CHAPTER

3 Logic

Riding a bicycle is a skill which, once learned, is rarely forgotten. What's more, bicycles are enough alike that if you can ride one bike, you can pretty much ride them all. This is an example of inductive reasoning, which is applying knowledge and experience learned in one situation to another situation. You will learn about inductive reasoning and how to apply it to mathematical problems.

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3.1 A Little Dash of Logic
Two Methods of Logical Reasoning

Objectives

In this lesson you will:

- Define inductive reasoning and deductive reasoning.
- Identify methods of reasoning.
- Compare and contrast methods of reasoning.
- Create examples using inductive and deductive reasoning.

Key Terms

- inductive reasoning
- deductive reasoning

Problem 1

1. Emma is watching her big sister do homework. She notices the following:

- $4^2 = 4 \times 4$
- nine cubed is equal to nine times nine times nine
- 10 to the fourth power is equal to four factors of 10 multiplied together

Emma concludes that raising a number to a power is the same as multiplying the number by itself as many times as indicated by the power. How did Emma reach this conclusion?

2. Ricky read that exponents mean repeated multiplication. He had to enter seven to the fourth power in a calculator but could not find the exponent button. So, he entered $7 \times 7 \times 7 \times 7$ instead. How did Ricky reach this conclusion?

3. Contrast the reasoning used by Emma and Ricky.

4. Was Emma's conclusion correct? Was Ricky's conclusion correct?

5. Jennifer's mother is a writing consultant. She was paid \$900 for a ten-hour job and \$1980 for a twenty two-hour job.

   a. How much does Jennifer's mother charge per hour?

   b. To answer Question 5(a), did you start with a general rule and make a conclusion or did you start with specific information and create a general rule?

6. Your friend Aaron tutors elementary school students. He tells you that the job pays \$8.25 per hour.

   a. How much does Aaron earn if he works 4 hours?

   b. To answer Question 6(a), did you start with a general rule and make a conclusion or did you start with specific information and create a general rule?

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Problem 2

The ability to use information to reason and make conclusions is very important in life and in mathematics. This lesson focuses on two methods of reasoning. You can construct the vocabulary for each type of reasoning by thinking about what prefixes, root words, and suffixes mean.

Look at the following information. Remember that a prefix is a beginning of a word. A suffix is an ending of a word.

- *in*—a prefix that can mean *toward* or *up to*
- *de*—a prefix that can mean *down from*
- *duc*—a base or root word meaning *to lead* and often *to think*, from the Latin word *duco*
- *-tion*—a suffix that forms a noun meaning *the act of*

1. Form a word that means "the act of thinking down from."

2. Form a word that means "the act of thinking toward or up to."

Inductive reasoning is reasoning that involves using specific examples to make a conclusion. Many times in life you must make generalizations about observations or patterns and apply these generalizations to unfamiliar situations. For example, you learn how to ride a bike by falling down, getting back up, and trying again. Eventually, you are able to balance on your own. After learning to ride your own bike, you can apply that knowledge and experience to ride an unfamiliar bike.

Deductive reasoning is reasoning that involves using a general rule to make a conclusion. For example, you learn the rule for which direction to turn a screwdriver: "righty tighty, lefty loosey." If you want to unscrew a screw, you apply the rule and turn counterclockwise.

3. Look back at Problem 1. Who used inductive reasoning?

4. Who used deductive reasoning?

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Problem 3

1. Your best friend reads a newspaper article that states that use of tobacco greatly increases the risk of cancer. He then notices that his neighbor Matilda smokes. He is concerned that Matilda has a high risk of cancer.

a. What is the specific information in this problem?

b. What is the general information in this problem?

c. What is the conclusion in this problem?

d. Did your friend use inductive or deductive reasoning to make the conclusion? Explain.

e. Is your friend's conclusion correct? Explain.

2. Molly returned from a trip to London and tells you, "It rains every day in England!" She explains that it rained each of the five days she was there.

a. What is the specific information in this problem?

b. What is the general information in this problem?

c. What is the conclusion in this problem?

d. Did Molly use inductive or deductive reasoning to make the conclusion? Explain.

e. Is Molly's conclusion correct? Explain.

3. You take detailed notes in history class and math class. A classmate is going to miss biology class tomorrow to attend a field trip. His biology teacher asks him if he knows someone in class who always takes detailed notes. He gives your name to the teacher. The biology teacher suggests he borrow your biology notes because he concludes that they will be detailed.

a. What conclusion did your classmate make? Why?

b. What type of reasoning did your classmate use? Explain.

c. What conclusion did the biology teacher make? Why?

d. What type of reasoning did the biology teacher use? Explain.

e. Will your classmate's conclusion always be true? Will the biology teacher's conclusion always be true? Explain.

4. The first four numbers in a sequence are 4, 15, 26, and 37.

a. What is the next number in the sequence? How did you calculate the next number?

b. What types of reasoning did you use and in what order to make the conclusion?

5. Write a short note to a friend explaining induction and deduction. Include definitions of both terms and examples that are very easy to understand.

Problem 4

There are two reasons why a conclusion may be false. Either the assumed information is false or the argument is not valid.

1. Derek tells his little brother that it will not rain for the next thirty days because he "knows everything." Why is this conclusion false?

2. Two lines are not parallel so the lines must intersect. Why is this conclusion false?

3. Write an example of a conclusion that is false because the assumed information is false.

4. Write an example of a conclusion that is false because the argument is not valid.

Be prepared to share your solutions and methods.

3.2 What's Your Conclusion?
Hypotheses, Conclusions, Conditional Statements, Counterexamples, Direct and Indirect Arguments

Objectives

In this lesson you will:

- Define a conditional statement.
- Identify the hypothesis and conclusion of a conditional statement.
- Construct direct and indirect arguments.

Key Terms

- conditional statement
- hypothesis
- conclusion
- proof by contrapositive
- direct argument
- counterexample
- indirect argument

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Problem 1

Read each pair of statements. Then write a valid conclusion.

1. Statement: Melanie's school guidance counselor tells her that if she applies for a scholarship, she will have a chance to receive it.
Statement: Melanie applies for a scholarship.
Conclusion:

2. Statement: If it rains, the baseball game will be cancelled.
Statement: It rains.
Conclusion:

3. Statement: If Suzanne misses the application deadline for the Vocational Training School, she will not be admitted.
Statement: Suzanne missed the application deadline.
Conclusion:

4. Statement: Marvin will know whether he enjoys waltz lessons if he attends his first waltz lesson.
Statement: Marvin attended his first waltz lesson.
Conclusion:

5. Statement: If having the most experience as a nuclear engineer had been the main requirement, then Olga would have gotten the job.
Statement: Olga did not get the job.
Conclusion:

Problem 2

Read each statement and conclusion. Then write the additional statement required to reach the conclusion.

1. Statement: If no evidence can be found linking a suspect to the scene of a crime, then the suspect will be found innocent.
   Statement:

   Conclusion: Therefore, the suspect was found innocent.

2. Statement: If the community service program chooses the litter removal project, then Mayor Elder will have the carnival in this neighborhood.
   Statement:

   Conclusion: Therefore, the community service program did not choose the litter removal project.

3. Statement: If clematis flowers are to survive, then they need sunlight.
   Statement:
   Conclusion: Therefore, the clematis flowers died.

4. Statement: The Secret Service will be at the dinner if the President shows up.
   Statement:
   Conclusion: Therefore, the President did not show up at the dinner.

5. Statement: If
   Statement: You have your umbrella.
   Conclusion: Therefore, it must have been raining.

6. Statement: If

   Statement: You ate a good breakfast.
   Conclusion: Therefore, you will not be hungry before lunch.

7. Statement: If

   Statement: You did your math homework.
   Conclusion: Therefore, your teacher is happy.

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Problem 3

A **conditional statement** is a statement that can be written in the form "If p, then q." The portion of the statement represented by p is the **hypothesis**. The portion of the statement represented by q is the **conclusion**.

For example, in the conditional statement "If a credit card has no annual fee and a low interest rate, then Samantha will consider applying for it," the hypothesis is "A credit card has no annual fee and a low interest rate" and the conclusion is "Samantha will consider applying for it."

1. The first statement of each question in Problems 1 and 2 is a conditional statement. For each, underline the hypothesis with a solid line. Then underline the conclusion with a dashed line.

   One way to prove the statement "If p, then q" is by using the argument "If not q, then not p." Another way to say this is "If q is false, then p is false." This type of argument is called **proof by contrapositive**.

2. Which conclusions in Problems 1 and 2 used proof by contrapositive? Print the word *contrapositive* beside each conclusion that used proof by contrapositive.

The remaining questions in Problems 1 and 2 use a form of proof called a **direct argument**. Each direct argument includes a conditional statement, a second statement formed by the hypothesis of the conditional statement, and a conclusion formed by the conclusion of the conditional statement.

Problem 4

A friend asks you to check her homework. One problem required her to simplify the expression $(x + y)^2$. She wrote $(x + y)^2 = x^2 + y^2$.

Your friend's work is a conditional statement even though it is not written in the form "If p, then q." However, it can be rewritten as "If x and y are numbers, then $(x + y)^2 = x^2 + y^2$."

1. Prove whether the conditional statement is true or false.

a. Choose two numbers for x and y, neither of which is 0.

b. Substitute your values for x and y into both sides of the equation. Then simplify each side of the equation.

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c. Is your friend's conditional statement true for all numbers? Explain.

A specific example that shows that a conditional statement is false is called a **counterexample**. Using a counterexample is a form of proof called an **indirect argument**.

2. How many counterexamples does it take to show that a conditional statement is false?

3. Is the statement "$a(b + c) = (a + b)c$ for all values of a, b, and c" true or false? If it is false, provide a counterexample.

4. Ardella notices that 3, 5, and 7 are all odd numbers and also all prime numbers. She proposes that all odd numbers are prime numbers.

   a. Did Ardella use inductive or deductive reasoning? Explain.

   b. Is Ardella correct? If she is incorrect, provide a counterexample.

Problem 5

1. What are two ways that deductive reasoning can result in a false conclusion?

2. What is the minimum that you need to do to prove that an assertion is false?

3. What kind of logical reasoning does not try to guarantee that its conclusion is true?

4. Write a false assertion. Then provide a counterexample to prove that it is false.

5. Write a conditional statement and underline the hypothesis with a solid line and the conclusion with a dashed line.

6. Write a direct argument using your conditional statement in Question 5. Remember that a direct argument includes a conditional statement, a second statement formed by the hypothesis of the conditional statement, and a conclusion formed by the conclusion of the conditional statement.

7. Write an indirect argument using your conditional statement in Question 5. Remember that an indirect argument uses the conditional statement, a second statement formed by the negation of the conclusion, and a conclusion formed by the negation of the hypothesis.

Be prepared to share your solutions and methods.

3.3 You Can't Handle the Truth (Table)

Converses, Inverses, Contrapositives, Biconditionals, Truth Tables, Postulates, and Theorems

Objectives

In this lesson you will:

- Explore the truth value of conditional statements.
- Use a truth table.
- Write the converse of a conditional statement.
- Write the inverse of a conditional statement.
- Write the contrapositive of a conditional statement.
- Write a biconditional statement.
- Differentiate between postulates and theorems.

Key Terms

- propositional form
- propositional variables
- truth value
- truth table
- converse
- inverse
- contrapositive
- logically equivalent
- biconditional statement
- postulate
- theorem

Previously, you learned that a conditional statement is a statement that can be written in the form "if *p*, then *q*." This form of a conditional statement is called the **propositional form.** The variable *p* represents the hypothesis and the variable *q* represents the conclusion. The variables *p* and *q* are **propositional variables.** The **truth value** of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then its truth value is considered "true."

Problem 1 Truth Tables

Consider the following direct argument.

Statement: If I am 18 years old, then I can vote.

Statement: I am 18 years old.

Conclusion: Therefore, I can vote.

1. What is the conditional statement?

2. What is the hypothesis, *p*? Underline the hypothesis with a solid line.

3. What is the conclusion, *q*? Underline the conclusion with a dashed line.

4. If *p* is true and *q* is true, then the truth value of a conditional statement is "true." Use these truth values to explain why the conditional statement is always true.

5. If *p* is true and *q* is false, then the truth value of a conditional statement is "false." Use these truth values to explain why the conditional statement is always false.

6. If *p* is false and *q* is true, then the truth value of a conditional statement is "true." Use these truth values to explain why the conditional statement could be true.

7. If p is false and q is false, then the truth value of a conditional statement is "true." Use these truth values to explain why the conditional statement could be true.

Questions 4 through 7 can be summarized using a **truth table** for p and q as shown. The first two columns of the truth table represent the possible truth values for p and q. The last column represents the truth value of the conditional statement ($p \mapsto q$). Notice that the truth value of a conditional statement is either "true" or "false," but not both.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

8. Consider the conditional statement "If an animal is a lion, then the animal is a mammal."

a. What is the hypothesis, p?

b. What is the conclusion, q?

c. Assume that p is true and q is true. What does that mean?

d. Could this statement be true? What is the truth value of the conditional statement when p is true and q is true?

e. Assume that p is true and q is false. What does that mean?

f. Could this statement be true? What is the truth value of the conditional statement when p is true and q is false?

g. Assume that p is false and q is true. What does that mean?

h. Could this statement be true? What is the truth value of the conditional statement when p is false and q is true?

i. Assume that p is false and q is false. What does that mean?

j. Could this statement be true? What is the truth value of the conditional statement when p is false and q is false?

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Problem 2 Converses

The **converse** of a conditional statement of the form "If p, then q" is the statement of the form "If q, then p." The converse is a new statement that results when the hypothesis and conclusion of the conditional statement are switched.

For each conditional statement written in propositional form, identify the hypothesis, p, and conclusion, q. Then write the converse of the conditional statement.

1. If a quadrilateral is a square, then the quadrilateral is a rectangle.

   a. Hypothesis, p:

   b. Conclusion, q:

   c. Is the conditional statement true? Explain.

   d. Converse:

   e. Is the converse true? Explain.

2. If an integer is even, then the integer is divisible by two.

a. Hypothesis, *p*:

b. Conclusion, *q*:

c. Is the conditional statement true? Explain.

d. Converse:

e. Is the converse true? Explain.

3. If a polygon has six sides, then the polygon is a pentagon.

a. Hypothesis, *p*:

b. Conclusion, *q*:

c. Is the conditional statement true? Explain.

d. Converse:

e. Is the converse true? Explain.

4. If two lines intersect, then the lines are perpendicular.

a. Hypothesis, *p*:

b. Conclusion, *q*:

c. Is the conditional statement true? Explain.

d. Converse:

e. Is the converse true? Explain.

5. What do you notice about the truth value of a conditional statement and the truth value of its converse?

Problem 3 Inverses

The **inverse** of a conditional statement of the form "If *p*, then *q*" is the statement of the form "If not *p*, then not *q*." The inverse is a new statement that results when the hypothesis and conclusion of the conditional statement are negated.

For each conditional statement written in propositional form, identify the hypothesis *p* and conclusion *q*. Then identify the negation of the hypothesis and conclusion and write the inverse of the conditional statement.

1. If a quadrilateral is a square, then the quadrilateral is a rectangle.

   a. Hypothesis, *p*:

   b. Conclusion, *q*:

   c. Is the conditional statement true? Explain.

   d. Not *p*:

   e. Not *q*:

   f. Inverse:

   g. Is the inverse true? Explain.

2. If an integer is even, then the integer is divisible by two.

   a. Hypothesis, *p*:

   b. Conclusion, *q*:

   c. Is the conditional statement true? Explain.

d. Not p:

e. Not q:

f. Inverse:

g. Is the inverse true? Explain.

3. If a polygon has six sides, then the polygon is a pentagon.

   a. Hypothesis, p:

   b. Conclusion, q:

   c. Is the conditional statement true? Explain.

   d. Not p:

   e. Not q:

   f. Inverse:

   g. Is the inverse true? Explain.

4. If two lines intersect, then the lines are perpendicular.

   a. Hypothesis, p:

   b. Conclusion, q:

   c. Is the conditional statement true? Explain.

   d. Not p:

   e. Not q:

   f. Inverse:

   g. Is the inverse true? Explain.

5. What do you notice about the truth value of a conditional statement and the truth value of its inverse?

Problem 4 Contrapositives

The **contrapositive** of a conditional statement of the form "if *p*, then *q*" is the statement of the form "if not *q*, then not *p*." The contrapositive is a new statement that results when the hypothesis and conclusion of the conditional statement are negated and switched.

For each conditional statement written in propositional form, identify the negation of the hypothesis and conclusion and write the contrapositive of the conditional statement.

1. If a quadrilateral is a square, then the quadrilateral is a rectangle.

   a. Is the conditional statement true? Explain.

   b. Not *p*:

   c. Not *q*:

   d. Contrapositive:

   e. Is the contrapositive true? Explain.

2. If an integer is even, then the integer is divisible by two.

   a. Is the conditional statement true? Explain.

   b. Not *p*:

   c. Not *q*:

   d. Contrapositive:

   e. Is the contrapositive true? Explain.

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3. If a polygon has six sides, then the polygon is a pentagon.

   a. Is the conditional statement true? Explain.

   b. Not *p*:

   c. Not *q*:

   d. Contrapositive:

   e. Is the contrapositive true? Explain.

4. If two lines intersect, then the lines are perpendicular.

   a. Is the conditional statement true? Explain.

   b. Not *p*:

   c. Not *q*:

   d. Contrapositive:

   e. Is the contrapositive true? Explain.

5. What do you notice about the truth value of a conditional statement and the truth value of its contrapositive?

Problem 5

1. Do you agree or disagree with each statement? If you disagree, provide a counterexample.

   a. If a conditional statement is true, then its converse is true.

   b. If a conditional statement is true, then its inverse is true.

   c. If a conditional statement is true, then its contrapositive is true.

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Two propositional forms are **logically equivalent** if they have the same truth values for corresponding values of the propositional variables.

2. Look at the four conditional statements used in Problems 2 through 4. Which conditional statement contained the most examples of logically equivalent relationships?

Problem 6

The negation of a statement, p, is logically equivalent to the statement "It is not true that p." The negation of a statement, p, is represented as "not p" or $\sim p$.

1. If the truth value of p is "true," what is the truth value of $\sim p$?

2. If the truth value of p is "false," what is the truth value of $\sim p$?

3. Complete the following truth table.

				Conditional	
p	$\sim p$	q	$\sim q$	$p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T		T			
T		F			
F		T			
F		F			

4. What do you notice about the last two columns?

5. The truth table proves that a conditional statement is logically equivalent to what other propositional form?

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Problem 7 Biconditional Statements

When a conditional statement and its converse are both true, they can be combined and written as a single statement using "if and only if." This new statement is called a **biconditional statement**.

For example:

Conditional Statement: If a quadrilateral has four right angles, then the quadrilateral is a rectangle.

Converse: If a quadrilateral is a rectangle, then the quadrilateral has four right angles.

The conditional statement and its converse are both true. So, they can be rewritten as a biconditional statement.

Biconditional: A quadrilateral has four right angles if and only if the quadrilateral is a rectangle.

For each conditional statement, write the converse. If possible, write a true biconditional statement. If it is not possible, explain why.

1. If a triangle has at least two congruent sides, then the triangle is isosceles.

a. Converse:

b. Biconditional:

2. If two lines are parallel, then the two lines do not intersect.

a. Converse:

b. Biconditional:

3. If two circles have equal length radii, then the two circles are congruent.

a. Converse:

b. Biconditional:

4. If a quadrilateral is a square, then the quadrilateral is a rectangle.

a. Converse:

b. Biconditional:

5. If an angle is bisected, then the angle is divided into two angles of equal measure.

a. Converse:

b. Biconditional:

Problem 8 Postulates and Theorems

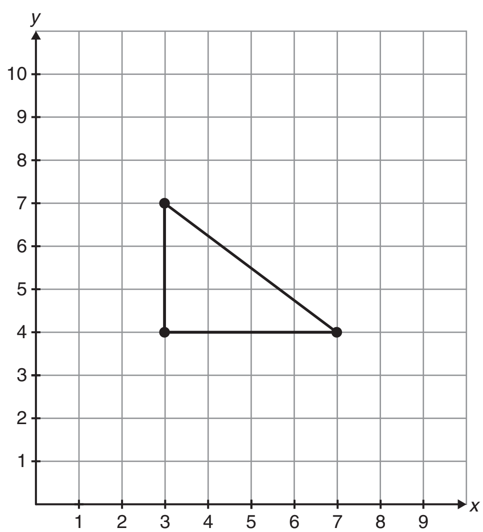
A **postulate** is a statement that is accepted without proof. A **theorem** is a statement that can be proven.

The Elements is a book written by the Ancient Greek mathematician Euclid. He used a small number of postulates to systematically prove many theorems.

Consider Euclid's first three postulates.

- Line Postulate: Exactly one line can be constructed through any two points.
- Line Intersection Postulate: The intersection of two lines is exactly one point.
- Midpoint Postulate: Exactly one midpoint can be constructed in any line segment.

One theorem that Euclid was able to prove is the Pythagorean Theorem, which summarizes the relationship between the three sides of a right triangle. One proof uses the area of squares. Use the graph below to explore proving the Pythagorean Theorem.



1. Construct three squares, each sharing one side of the right triangle.
2. What do you need to know to calculate the area of each square?

3. The length of the horizontal side of the triangle is 4 units. The length of the vertical side of the triangle is 3 units. The length of the longest side of the triangle is 5 units. What is the area of each square?

4. How does the sum of the areas of the small and medium squares compare to the area of the large square?

5. Consider a triangle with side lengths of 6 units, 8 units, and 10 units. What is the area of each square?

6. How does the sum of the areas of the small and medium squares compare to the area of the large square?

7. Consider a right triangle with side lengths of a units, b units, and c units. What is the area of each square?

8. How does the sum of the areas of the small and medium squares compare to the area of the large square?

9. State the Pythagorean Theorem.

10. How many sets of numbers would you need to test to prove this theorem? Explain. Is this possible?

11. How can you prove that a theorem is true for all numbers?

Be prepared to share your solutions and methods.

3.4 Proofs Aren't Just for Geometry

Introduction to Direct and Indirect Proof with the Properties of Numbers

Objectives

In this lesson you will:

- Use the commutative, associative, identity, and inverse laws for addition and multiplication.
- Use the distributive law.
- Use direct proof to prove a theorem.
- Use indirect proof to prove a theorem.

Key Terms

- commutative law
- associative law
- identity law
- inverse law
- distributive law
- proof by contradiction

Problem 1 Direct Proofs and Number Laws

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1. Some conditional statements can be proven using a direct proof. Read the conditional statement and each step of the direct proof. For each step, explain what changed from the previous step.

Conditional Statement: If $a + bc = c(b + a) + a$, then $a = 0$ or $c = 0$

Steps	**What Changed?**
$a + bc = cb + ca + a$	
$a + bc = bc + ca + a$	
$a + bc = ca + a + bc$	
$a + bc - bc = ca + a + bc - bc$	
$a = ca + a$	
$a - a = ca + a - a$	
$0 = ca$	
$a = 0$ or $c = 0$	

Complete the table to summarize the real number laws. The following laws are true for any real numbers a, b, and c.

2.

Name of Law	Symbolic Representation of Law Under Addition	Symbolic Representation of Law Under Multiplication
Commutative	$a + b = b + a$	
Associative		$a(bc) = (ab)c$
Identity	$a + 0 = a$	$a \cdot 1 = a$
Inverse		$a \cdot \frac{1}{a} = 1 \ (a \neq 0)$
Distributive	$a(b + c) = ab + ac$	

3. Complete the direct proof from Question 1. Use the names of the laws from the table. If you cannot find a law that is a good fit, write a statement that summarizes the rule or property of real numbers.

Conditional Statement: If $a + bc = c(b + a) + a$, then $a = 0$ or $c = 0$

Steps **Reasons**

$a + bc = cb + ca + a$

$a + bc = bc + ca + a$

$a + bc = ca + a + bc$

$a + bc - bc$
$= ca + a + bc - bc$

$a = ca + a$

$a - a = ca + a - a$

$0 = ca$

$a = 0$ or $c = 0$

Consider the associative laws $(a + b) + c = a + (b + c)$, and $a(bc) = (ab)c$.

The associative law can be stated in words as:

When three terms are added, the first two terms can be grouped or the last two terms can be grouped.

When three factors are multiplied, the first two factors can be grouped or the last two factors can be grouped.

4. State the commutative law in words.

5. State the identity law in words.

6. State the inverse law in words.

7. State the distributive law in words.

Problem 2 Indirect Proofs

In Problem 1, you used a direct proof to prove the theorem "If $a + bc = c(b + a) + a$, then $a = 0$ or $c = 0$." This theorem can also be proven using an indirect proof called **proof by contradiction**.

To prove a statement using proof by contradiction, assume that the conclusion is false. Then show that the hypothesis is false or a contradiction. This is equivalent to showing that if the hypothesis is true, then the conclusion is also true.

Begin by assuming that $a \neq 0$ and $c \neq 0$. So, let $a = 2$ and $c = 2$. Substitute these values into the equation and simplify.

Complete the indirect proof. Use the names of the real number laws.

Steps	**Reasons**
$2 + 2b = 2(b + 2) + 2$	Assumption (negation of the conclusion)
$2 + 2b = 2b + 4 + 2$	
$2 + 2b = 4 + 2 + 2b$	
$0 = 4$	

This is a contradiction because $0 \neq 4$. So, the theorem must be true.

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Problem 3 Indirect Proofs

1. Examine Kate's solution to a math problem.

   $\frac{ab + c}{a} = b + c$ for all real numbers a, b, and c

   a. Is Kate's solution correct?

   b. Prove your answer to part (a).

2. Prove or disprove the statement $\frac{ab + ac}{a} = b + c$.

3. Identify the error in the following proof.

If $a = 0$, then $5 = 7$.

1. $0 = 0$	All numbers equal themselves.
2. $5 \cdot 0 = 7 \cdot 0$	Zero times any number is equal to zero.
3. $5a = 7a$	Let $a = 0$.
4. $5a + ax = 7a + ax$	Algebraic equations remain true if you perform the same operation on both sides.
5. $a(5 + x) = a(7 + x)$	Distributive law of multiplication with respect to addition.
6. $\frac{a(5 + x)}{a} = \frac{a(7 + x)}{a}$	Algebraic equations remain true if you perform the same operation on both sides.
7. $5 + x = 7 + x$	Inverse law of multiplication
8. $5 = 7$	Algebraic equations remain true if you perform the same operation on both sides.

Problem 4

Complete the table to summarize the real number laws. The following laws are true for any real numbers *a*, *b*, and *c*.

Don't look back!

Name of Law	Symbolic Representation of Law Under Addition	Symbolic Representation of Law Under Multiplication

Be prepared to share your solutions and methods.