   d. $a_n = 5\left(\frac{1}{2}\right)^n$

2. Write an explicit formula for each sequence.

   a. 2, 4, 8, 16, . . .

   b. 4, 9, 14, 19, . . .

   c. 5, 12, 19, 26, . . .

   d. 2, 5, 10, 17, . . .

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Problem 4 Recursive Formulas

For some sequences, it is easier to describe the pattern in terms of what operations are performed to calculate the next term from the previous term. A **recursive formula** is a formula for defining all terms of a sequence in terms of the previous terms.

For example, consider the sequence 10, 15, 20, 25, … . Each term of the sequence is calculated by adding 5 to the previous term. The sequence can be defined recursively as:

$a_1 = 10, a_n = a_{n-1} + 5$

Notice that a recursive formula has two parts: the first term and a formula for calculating each term.

1. Use each recursive formula to generate the first four terms of the sequence.

a. $a_1 = 3, a_n = 2a_{n-1}$

b. $a_1 = 23, a_n = a_{n-1} + 8$

c. $a_1 = 1, a_n = 3a_{n-1} - 2$

d. $a_1 = 7, a_n = 3a_{n-1} - 2$

e. $a_1 = 2, a_n = (a_{n-1})^2$

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2. Write a recursive formula for each sequence.

a. 3, 7, 11, 15, . . .

b. 3, 7, 15, 31, . . .

c. 1, 2, 5, 26, . . .

d. $\frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, \ldots$

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Be prepared to share your solutions and methods.

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2.3 Adding or Multiplying
Arithmetic and Geometric Sequence

Objectives

In this lesson you will:

- Determine the initial term and the common difference for arithmetic sequences.
- Define arithmetic sequences using explicit and recursive formulas.
- Determine the initial term and the common ratio for geometric sequences.
- Define geometric sequences using explicit and recursive formulas.

Key Terms

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

2

Problem 1 Arithmetic Sequences

1. Calculate the first four terms of each sequence.

a. $a_n = 5n$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

b. $a_1 = 2 \quad a_n = a_{n-1} + 3$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

c. $a_n = 2n + 4$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

d. $a_1 = 4 \quad a_n = a_{n-1} - 3$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

2. Write a recursive formula for each sequence.

a. 10, 20, 30, 40, . . . $a_1 =$ $a_n =$

b. −3, −5, −7, −9, . . . $a_1 =$ $a_n =$

c. 1, 0, −1, −2, . . . $a_1 =$ $a_n =$

d. 2, 9, 16, 23, . . . $a_1 =$ $a_n =$

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3. Write an explicit formula for each sequence.

a. 4, 8, 12, 16, . . .

b. 3, 5, 7, 9, . . .

c. 2, 9, 16, 23, . . .

d. 3, 0, −3, −6, . . .

4. What is similar about the sequences in Questions 1 through 3?

2

An **arithmetic sequence** is a sequence in which each term is calculated from the preceding term by adding a constant. This constant is called the **common difference**. An arithmetic sequence is defined by the initial term a_1 and the common difference, d.

For example, consider the sequence that represents the cost of ordering shirts: 23, 31, 39, 47, ... The term number represents the number of shirts ordered and the value of the term represents the cost of ordering that many shirts. The initial term is 23. This represents the cost of ordering 1 shirt. The common difference is 8. For each additional shirt that is ordered, the cost is increasing by $8.

5. Write an explicit formula for the sequence that represents the cost of ordering shirts.

6. Write the formula from Question 5 as a function.

7. What is the y-intercept of the function? Is this the same as the initial term of the sequence? Why or why not?

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8. Generate the first four terms of each arithmetic sequence. Then write a recursive formula and an explicit formula.

a. Initial term of 3 and common difference of 2

b. Initial term of 3 and common difference of −2

c. Initial term of 10 and common difference of −3

d. Initial term of a_1 and common difference of d.

9. For each arithmetic sequence, identify the common difference. Then write a recursive formula and an explicit formula.

a. 3, 6, 9, 12, . . .

b. 3, 5, 7, 9, . . .

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c. 12, 7, 2, −3, . . .

d. 1, 9, 17, 25, . . .

e. $\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}, \ldots$

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Problem 2 Geometric Sequences

1. Calculate the first four terms of each sequence.

Take Note

$a^0 = 1$ by definition of a zero exponent.

a. $a_n = 3 \cdot 2^{n-1}$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

b. $a_1 = 2 \quad a_n = 3a_{n-1}$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

c. $a_n = -2\left(\frac{1}{2}\right)^{n-1}$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

d. $a_1 = -1 \quad a_n = -2a_{n-1}$

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

2. Write a recursive formula for each sequence.

a. 1, 10, 100, 1000, . . . a_1 a_n

b. $3, -1, \frac{1}{3}, -\frac{1}{9}, \ldots$ a_1 a_n

c. $3, 2, \frac{4}{3}, \frac{8}{3}, \ldots$ a_1 a_n

d. 10, 1, 0.1, 0.01, . . . a_1 a_n

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3. Write an explicit formula for each sequence.

   a. 1, 2, 4, 8, . . .

   b. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$

   c. 2, −6, 18, −54, . . .

   d. 11, 1.1, 0.11, 0.011, . . .

4. What is similar about the sequences in Questions 1 through 3?

A **geometric sequence** is a sequence in which each term is calculated from the preceding term by multiplying by a constant. This constant is called the **common ratio**. A geometric sequence is defined by the initial term, g_1, and the common ratio, r.

5. Generate the first four terms of each geometric sequence. Then write a recursive formula and an explicit formula.

   a. Initial term of −2 and common ratio of 3

   b. Initial term of 1 and common ratio of $\frac{1}{2}$

   c. Initial term of 100 and common ratio of $\frac{1}{4}$

   d. Initial term of g_1 and common ratio of r

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6. For each geometric sequence, identify the common ratio. Then write a recursive formula and an explicit formula.

a. 1, 6, 36, 216, . . .

b. $\frac{1}{4}, -\frac{1}{2}, 1, -2$

c. 12, 6, 3, $\frac{3}{2}$, . . .

d. $\frac{9}{4}, -\frac{3}{2}, 1, -\frac{2}{3}$, . . .

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Problem 3 Classifying Sequences

Classify each sequence as arithmetic, geometric, or neither.

For each arithmetic sequence, identify the common difference. Then write a recursive formula and an explicit formula.

For each geometric sequence, identify the common ratio. Then write a recursive formula and an explicit formula.

1. $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \ldots$

2. 12, 14.5, 17, 19.5, . . .

3. 1, 3, 7, 15, . . .

4. 12, 8, 4, 0

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5. 0, 3, 8, 15, . . .

6. −1, 5, −25, 125, . . .

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Be prepared to share your solutions and methods.

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2.4 Home, Home on the Domains and Ranges
Domains and Ranges of Algebraic Functions

Objective

In this lesson you will:

- Determine the domains and ranges of functions.

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Problem 1

A company makes boxes of various sizes. Each box is a rectangular prism with a height of 8 inches and a volume of 1200 cubic inches.

1. Let x represent the width of a box. Write a function $f(x)$ to model the length of the box.

2. What type of function is $f(x)$?

3. Use $f(x)$ to calculate the following.

a. What is the length of a box with a width of 10 inches?

b. $f(15)$

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c. $f(1)$

d. What is the width of a box with a length of 5 inches?

e. What is the value of x if $f(x) = 100$?

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f. What is the value of x if $f(x) = 2$?

g. Is it possible to make a box so that the length and width are equal? If so, what is the common value of the length and width?

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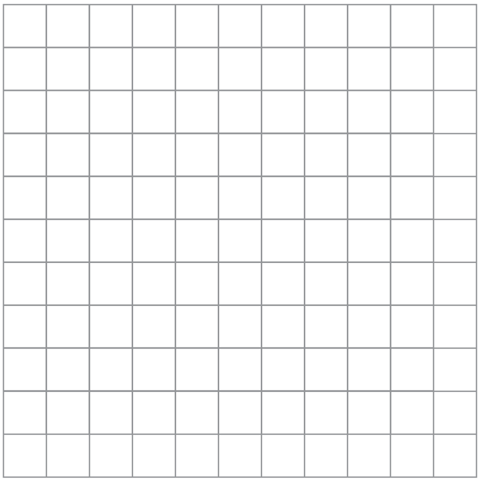
4. Enter the values from Question 3 in the following table.

Labels		
Units		
Expression		

5. Create a scatter plot of the function on the grid shown.



6. Draw a curve that represents all possible lengths and widths. Why does it make sense to connect the points of the scatter plot?

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7. What happens to the length as the width gets very large?

8. Is it possible to make a box with a length of zero inches? How do you know based on the problem situation? How do you know based on the graph?

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9. What happens to the length as the width gets very small?

10. Is it possible to make a box with a width of zero inches? How do you know based on the problem situation? How do you know based on the graph?

11. Does the graph intersect the *x*- or *y*-axis? What does this mean in terms of the problem situation?

12. What are the domain and range of $f(x)$ in this problem situation?

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Problem 2

1. Use the function from Problem 1 to calculate the following.

a. $f(-15)$

b. $f(-1.5)$

c. $f(-25)$

d. $f(x) = -5$

e. $f(x) = -10$

f. $f(x) = -1$

2. Do the values from Question 1 make sense in terms of the problem situation? Explain.

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3. Graph the function using a graphing calculator. Set the x-bounds of the graph from −100 to 100.

4. How is the graph in Question 3 different from the graph in Problem 1 Question 5? How is it similar?

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5. Is x ever equal to 0? Explain using the graph and the function.

6. Is $f(x)$ ever equal to 0? Explain using the graph and the function.

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7. Examining your graph, what appears to be happening as x approaches zero from the right? from the left?

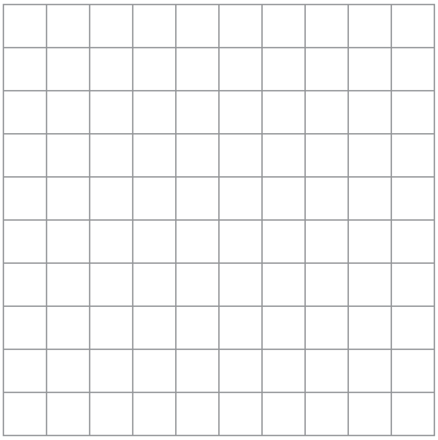
8. Does the graph intersect the x- or y-axis? Explain.

9. What are the domain and range of the function?

Problem 3

Graph each function. Then identify the type of function and determine the domain and range of the function.

1. $g(x) = 2x - 3$



Type of function:

Domain:

Range:

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2. $f(x) = \sqrt{2x}$

Type of function:

Domain:

Range:

3. $h(x) = |x - 1|$

Type of function:

Domain:

Range:

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4. $g(x) = -2x^2$

Type of function:

Domain:

Range:

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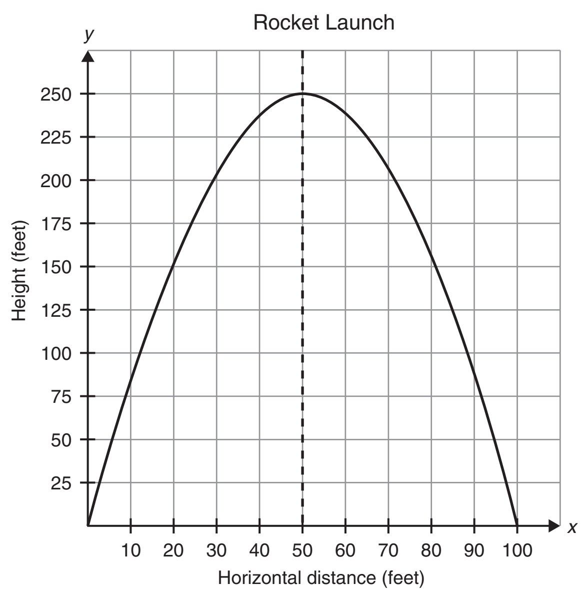
2.5 Rocket Man
Extrema and Symmetry

Objectives
In this lesson you will:
- Determine *x*- and *y*-intercepts of graphs.
- Calculate extreme points of graphs.
- Identify lines of symmetry of graphs.

Problem 1

The graph shows the horizontal distance from launch and the height of a rocket.



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1. What is the height of the rocket when the horizontal distance from launch is

   a. 20 feet?

   b. 40 feet?

   c. 90 feet?

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2. What is the horizontal distance of the rocket from launch when the height is

   a. 125 feet?

   b. 200 feet?

   c. 300 feet?

3. What is the rocket's horizontal distance from launch when it is on the ground?

4. What is the maximum height of the rocket? What is the rocket's horizontal distance from launch when it is at this maximum height?

5. What term is used to describe the point from Question 4? Why?

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6. For what values of x is the graph increasing? Decreasing?

7. Is the graph increasing or decreasing when $x = 50$?

8. Draw a vertical line at $x = 50$. What term is used to describe this line? Why?

9. Examine your answers to each part of Questions 2 and 3. What do you notice about your answers and the line you drew in Question 8?

10. Do all functions have **extreme points**? Do all functions have **lines of symmetry**? Explain.

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11. Can a function have a line of symmetry but not an extreme point? If so, describe the function.

12. Can a function have an extreme point but not a line of symmetry? If so, describe the function.

Problem 2

For each function:

- Sketch a graph.
- Determine the domain and range.
- Determine the *x*- and *y*-intercept(s) and label each on the graph.
- Determine any vertical or horizontal lines of symmetry and draw each on the graph.
- Determine all extreme points and label each on the graph.

1. $f(x) = |2x - 4|$

Domain:

Range:

x-intercept(s):

y-intercept(s):

Line of symmetry:

Extreme point:

2. $f(x) = x^2 - 6x - 27$

Domain:

Range:

x-intercept(s):

y-intercept(s):

Line of symmetry:

Extreme point:

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3. $f(x) = 2 + \sqrt{x}$

Domain:

Range:

x-intercept(s):

y-intercept(s):

Line of symmetry:

Extreme point:

4. $f(x) = 5x - 6$

Domain:

Range:

x-intercept(s):

y-intercept(s):

Line of symmetry:

Extreme point:

5. $f(x) = \dfrac{20}{x}$

Domain:

Range:

x-intercept(s):

y-intercept:

Line of symmetry:

Extreme point:

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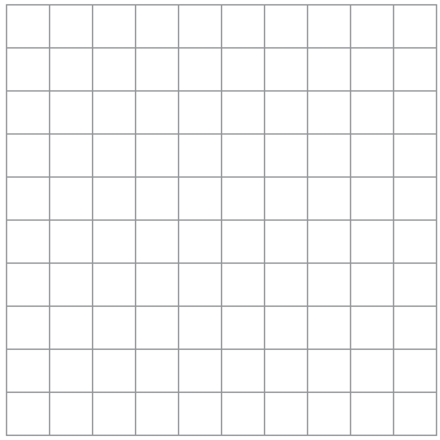
6. $f(x) = x^3 - x$



Domain:

Range:

x-intercept(s):

y-intercept(s):

Line of symmetry:

Extreme point:

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Be prepared to share your solutions and methods.

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2.6 Changing Change
Rates of Change of Functions

Objectives

In this lesson you will:

- Calculate average rates of change of functions.
- Understand the relationship between average rate of change and slope.
- Describe the average rates of change for different functions.

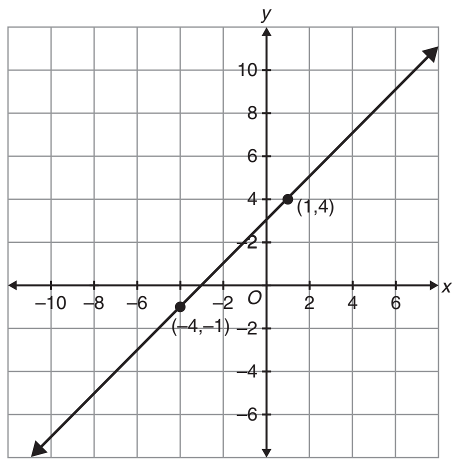
Key Term

- average rate of change

2

Problem 1

The graph of the function $g(x) = x + 3$ is shown on the grid. The points (1, 4) and (−4, −1) are labeled.



1. What type of function is $g(x)$?

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2. Complete the following table.

a. Locate three other points and add these points to the table.

b. Calculate the change in the x-values of consecutive points, Δx, by subtracting the x-value in the preceding row from the x-value in the current row.

c. Calculate the change in the y-values of consecutive points, Δy, by subtracting the y-value in the preceding row from the y-value in the current row.

d. Calculate the average rate of change between consecutive points, $\frac{\Delta y}{\Delta x}$.

x	y	Δx	Δy	$\frac{\Delta y}{\Delta x}$
1	4			
−4	−1	−5	−5	1
−3				

3. What do you notice about the average rates of change for $g(x)$?

4. Complete the table for the linear function $f(x) = -2x + 1$.

x	y	Δx	Δy	$\frac{\Delta y}{\Delta x}$
1				
2				
−3				
0				
−2				

5. What do you notice about the average rates of change for $f(x)$?

6. What can you conclude about the average rates of change for linear functions?

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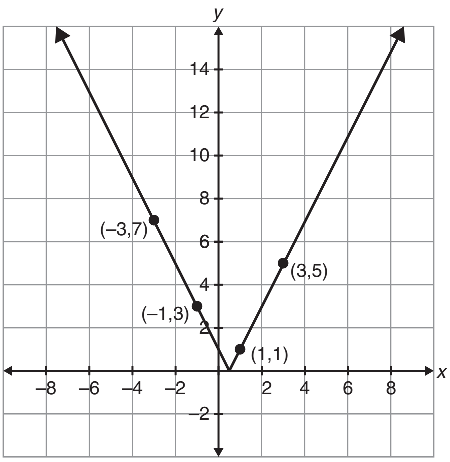
7. What is another term for the average rate of change for a linear function?

Problem 2

The graph of the function $k(x) = |2x - 1|$ is shown on the grid. The points (1, 1), (3, 5), (−1, 3), and (−3, 7) are labeled.



1. What type of function is $k(x)$?

2. Calculate the average rate of change between the two leftmost points.

3. Calculate the average rate of change between the two rightmost points.

4. What can you conclude about the average rate of change for the right part of the graph?

5. What can you conclude about the average rate of change for the left part of the graph?

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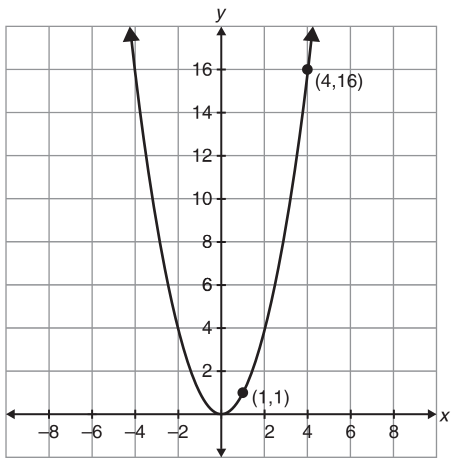
6. What can you conclude about the average rates of change for absolute value functions?

Problem 3

The graph of the function $p(x) = x^2$ is shown on the grid. The points (1, 1) and (4, 16) are labeled.



1. What type of function is $p(x)$?

2. Calculate the average rate of change between the points (1, 1) and (4, 16).

3. Draw a line between the points (1, 1) and (4, 16). What is the slope of the line?

4. How does the slope of the line compare to the average rate of change between the two points?

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5. Evaluate $p(x)$ for $x = 2$. Represent the result as an ordered pair.

6. Calculate each average rate of change.

a. Between the point from Question 5 and (1, 1)

a. Between the point from Question 5 and (4, 16)

7. Draw a line between the point from Question 5 and (1, 1). Draw another line between the point from Question 5 and (4, 16). What is the slope of each line?

8. How does the slope of each line compare to the average rate of change between the two points?

9. What do you think will be true about the average rate of change between two points to the left of the minimum point?

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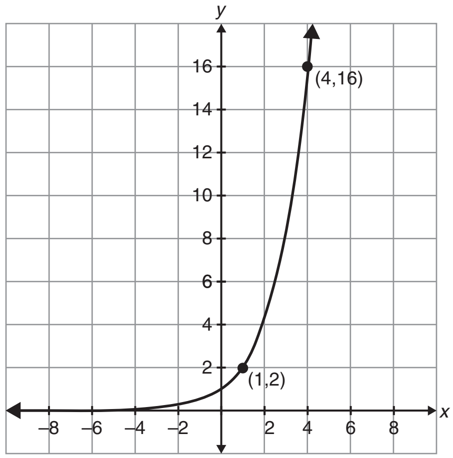
10. What can you conclude about the average rates of change for quadratic functions?

Problem 4

The graph of the function $q(x) = 2^x$ is shown on the grid. The points (1, 2) and (4, 16) are labeled.



1. What type of function is $q(x)$?

2. Calculate the average rate of change between the points (1, 2) and (4, 16).

3. Draw a line between the points (1, 2) and (4, 16). What is the slope of the line?

4. How does the slope of the line compare to the average rate of change between the two points?

5. Evaluate $q(x)$ for $x = 3$. Represent the result as an ordered pair.

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6. Calculate each average rate of change.

   a. Between the point from Question 5 and (1, 2).

   b. Between the point from Question 5 and (4, 16).

7. Draw a line between the point from Question 5 and (1, 2). Draw another line between the point from Question 5 and (4, 16). What is the slope of each line?

8. How does the slope of each line compare to the average rate of change between the two points?

9. What can you conclude about the average rates of change for exponential functions?

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Problem 5

Complete the table to summarize the properties of several basic functions.

Function Type	Linear	Absolute Value	Quadratic
Equation of Basic Function			
Graph			
Domain and Range			
Extrema			
Intervals of Increasing and Decreasing			
Vertical Line of Symmetry			

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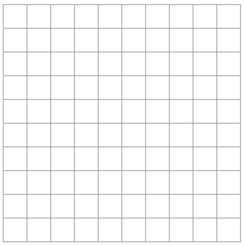
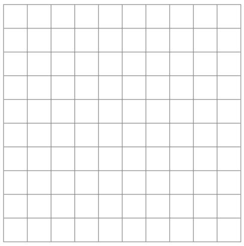
Function Type	Cubic	Indirect Variation
Equation of Basic Function		
Graph		
Domain and Range		
Extrema		
Intervals of Increasing and Decreasing		
Vertical Line of Symmetry		

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Function Type	Square Root	Exponential
Equation of Basic Function		
Graph		
Domain and Range		
Extrema		
Intervals of Increasing and Decreasing		
Vertical Line of Symmetry		

Be prepared to share your solutions and methods.

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