CHAPTER

4 Modeling with Functions

Tiles were first used to make roofs in ancient Greece, where they were popular for their fire-resistant properties. Modern tiles are used in many building and decorative applications. You will learn how to use geometric patterns, numeric patterns, and algebraic functions to calculate the number of tiles needed to construct a patterned tile floor.

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4.1 Squares and More
Using Patterns to Generate Algebraic Functions

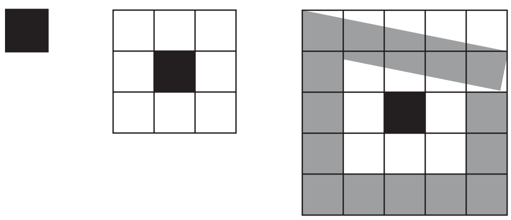
Objectives

In this lesson you will:

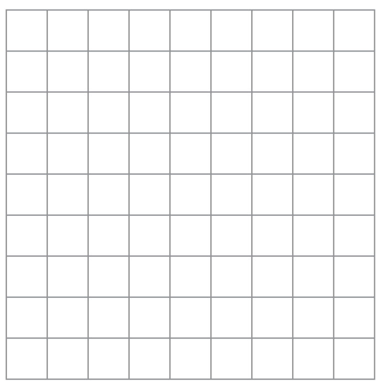
- Generate algebraic functions using numeric and geometric patterns.
- Represent algebraic functions in different forms.

Problem 1

Terrance owns a flooring company. His latest job involves tiling a square room that is 101 feet by 101 feet. The customer requested a pattern of alternating black, white, and grey tiles as shown. Each tile is one square foot.



1. Sketch the design for a square floor that is 9 feet by 9 feet.



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2. Complete the following table to summarize the number and color of tiles used.

Tiles Along Edge of Floor	1	3	5	7	9
Black tiles	1	1	1		
White tiles	0	8	8		
Grey tiles	0	0	16		
New tiles	1	8	16		
Total tiles	1	9	25		

3. How many times will Terrance need to repeat this pattern to complete the room?

4. Continue the table:

Tiles Along Edge of Floor	11	13	15
Black tiles			
White tiles			
Grey tiles			
New tiles			
Total tiles			

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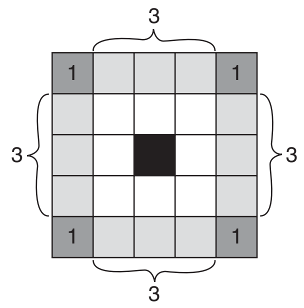
5. Why is the number of tiles along the edge of the floor increasing by 2 each time?

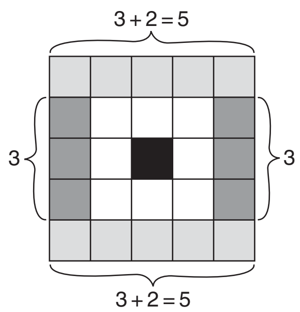
6. How many total tiles will Terrance need to cover the entire room? Explain.

Terrance needs to know how many tiles are required to complete the job. He asks several co-workers how many new tiles must be added to a square with side length *n* tiles to build the next square. Each begins by explaining how many tiles should be added to a square with a side length of 3 tiles. Then each generalizes.

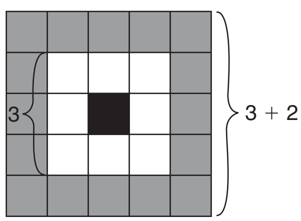
7. Wilma says that you must add 3 tiles to each of the four sides of the white square, which is $4 \cdot 3$ tiles. Then you must add 1 tile at each corner. So, the number of additional tiles added to a 3×3 square is $4 \cdot 3 + 4$. Using Wilma's pattern, write an expression for the number of tiles that must be added to an $n \times n$ square.



8. Howard says that you must add 5 tiles to two of the sides and 3 tiles to the other two sides. So, the number of additional tiles added to a 3×3 square is $2(3 + 2) + 2 \cdot 3$. Using Howard's pattern, write an expression for the number of tiles that must be added to an $n \times n$ square.



9. Kenesha says that you really have two squares. The original square has $3 \cdot 3$ tiles. The newly formed square has $5 \cdot 5$ tiles. So, the number of additional tiles added to a 3×3 square is $5 \cdot 5 - 3 \cdot 3$. Using Kenesha's pattern, write an expression for the number of tiles that must be added to an $n \times n$ square.



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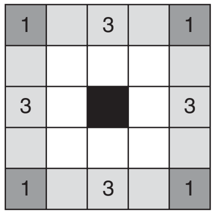
10. Finally, Lebron says that you need to add 3 tiles four times and then add the four corners. So, the number of additional tiles added to a 3×3 square is $3 + 3 + 3 + 3 + 4$. Using Lebron's pattern, write an expression for the number of tiles that must be added to an $n \times n$ square.



11. Use the expressions from Questions 7–10 to calculate the number of tiles that must be added to squares with side lengths of 11 tiles and 13 tiles.

a. Wilma's formula:

b. Howard's formula:

c. Kenesha's formula:

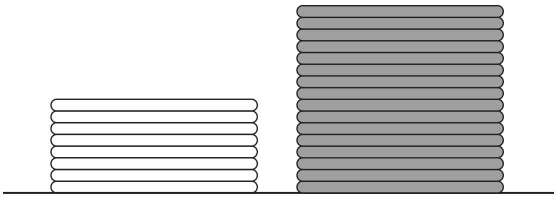
d. Lebron's formula:

12. Whose expression is correct? Explain.

13. Show why the expressions are equivalent.

Problem 2

Now that Terrance knows how many tiles must be added to each square, he wants to create stacks of tiles so that the tiles for each new row added to the black center tile are stacked together. First, Terrance must know the color of the tiles in each stack.



1. Complete the table.

Stack Number	**1**	**2**	**3**	**4**	**5**
Color	white	grey			

2. List the stack numbers that contain white tiles.

3. The white stack numbers can be represented by an arithmetic sequence. Write an explicit formula to represent a general term, a_w, of this sequence.

4. List the stack numbers that contain grey tiles.

5. The grey stack numbers can be represented by an arithmetic sequence. Write an explicit formula to represent a general term, a_g, of this sequence.

6. List the stack numbers that contain black tiles.

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7. The black stack numbers can be represented by an arithmetic sequence. Write an explicit formula to represent a general term, a_b, of this sequence.

8. Use the explicit formulas to determine the color of the tiles in each stack.

a. 33rd stack:

b. 41st stack:

c. 28th stack:

Problem 3

Finally, Terrance must know the number of tiles in each stack.

1. Complete the table.

Stack Number	**1**	**2**	**3**	**4**	**5**	***n***
Tiles Along Edge of Floor	3	5	7			
Number of Tiles in Stack	8	16				

2. Write an expression for the number of tiles along the edge of the floor for the *n*th stack.

3. How many stacks are required to complete the job? Explain.

4. Write an expression for the number of tiles in the *n*th stack.

5. How many tiles are in the last stack? Explain.

6. For each stack, determine the number of tiles in the stack and the side length of the square that is created using that stack.

a. Stack 26:

b. Stack 43:

c. Stack 36:

Be prepared to share your solutions and methods.

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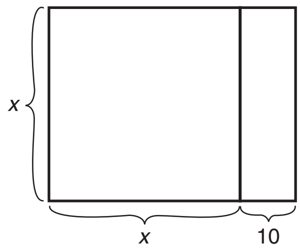
4.2 Areas and Areas
Using Multiple Representations of Algebraic Functions

Objectives
In this lesson you will:
- Represent functions using words, tables, equations, graphs, and diagrams.
- Analyze functions using multiple representations.

Problem 1

A company owns a large plot of land. They want to divide the large plot into smaller plots consisting of square lots, x feet on a side, with a 10-foot-wide driveway along one side as shown.



1. What is the area of each square lot? Label the diagram.

2. What is the area of each driveway? Label the diagram.

3. Use area composition to write an expression for the area of the square lot and the driveway.

4. What is the length of the square lot and the driveway?

5. What is the width of the square lot and the driveway?

6. Use the length and width from Questions 4 and 5 to write an expression for the area of the square lot and the driveway.

7. You wrote the total area in two different ways in Questions 3 and 6. Show how these expressions are equivalent.

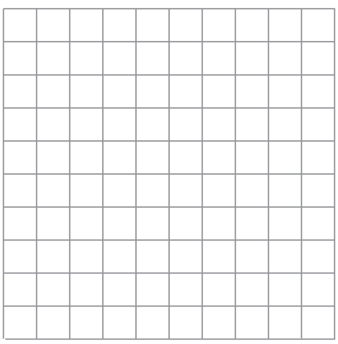
8. Complete the table.

Width of Square Lot	Length of Plot	Area of Square Lot	Area of Driveway	Total Area of Plot
10	20	100	100	200
30				
50				
100				
x				

9. Write a function $A(x)$ to represent the total area of the lot and the driveway for a lot with side length x.

10. What are the domain and range of $A(x)$ in terms of the problem situation?

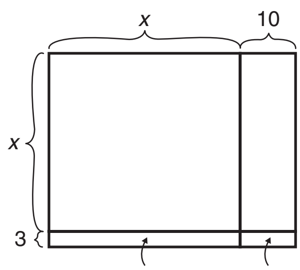
11. Graph $A(x)$.



12. List the different ways that the problem situation was represented.

Problem 2

A second developer is dividing a similar plot of land into smaller plots consisting of square lots, *x* feet on a side, with a 10-foot-wide driveway along one side and a 3-foot-wide walkway along another side as shown.



1. What is the area of each square lot? Label the diagram.

2. What is the area of each driveway? Label the diagram.

3. What is the area of each walkway adjacent to the square lot? Label the diagram.

4. What is the area of each walkway adjacent to the driveway? Label the diagram.

5. Use area composition to write an expression for the total area of the square lot, the driveway, and the walkway.

6. What is the length of the square lot, driveway, and walkway?

7. What is the width of the square lot, driveway, and walkway?

8. Use the length and width from Questions 6 and 7 to write an expression for the total area of the square lot, the driveway, and the walkway.

9. You wrote the total area in two different ways in Questions 5 and 8. Show how these expressions are equivalent.

10. Complete the table.

Width of Square Lot	Width of Plot	Length of Plot	Area of Square Lot	Area of Walkway	Area of Driveway	Total Area
10	13	20	100	60	100	260
30						
50						
100						
x						

11. Write a function $A(x)$ to represent the total area of the lot, driveway, and walkway for a lot with side length x.

12. What are the domain and range of $A(x)$ in terms of the problem situation?

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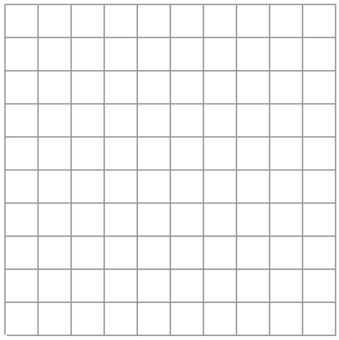
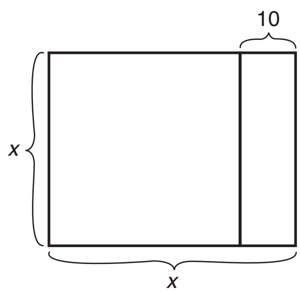
13. Graph $A(x)$.



Problem 3

A third developer is dividing a similar plot of land into smaller plots consisting of square lots, x feet on a side, with a 10-foot-wide driveway within the square lot as shown.



1. What is the area of each square lot? Label the diagram.

2. What is the area of each driveway? Label the diagram.

3. Use area composition to write an expression for the area of the plot not covered by the driveway. Label the diagram.

4. What is the length of the plot not covered by the driveway?

5. What is the width of the plot not covered by the driveway?

6. Use the length and width from Questions 4 and 5 to write an expression for the area of the plot not covered by the driveway.

7. You wrote the total area in two different ways in Questions 3 and 6. Show how these expressions are equivalent.

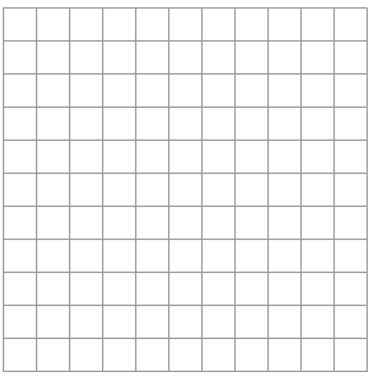
8. Complete the table.

Width of Square Lot	Length of Plot Not Covered by the Driveway	Area of Square Lot	Area of Driveway	Area of Plot Not Covered by the Driveway
20	10	400	200	200
30				
50				
100				
x				

9. Write a function $A(x)$ to represent the area of the plot not covered by the driveway for a lot with side length x.

10. What are the domain and range of $A(x)$ in terms of the problem situation?

11. Graph $A(x)$.



Be prepared to share your solutions and methods.

4.3 Models for Polynomials
Operations with Polynomials

Objective
In this lesson you will:
- Use area models to add, subtract, and multiply polynomials.

Key Terms
- polynomial
- monomial
- binomial
- trinomial

Problem 1

A **polynomial** is an expression formed by adding and subtracting terms of the form ax^n, where a is any number and n is a whole number. For example, the expressions $2x$, $x^2 + 2x + 5$, and $3x^4 - x^2 + 1$ are polynomials.

Some polynomials have special names based on the number of terms as shown in the table.

Polynomial Name	Number of Terms	Examples
monomial	one (mono)	$2x$, 5, $4x^3$
binomial	two (bi)	$x - 5$, $4x^3 + 2x$
trinomial	three (tri)	$4x^3 + x - 5$, $2x^2 + 7x - 3$

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In the last lesson, you used area models to represent polynomials such as $x^2 + 3x + 10x + 30$, and $x^2 - 10x$.

1. For each sum or difference, sketch the resulting model. Then calculate the sum or difference.

a. $(2x + 5) + (4x + 4) =$

Take Note

To distribute the negative sign:
$-(4x + 2) = -(4x) - (2) = -4x - 2$

b. $(5x + 6) - (4x + 2) =$

c. $(3x^2 + 5) - (x^2 + 2) =$

d. $(x^2 + 5x) + (x^2 + 4x + 4) =$

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2. Based on your results from Question 1, what must be true to add or subtract two terms?

3. Calculate each sum or difference without sketching a model.

   a. $(7x + 4) + (3x + 1) =$

   b. $(3x + 8) - (3x + 2) =$

   c. $(x^2 + 2x) + (7x + 1) =$

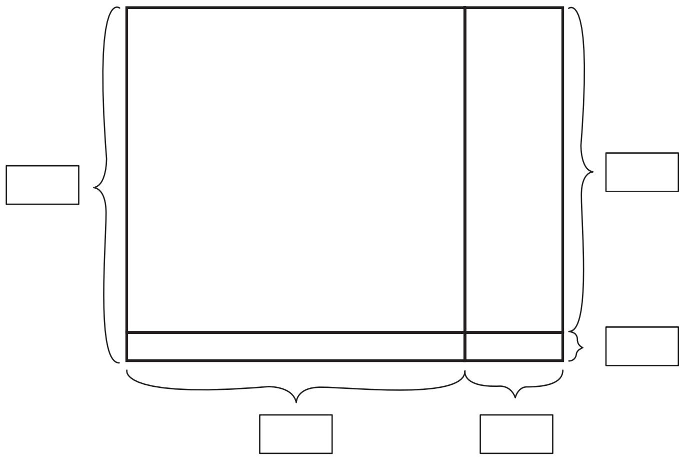
   d. $(2x^2 + 4x) - (x^2 + 3x) =$

   e. $(5x + 3) - (2x + 1) =$

Problem 2

A developer is dividing a plot of land into smaller plots consisting of square lots, *x* feet on each side, with a 10-foot-wide driveway within one side and a 3-foot walkway within another side as shown. The remaining portion of the lot will be used to build a home.

1. Label the dimensions on the diagram of this plot.



2. What is the area of each square lot?

3. What is the area of each driveway?

4. What is the area of each walkway?

5. Use area composition to write an expression for the total area of the plot used for building a home.

6. What is the length of the plot used for building a home?

7. What is the width of the plot used for building a home?

8. Use the length and width from Questions 6 and 7 to write an expression for the total area of the plot used for building a home.

9. Complete the table.

Width of Square Lot	Width of House Plot	Length of House Plot	Area of Square Lot	Area of Driveway	Area of Walkway	Area of House Plot
50	40	47	2500	470	150	1880
x						

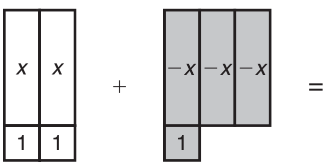
10. Write a function $A(x)$ to represent the total area of the plot used for building a home for a lot with side length x.

11. What are the domain and range of $A(x)$ in terms of the problem situation?

Problem 3

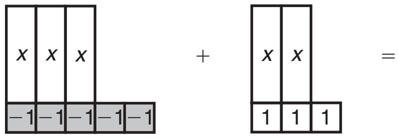
Using an area model for multiplying two polynomials involving subtraction is more difficult than multiplying two polynomials involving only addition. We will use a different shade to represent negative areas.

For example, $(2x + 2) + (-3x - 1) = 2x + 2 - 3x - 1 = -x + 1$.

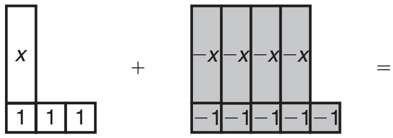


1. For each sum or difference, sketch the resulting model. Then calculate the sum or difference.

a. $(3x - 5) + (2x + 3) =$

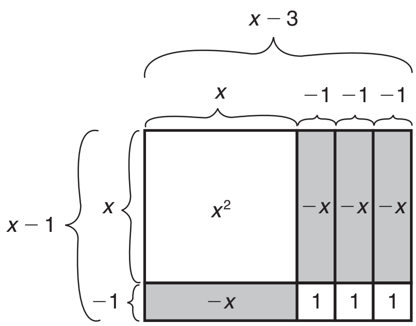


b. $(x + 3) + (-4x - 5) =$



Using a different shade to represent negative areas can also be used when multiplying polynomials.

For example, $(x - 1)(x - 3) = x^2 - x - x - x - x + 1 + 1 + 1 = x^2 - 4x + 3$.



2. For each product, sketch the resulting model. Then calculate the product.

a. $(x + 2)(x + 3) =$

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b. $(x + 3)(x - 5) =$

c. $(x - 2)(x - 4) =$

d. $(x - 2)(x + 2) =$

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Be prepared to share your solutions and methods.

4.4 Another Factor
Dividing and Factoring Quadratic Trinomials

Objectives

In this lesson you will:

- Multiply polynomials using area models, multiplication tables, and the distributive property.
- Divide polynomials using area models, multiplication tables, and long division.
- Factor quadratic trinomials using area models.

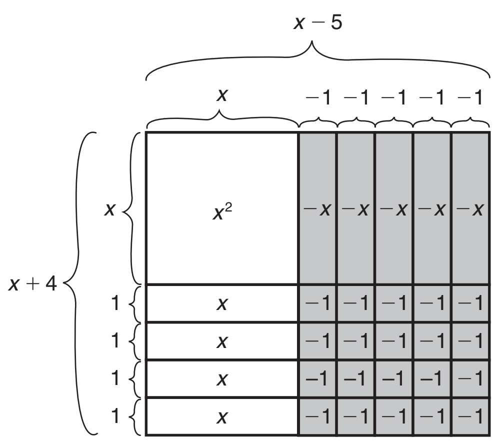
Key Terms

- quadratic trinomial
- factoring

Problem 1 Models for Multiplication

Previously, you learned how to multiply polynomials using an area model. An area model can be used to multiply $(x + 4)(x - 5)$ as follows.

$(x + 4)(x - 5) = x^2 + 4x - 5x - 20 = x^2 - x - 20$



A multiplication table is a model for multiplying polynomials that is similar to an area model. A multiplication table can be used to multiply $(x + 4)(x - 5)$ as follows.

$(x + 4)(x - 5)$

$\cdot$	x	4
x	x^2	$4x$
-5	$-5x$	-20

$(x + 4)(x - 5) = x^2 + 4x - 5x - 20 = x^2 - x - 20$

An area model and a multiplication table are visual representations of the distributive property. The distributive property can be used to multiply $(x + 4)(x - 5)$ as follows.

$(x + 4)(x - 5) =$ Distribute $(x + 4)$

$=$ Distribute x and -5

$=$

$=$

1. Perform each multiplication using the method specified.

a. Use an area model to multiply $(x - 3)(x - 2)$.

b. Use the distributive property to multiply $(x + 8)(x - 7)$.

c. Use a multiplication table to multiply $(x + 5)(x - 10)$.

2. Perform each multiplication using any method.

   a. $(x - 7)(x - 9) =$

   b. $(x + 1)(x - 1) =$

   c. $(x + 12)(x + 12) =$

   d. $(x - 7)(x - 7) =$

   e. $(x - 6)(x + 6) =$

   f. $(x - 11)(x + 20) =$

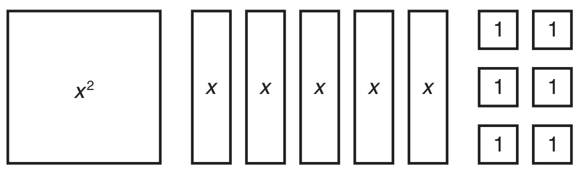
In Questions 1 and 2, you multiplied two binomials. Each product was a trinomial with an x^2 term. A trinomial that has an exponent of 2 as the largest power in any term is called a **quadratic trinomial**.

Problem 2 Dividing Polynomials with an Area Model

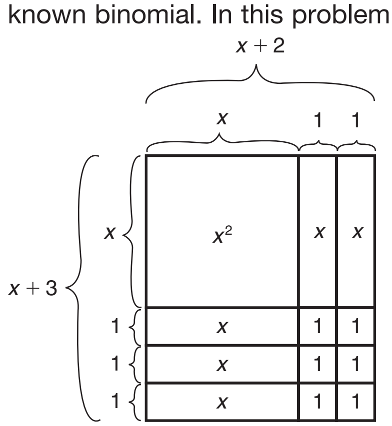
Some quadratic trinomials can be written as the product of two binomials. It may be useful to write a trinomial as a product to solve some problems. If one binomial is known, then you can use several methods to calculate the other binomial.

An area model can be used to divide $(x^2 + 5x + 6) \div (x + 2)$ as follows.

First, represent each part of the trinomial as a piece of the area model. In this problem, $x^2 + 5x + 6$ consists of 1 x^2 term, 5 x terms, and 6 constant terms.



Second, use the pieces to form a rectangle with one side length equal to the known binomial. In this problem, form a rectangle with a side length of $x + 2$.



$(x^2 + 5x + 6) \div (x + 2) = x + 3$

The length of the other side of the rectangle is the quotient.

1. Perform each division using an area model.

a. $(x^2 + 3x + 2) \div (x + 2) =$

b. $(x^2 - 3x + 2) \div (x - 2) =$

c. $(x^2 - x - 6) \div (x + 2) =$

Problem 3 Dividing Polynomials with a Multiplication Table

A multiplication table can be used to divide $(x^2 - x - 12) \div (x + 3)$ as follows.

Enter the known binomial along the top row of the multiplication table. Enter the x^2 term and the constant term of the trinomial within the table. In this problem, enter the binomial $x + 3$ along the top row. Then enter the x^2 term and the constant term.

$\cdot$	x	3
	x^2	
		-12

Enter additional values one at a time. In this problem, the term x^2 is the result of multiplying x and some other quantity. The unknown quantity is x because x times x is x^2.

$\cdot$	x	3
x	x^2	
		-12

Multiply the x in the left column by 3 in the top row. Enter the product of $3x$.

$\cdot$	x	3
x	x^2	$3x$
		-12

The x term of the trinomial is $-x$, which is the result of adding $3x$ and some other quantity. The unknown quantity is $-4x$ because the sum of $-4x$ and $3x$ is $-x$.

$\cdot$	x	3
x	x^2	$3x$
	$-4x$	-12

The term $-4x$ is the result of multiplying x and some other quantity. The unknown quantity is -4 because x times -4 is $-4x$.

$\cdot$	x	3
x	x^2	$3x$
-4	$-4x$	-12

$(x^2 - x - 12) \div (x + 3) = x - 4$

The left column represents the quotient.

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1. Perform each division using a multiplication table.

   a. $(x^2 + 4x - 5) \div (x - 1) =$

   b. $(x^2 - 7x + 12) \div (x - 4) =$

   c. $(x^2 + x - 20) \div (x + 5) =$

Problem 4 Dividing Polynomials using Long Division

A third method for dividing a trinomial by a binomial is long division. Long division can be used to divide $(x^2 - 5x - 14) \div (x + 2)$ as follows.

$$
\begin{array}{r@{}l}
 & x \\
x + 2 & \overline{)x^2 - 5x - 14} \\
 & \underline{x^2 + 2x} \\
 & \phantom{)x^2 +{}} -7x
\end{array}
$$

Divide, multiply and subtract

$$
\begin{array}{r@{}l}
 & x - 7 \\
x + 2 & \overline{)x^2 - 5x - 14} \\
 & \underline{x^2 + 2x} \\
 & \phantom{)x^2 +{}} -7x - 14 \\
 & \phantom{)x^2 +{}} \underline{-7x - 14} \\
 & \phantom{)x^2 + -7x - 1{}} 0
\end{array}
$$

Bring down, divide, multiply and subtract

$(x^2 - 5x - 14) \div (x + 2) = x - 7$

1. Perform each division using long division.

   a. $(x^2 - 7x + 10) \div (x - 5) =$

   b. $(x^2 - 3x - 28) \div (x - 7) =$

2. Perform each division using any method.

   a. $(x^2 - 25) \div (x - 5) =$

   b. $(x^2 - 20x + 100) \div (x - 10) =$

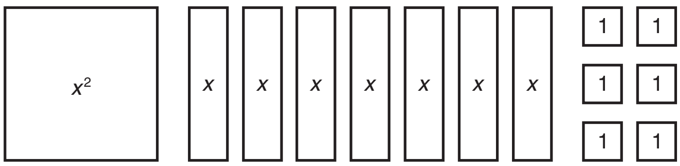
   c. $(x^2 - 4x - 77) \div (x + 7) =$

Problem 5 Factoring Trinomials with an Area Model

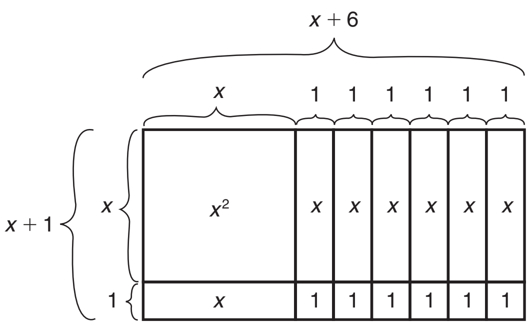
Sometimes you may want to write a quadratic trinomial as the product of two binomials but you may not know either of the binomials. Each binomial is a factor of the trinomial. The process of writing a trinomial as the product of two binomials is called **factoring**.

An area model can be used to factor $x^2 + 7x + 6$ as follows.

First, represent each part of the trinomial as a piece of the area model. In this problem, $x^2 + 7x + 6$ consists of 1 x^2 term, 7 x terms, and 6 constant terms.



Second, use all of the pieces to form a rectangle. In this problem, the parts can only be arranged in one way.



The length and width of the rectangle represents the two factors. Write the trinomial as the product of these two factors.

$x^2 + 7x + 6 =$

Factor each trinomial using an area model.

1. $x^2 + 5x + 4 =$

4

2. $x^2 + 5x - 6 =$

3. $x^2 - 3x - 10 =$

4

4. $x^2 - 6x + 9 =$

Be prepared to share your solutions and methods.

4

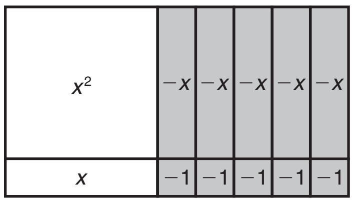
4.5 More Factoring
Factoring Quadratic Trinomials

Objective
In this lesson you will:
- Factor quadratic trinomials.

Key Term
- general form of a quadratic trinomial

Problem 1

An area model can be an effective tool for factoring trinomials for relatively small numbers. For example, $x^2 - 4x - 5$ requires only 12 area pieces to factor: one x^2, five $-x$'s, one x, and five -1's.



$x^2 - 4x - 5 = (x - 5)(x + 1)$

For trinomials with larger numbers, an area model has limited usefulness. For example, $x^2 - 10x - 24$ requires 39 area pieces to factor: one x^2, twelve $-x$'s, two x's, and twenty four -1's. For problems like this, another method is needed to factor.

The **general form of a quadratic trinomial** is $ax^2 + bx + c$, where a, b, and c are constants. In this lesson, you will be factoring quadratic trinomials where $a = 1$, which are factored as $x^2 + bx + c = (x + r_1)(x + r_2)$.

1. Look back at the quadratic trinomials that you factored using an area model. What do you notice about the constant term of the trinomial, c, and the constant of the binomial factors, r_1 *and* r_2?

2. Perform each multiplication.

a. $(x + 1)(x + 6) =$

b. $(x - 1)(x - 6) =$

c. $(x + 3)(x + 2) =$

d. $(x - 3)(x - 2) =$

e. $(x + 1)(x - 6) =$

f. $(x - 1)(x + 6) =$

g. $(x - 3)(x + 2) =$

h. $(x + 3)(x - 2) =$

3. Use the answers from Question 2 to answer each question.

a. How are r_1 and r_2 related to c?

b. How are r_1 and r_2 related to b?

Your answers to Question 3 point to a method for factoring any quadratic trinomial of the form $x^2 + bx + c$.

For example, to factor $x^2 - 4x - 5$ perform the following.

- List the factor pairs of the constant term c.

  In this problem the constant term is -5. Factor pairs of -5 are: -1 and 5, 1 and -5.

- Calculate the sum of each factor pair.

  $-1 + 5 = 4$

  $1 + (-5) = -4$

- Select the factor pair whose sum is equal to the coefficient of the x term, b.

- In this problem the x term is $-4x$. The coefficient of this term is -4. Select the factor pair whose sum is equal to -4. The factor pair is 1 and -5.

- Write the factors of the quadratic trinomial.

  $x^2 - 4x - 5 = (x - 5)(x + 1)$

4

4. Factor $x^2 - 10x - 24$ using this method.

a. List the factor pairs of the constant term c.

b. Calculate the sum of each factor pair:

c. Select the factor pair whose sum is equal to the coefficient of the x term, b.

d. Write the factors of the quadratic trinomial.

4

Problem 2

Factor each trinomial using the method from Problem 1.

1. $x^2 - 10x + 24$

2. $x^2 + 5x - 24$

3. $x^2 - 3x - 28$

4. $x^2 - 12x - 28$

5. $x^2 - 10x + 25$

4

6. $x^2 - 25$

7. $x^2 - 14x + 45$

8. $x^2 - 17x + 52$

4

Factor each trinomial.

9. $x^2 - 16x + 15 =$

10. $x^2 - x - 12 =$

11. $x^2 + 12x + 20 =$

12. $x^2 - 49 =$

13. $x^2 - 12x + 36 =$

14. $x^2 + x - 42 =$

15. $x^2 - 16x + 48 =$

16. $x^2 - 9x + 18 =$

Be prepared to share your solutions and methods.

4

4.6 Radically Speaking!

Operations with Square Roots

Objectives

In this lesson you will:

- Simplify square roots.
- Multiply square roots.

Key Terms

- perfect square
- radical symbol
- radicand

Problem 1

Previously, you factored polynomials in the form $x^2 + bx + c$. For example, $x^2 - 25$ can be factored as $(x - 5)(x + 5)$.

Some polynomials, such as $x^2 - 20$, are not factorable using integers. However, they may be factorable using square roots. For example, $x^2 - 20$ can be factored using square roots as $(x - \sqrt{20})(x + \sqrt{20})$.

1. Factor each polynomial.

a. $x^2 - 1 =$

b. $x^2 - 100 =$

c. $x^2 - 2 =$

d. $x^2 - 8 =$

e. $x^2 - 75 =$

f. $x^2 - 144 =$

Numbers like 25 are **perfect squares**. A perfect square is a number that can be written as an integer squared. For example, 25 can be written as 5^2. The square root of a number is the number that can be multiplied by itself to result in the original number. The square root of 25 is 5 because $5^2 = 25$. The **radical symbol** $\sqrt{}$ is the mathematical symbol used to indicate a square root. The **radicand** is the expression under the radical symbol.

4

2. List the first 10 perfect squares and the square root of each.

Square roots are also useful when using the Pythagorean Theorem to calculate the third side of a right triangle.

For example, if the lengths of the legs of ΔABC are 4 feet and 6 feet, the Pythagorean Theorem can be used to calculate the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

$$4^2 + 6^2 = c^2$$

$$16 + 36 = c^2$$

$$52 = c^2$$

$$\sqrt{52} = c$$

Take Note

Pythagorean Theorem: In any right triangle, the sum of the squares of the two shorter sides, the legs, is equal to the square of the longest side, the hypotenuse.

3. Calculate the missing side of each right triangle.

a. $a = 4$, $b = 7$, $c = ?$

4

b. $a = 6$, $b = 8$, $c = ?$

c. $a = 4$, $b = ?$, $c = 6$

d. $a = ?$, $b = 24$, $c = 25$

4

Problem 2

For non-zero numbers a and b, $\sqrt{a}\,\sqrt{b} = \sqrt{ab}$. For example, $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ and $\sqrt{7} \cdot \sqrt{7} = \sqrt{49} = 7$.

1. Calculate each product.

a. $\sqrt{4} \cdot \sqrt{4} =$

b. $\sqrt{17} \cdot \sqrt{17} =$

c. $\sqrt{3} \cdot \sqrt{3} =$

d. $\sqrt{7} \cdot \sqrt{5} =$

2. Calculate each product.

a. $\sqrt{50} \cdot \sqrt{2} =$

b. $\sqrt{8} \cdot \sqrt{2} =$

c. $\sqrt{32} \cdot \sqrt{2} =$

d. $\sqrt{75} \cdot \sqrt{3} =$

To simplify a square root, write the radicand as the product of a perfect square and another factor. For example, simplify $\sqrt{12}$ as follows.

$\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

A radical is considered completely simplified if there are no additional perfect square factors in the radicand.

3. Simplify each radical completely.

a. $\sqrt{75} =$

b. $\sqrt{27} =$

c. $\sqrt{72} =$

d. $\sqrt{48} =$

e. $\sqrt{500} =$

f. $\sqrt{175} =$

4

Take Note

You may find it easier to simplify before multiplying.

4. Calculate each product and simplify completely.

a. $\sqrt{6}\sqrt{12} =$

b. $\sqrt{10}\sqrt{5} =$

c. $\sqrt{15}\sqrt{45} =$

d. $\sqrt{50}\sqrt{14} =$

e. $\sqrt{500}\sqrt{45} =$

f. $\sqrt{3}(\sqrt{6} - \sqrt{15}) =$

g. $\sqrt{12}(\sqrt{6} + \sqrt{15}) =$

h. $\sqrt{63}\sqrt{14} =$

i. $\sqrt{10}\sqrt{35}\sqrt{14} =$

j. $\sqrt{33}\sqrt{44}\sqrt{15} =$

Be prepared to share your solutions and methods.

4.7 Working with Radicals
Adding, Subtracting, Dividing, and Rationalizing Radicals

Objectives

In this lesson you will:

- Add and subtract square roots.
- Divide square roots.
- Simplify square roots by rationalizing denominators.

Key Terms

- rational numbers
- irrational numbers
- rationalizing the denominator

Problem 1

When adding and subtracting polynomials, only terms that involve the same power, like $3x^2$ and $-5x^2$, can be added or subtracted.

When adding and subtracting square roots, only roots that have the same radicand can be added or subtracted. For example, $4\sqrt{2} + 2\sqrt{3} - \sqrt{2} = 3\sqrt{2} + 2\sqrt{3}$.

1. Calculate each sum or difference.

a. $4\sqrt{2} + 5\sqrt{2} =$

b. $3\sqrt{3} - 5\sqrt{3} =$

c. $3\sqrt{3} - 5\sqrt{2} =$

d. $7\sqrt{3} - 6\sqrt{3} - 2\sqrt{3} =$

e. $-3\sqrt{3} - 6\sqrt{5} - 5\sqrt{5} =$

f. $4\sqrt{2} + 2\sqrt{3} - 7\sqrt{5} =$

2. Simplify each radical. Then calculate each sum or difference.

a. $\sqrt{8} + 5\sqrt{2} =$

b. $\sqrt{18} - \sqrt{50} =$

c. $2\sqrt{18} + \sqrt{12} =$

d. $3\sqrt{28} - 5\sqrt{175} + 3\sqrt{7} =$

4

e. $7\sqrt{8} + 5\sqrt{32} - 3\sqrt{45} =$

f. $\sqrt{125} + 5\sqrt{7} - 3\sqrt{500} =$

Problem 2

Rational numbers are numbers that can be written in the form $\frac{a}{b}$, where a and b are integers. All integers and fractions are rational numbers. Numbers that are not rational are called **irrational numbers**. Numbers like π and square roots such as $\sqrt{2}$ are irrational numbers.

You can prove that $\sqrt{2}$ is irrational using an indirect proof.

Assume that $\sqrt{2}$ is a rational number and can be written in the form $\sqrt{2} = \frac{a}{b}$, where a and b are integers, one number odd and the other even.

1. Enter the reasons for each of the steps in the proof.

$\sqrt{2} = \frac{a}{b}$

$b(\sqrt{2}) = b\left(\frac{a}{b}\right)$

$\sqrt{2}b = a$

$(\sqrt{2}b)^2 = a^2$

$2b^2 = a^2$

$\therefore a^2$ must be even	Definition of even number
$\therefore a$ must be even	The square root of an even number must be even
Let $a = 2n$	Definition of even number
$a^2 = 4n^2$	
$2b^2 = 4n^2$	
$b^2 = 2n^2$	
$\therefore b^2$ must be even	
b must be even	The square root of an even number must be even

But this is a contradiction.

So $\sqrt{2} \neq \frac{a}{b}$ and $\sqrt{2}$ is not rational.

$\therefore \sqrt{2}$ is irrational

The decimal approximations of $\sqrt{2}$ and $\sqrt{3}$ are:

$\sqrt{2} \approx$ 1.4142135623 7309504880 1688724209 6980785696 7187537694 8073176679 7379907324 78462 …

$\sqrt{3} \approx$ 1.7320508075 6887729352 7446341505 8723669428 0525381038 0628055806 9794519330 16909 …

Radicals like $\sqrt{2}$ and $\sqrt{3}$ are decimals with an infinite number of decimal places. It is not possible to calculate exact results with the decimal representation. Dividing by an approximation of an irrational number often creates rounding errors. But multiplying by an approximation of an irrational number does not introduce as many rounding errors. So, it is desirable to rewrite expressions involving irrational numbers to involve multiplication instead of division.

Problem 3

Rationalizing the denominator is a process of rewriting an expression so that the denominator does not contain a radical.

To rationalize the denominator, multiply by a form of one that results in a perfect square in the radicand of the denominator. For example, to simplify $\frac{5}{\sqrt{2}}$, perform the following.

$$\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{4}} = \frac{5\sqrt{2}}{2}$$

A radical expression is not considered completely simplified if a radical exists in a denominator.

1. Simplify each expression by rationalizing the denominator.

a. $\frac{4}{\sqrt{2}} =$

b. $-\frac{6}{\sqrt{3}} =$

c. $\frac{35}{\sqrt{14}} =$

d. $\frac{2\sqrt{5}}{\sqrt{15}} =$

4

2. Simplify each expression completely.

a. $\frac{2\sqrt{5}}{3} + \frac{\sqrt{125}}{3} =$

b. $2\sqrt{7} - 2\sqrt{63} =$

c. $\frac{3\sqrt{5}}{4} \cdot \frac{2\sqrt{5}}{\sqrt{3}} =$

d. $\frac{3\sqrt{525}}{\sqrt{28}} =$

e. $-\frac{\sqrt{72}}{5\sqrt{3}} =$

f. $\frac{2\sqrt{5}}{7} \div \frac{3\sqrt{5}}{\sqrt{3}} =$

g. $\frac{5\sqrt{5}}{7} \cdot \frac{7\sqrt{15}}{2\sqrt{12}} =$

h. $\frac{10\sqrt{99}}{\sqrt{45}} =$

i. $\frac{5\sqrt{55}}{6} \cdot \frac{3\sqrt{75}}{2\sqrt{24}} =$

Be prepared to share your solutions and methods.

4.8 Rain Gutters
Modeling with Functions

Objectives

In this lesson you will:

- Use multiple representations of functions to model and solve problems.
- Use multiple representations of functions to analyze problems.

Problem 1

A contractor has asked you for some help. The contractor is making customized rain gutters for a house. To form the gutters, he uses long rectangular sheets of metal and bends two sides up. An end view of the gutter is shown.

The contractor is using metal sheets that are 8.5 inches wide. The length of each metal sheet is not important for this problem. The contractor wants to know the relationship between the side length and bottom width of each gutter. He also wants to know all of the possible side-length and bottom-width measurements that can be used to construct gutters.

1. Use sheets of paper that are 8.5 inches wide to construct 5 different gutters. Make some gutters short and wide; make others tall and narrow. In the table below, record the side-length and bottom-width measurements of each gutter you construct.

Labels	**Gutter**	**Side Length**	**Bottom Width**
Units		**Inches**	**Inches**
	Gutter 1		
	Gutter 2		
	Gutter 3		
	Gutter 4		
	Gutter 5		

2. Complete the table. If necessary, construct models of each gutter.

Side Length	Bottom Width
Inches	Inches
0	
0.25	
0.5	
0.75	
1	
1.25	
1.5	
1.75	
2	
2.25	
2.5	
2.75	
3	
3.25	
3.5	
3.75	
4	
4.25	

4

3. Based on the table, describe the relationship between the side length and bottom width.

4. As the side length increases by a quarter inch, how does the bottom width change?

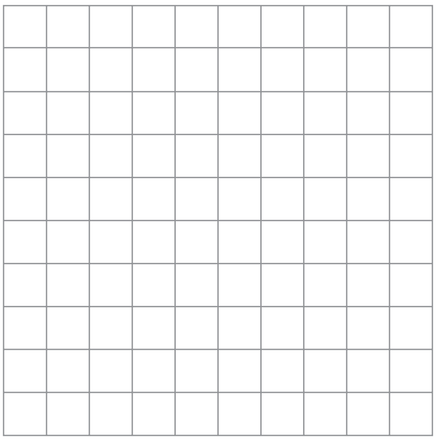
5. As the side length increases by one half inch, how does the bottom width change?

6. As the side length increases by one inch, how does the bottom width change?

7. Describe how to calculate the bottom width for any side length.

8. Define a function $w(l)$ for the bottom width for a side length of l inches.

9. Graph the function $w(l)$.

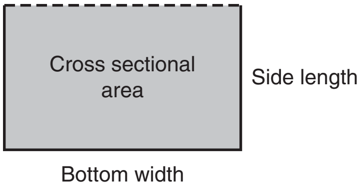


10. What are the range and domain of $w(l)$?

11. What type of function is $w(l)$?

Problem 2

The cross-sectional area of a gutter is important because a larger area can carry more rain.



1. Calculate the cross-sectional area for a gutter with a side length of:

   a. 2 inches.

   b. 3 inches.

   c. 2.5 inches.

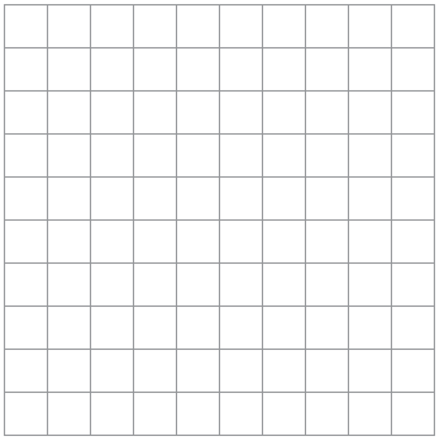
4

2. Define a function $A(l)$ for the cross sectional area of a gutter with a side length of l inches.

3. Complete the table.

Labels	Side Length	Cross-Sectional Area
Units	Inches	Square Inches
Expression	0	
	0.5	
	1	
	1.5	
	2	
	2.5	
	3	
	3.5	
	4	

4. Graph the function $A(l)$



5. What type of function is $A(l)$?

6. Set the function $A(l)$ equal to 0. Calculate the values of l by factoring the equation and setting each factor to 0.

Take Note

If $ab = 0$ then $a = 0$ or $b = 0$ or both are 0.

7. What are the intercepts of $A(l)$? What does each mean in terms of the problem? Label each intercept on the graph.

8. Where is the vertex? What does it mean in the problem? Label the vertex on the graph.

9. What are the domain and range of $A(l)$?

10. What is the equation of the axis of symmetry? Label the axis of symmetry on the graph.

Be prepared to share your solutions and methods.

4

4.9 More Areas
More Modeling with Functions

Objectives
In this lesson you will:
- Use multiple representations of functions to model and solve problems.
- Use multiple representations of functions to analyze problems.

Problem 1

A developer is planning plots of land. Each plot consists of a square lot that is x feet on each side. A 3-foot walkway surrounds the square lot on three sides. A 10-foot driveway surrounds the square lot on the fourth side.

1. Draw a diagram of the plot that includes the square lot, walkway, and driveway. Label all dimensions in the diagram.

What is the area of the square lot?

2. What is the area of the walkway? Write the area as a simplified expression.

3. What is the area of the driveway? Write the area as a simplified expression.

4

4. What is the combined area of the walkway and the driveway? Write the area as a simplified expression.

5. What is the total area of the plot? Write the area as a simplified expression.

6. What is the width of the plot?

7. What is the length of the plot?

8. Use the length and width from Questions 7 and 8 to write an expression for the area of the plot.

9. You wrote the area of the plot in two different ways in Questions 5 and 8. Show how these expressions are equivalent.

10. Write an expression for the area of the plot minus the area of the square lot. Write this area as a simplified expression.

11. What does the expression in Question 10 represent in the problem?

12. The square lot is 100 feet on each side. Calculate each area.

a. Area of the square lot

b. Area of the walkway

c. Area of the driveway

d. Area of the plot

Problem 2

A developer is planning plots of land. Each plot consists of a rectangular lot with a width of x feet and a length of $x + 6$ feet. A 3-foot walkway is adjacent to each of the shorter sides of the rectangular lot. A 10-foot driveway is adjacent to each of the longer sides of the rectangular lot. The driveways extend to the end of the walks.

1. Draw a diagram of the plot that includes the square lot, walkway, and driveway. Label all dimensions in the diagram.

2. What is the area of the rectangular lot? Write the area as a simplified expression.

3. What is the area of the walkways? Write the area as a simplified expression.

4

4. What is the area of the driveways?

5. What is the combined area of the walkway and the driveway? Write the area as a simplified expression.

6. What is the total area of the plot? Write the area as a simplified expression.

7. What is the width of the plot?

8. What is the length with the plot?

9. Use the length and width from Questions 7 and 8 to write an expression for the area of the plot.

10. You wrote the area of the plot in two different ways in Questions 6 and 9. Show how these expressions are equivalent.

4

11. Write an expression for the area of the plot minus the area of the rectangular lot. Write this area as a simplified expression.

12. What does the expression in Question 11 represent in the problem?

13. A rectangular lot has a width of 100 feet. Calculate each area.

a. Area of the rectangular lot

b. Area of the walkways

c. Area of the driveways

d. Area of the plot

14. The total area of layout plot is 3120 square feet. Calculate the dimensions of the rectangular lot by performing the following steps.

a. Set the expression for the total area of the plot equal to 3120.

b. Transform the equation so that one side is equal to zero.

c. Factor the expression on the other side of the equation.

d. Set each factor equal to zero and solve.

Be prepared to share your solutions and methods.