CHAPTER

5 Properties of Triangles

The largest food industry in the world is growing grapes. About 25 million acres of land are used to produce over 72 million tons of grapes each year. You will use congruent triangles to construct posts for growing grape vines.

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5.1 Properties of Triangles
Angle Relationships in a Triangle

Objectives
In this lesson you will:
- Classify triangles using the measures of the interior angles.
- Determine the relationship between the measures of interior angles of a triangle and the lengths of sides of a triangle.
- Identify the remote interior angles of a triangle.
- Identify the exterior angle of a triangle.
- Solve problems using the relationship between an exterior angle of a triangle and remote interior angles.
- Prove the Exterior Angle Inequality Theorem.

Key Terms
- acute triangle
- obtuse triangle
- right triangle
- equiangular triangle
- exterior angle
- remote interior angles
- Exterior Angle Inequality Theorem

Problem 1 Classifying Triangles

One way to classify a triangle is by the measure of the interior angles. These classifications include acute triangles, obtuse triangles, and right triangles.

1. An **acute triangle** is a triangle that has three acute angles.

a. Draw an acute triangle with three interior angles of different measure. Measure each interior angle.

Take Note

The Triangle Sum Theorem states that the sum of the measures of the interior angles of a triangle is 180°.

b. Measure the length of each side of the triangle.

c. The longest side of the triangle is opposite which interior angle?

d. The shortest side of the triangle is opposite which interior angle?

2. An **obtuse triangle** is a triangle that has one obtuse angle.

a. Draw an obtuse triangle with three interior angles of different measure. Measure each interior angle.

b. Measure the length of each side of the triangle.

c. The longest side of the triangle is opposite which interior angle?

d. The shortest side of the triangle is opposite which interior angle?

3. A **right triangle** is a triangle that has one right angle.

a. Draw a right triangle with three interior angles of different measure. Measure each interior angle.

b. Measure the length of each side of the triangle.

c. The longest side of the triangle is opposite which interior angle?

d. The shortest side of the triangle is opposite which interior angle?

4. An **equiangular triangle** is a triangle that has three equal angles.

   a. Draw an equiangular triangle. Measure each interior angle.

   b. Measure the length of each side of the triangle.

   c. What do you notice about the measures of the three interior angles of an equiangular triangle?

   d. What do you notice about the lengths of the three sides of an equiangular triangle?

5. Triangle ABC has interior angles measuring 57°, 62°, and 61°, respectively. Without drawing a picture, describe how to locate the longest and shortest sides of the triangle in terms of the measures of the interior angles.

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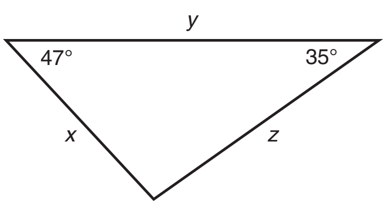
6. For each triangle, list the sides from shortest to longest.

   a.



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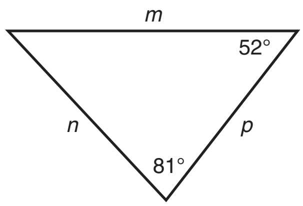

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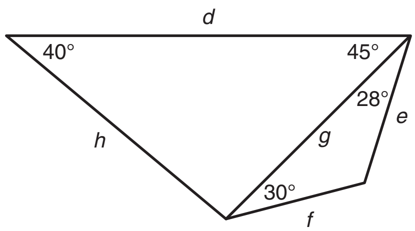
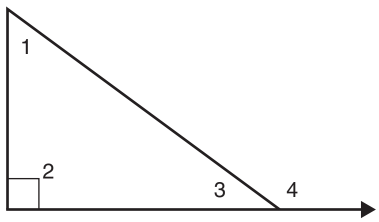


Problem 2 Exterior Angles of a Triangle

1. Use the figure shown to answer each question.



a. Name the interior angles of the triangle.

b. Name an **exterior angle** of the triangle.

c. What do you need to know to answer parts a and b?

d. What is $m\angle 1 + m\angle 2 + m\angle 3$? How do you know?

e. What is $m\angle 3 + m\angle 4$? How do you know?

f. Explain why $m\angle 1 + m\angle 2 = m\angle 4$.

2. What does the word "remote" mean in the sentence, "The treasure is buried on a remote island"?

3. Why would $\angle 1$ and $\angle 2$ be referred to as **remote interior angles** with respect to the exterior angle?

4. Write the equation $m\angle 4 = m\angle 1 + m\angle 2$ as a sentence using the words sum, remote interior angles, and exterior angle.

5. Is the equation and sentence from Question 4 a postulate or a theorem? Why?

6. Classify the triangle in Question 1 based on the angle measures.

7. Consider the relationship between the measure of an exterior angle of an acute triangle and the sum of the measures of the two remote interior angles. Is the equation from Question 4 still true? Explain.

8. Consider the relationship between the measure of an exterior angle of an obtuse triangle and the sum of the measures of the two remote interior angles. Is the equation from Question 4 still true? Explain.

9. You can use patty paper to explore the relationship between an exterior angle and the remote interior angles of a triangle. Perform the following steps.

Step 1: Place the patty paper over the triangle in Problem 1. Copy ∠1. Then copy ∠2 so that ∠1 and ∠2 share a common side.

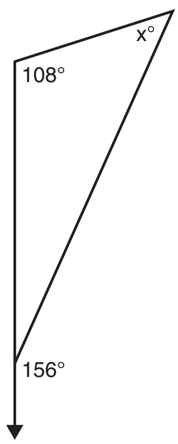
Step 2: Hold the patty paper over ∠4 so that one side of ∠4 lines up with side of ∠1 and the other side of ∠4 lines up with a side of ∠2.

What do you notice?

Problem 3

Solve for *x* in each triangle.

1.



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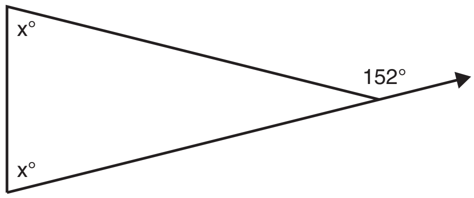
2.



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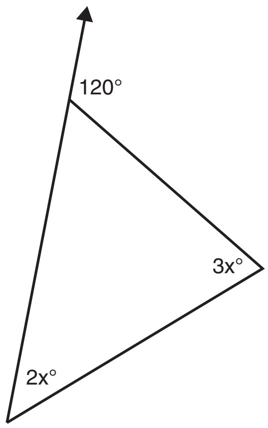
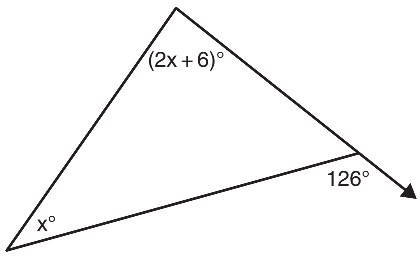


4.



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Problem 4 Exterior Angle Inequality Theorem

The **Exterior Angle Inequality Theorem** states,

The measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles.

1. Draw a triangle *ABC* with exterior angle *ACD*.

2. Use the diagram to write an inequality that states the Exterior Angle Inequality Theorem.

3. Use the diagram to explain why the Exterior Angle Inequality Theorem is true for all triangles.

4. Why is the Exterior Angle Inequality considered a theorem rather than a postulate?

Be prepared to share your solutions and methods.

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5.2 Properties of Triangles
Side Relationships of a Triangle

Objectives
In this lesson you will:
- Classify triangles using the lengths of the sides.
- Determine the relationship between the lengths of the sides of a triangle and the measures of the interior angles of a triangle.
- Determine and apply the Triangle Inequality Theorem.

Key Terms
- scalene triangle
- isosceles triangle
- equilateral triangle

Problem 1 Classifying Triangles

One way to classify a triangle is by the lengths of the sides. These classifications include scalene triangles, isosceles triangles, and equilateral triangles.

1. A **scalene triangle** is a triangle that has three sides of different lengths.

   a. Draw a scalene triangle. Measure each side.

> **Take Note**
> The Triangle Sum Theorem states that the sum of the measures of the interior angles of a triangle is 180°.

   b. Measure each interior angle of the triangle.

   c. The largest interior angle of the triangle is opposite which side?

   d. The smallest interior angle of the triangle is opposite which side?

2. An **isosceles triangle** is a triangle that has two or more equal-length sides.

   a. Draw an isosceles triangle with two equal sides. Measure each side.

   b. Measure each interior angle of the triangle.

   c. The largest interior angle of the triangle is opposite which side?

   d. The smallest interior angle of the triangle is opposite which side?

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3. An **equilateral triangle** is a triangle that has three equal length sides.

   a. Draw an equilateral triangle. Measure each side.

b. Measure each interior angle of the triangle.

c. The largest interior angle of the triangle is opposite which side?

d. The smallest interior angle of the triangle is opposite which side?

4. Triangle *ABC* has side lengths measuring 4 centimeters, 5 centimeters, and 6 centimeters, respectively. Without drawing a picture, describe how to locate the largest and smallest interior angles of the triangle in terms of the side lengths.

Problem 2

1. Sarah claims that segments of any three lengths will form a triangle. Sam believes that some combinations of segment lengths will not form a triangle. Who is correct? Explain.

2. Sam claims that he can look at the lengths of three segments and know immediately if the segments can form a triangle. Learn Sam's secret by working through the following.

Step 1: Break a piece of strand pasta at two random points. Measure the length of each pasta piece.

Step 2: Attempt to form a triangle by placing the three pieces end to end. Record your results in the first row of the following table.

Step 3: Collect the results from your classmates.

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Form a Triangle?

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3. How many students are in your class?

4. How many students were able to form a triangle using their pasta pieces?

5. How many students were not able to form a triangle using their pasta pieces?

6. Estimate the probability that a student in your class was able to form a triangle using their pasta pieces.

7. Examine the table from Question 2. Compare the lengths that formed a triangle and the lengths that did not form a triangle.

a. What do you notice?

b. What must be true about the lengths for a triangle to be formed?

c. What must be true about the lengths for a triangle to not be formed?

8. Predict whether a triangle could be formed from each set of segment lengths.

a. 2 cm, 5.1 cm, 2.4 cm

b. 9.2 cm, 7 cm, 1.9 cm

Problem 3

A graphing calculator can also be used to simulate the activity from Problem 2.

1. Assume that the length of a piece of pasta is 100 centimeters. Use your calculator to generate two random numbers between 0 and 100 representing the breaking points of the pasta piece. How can you calculate the length of the third pasta piece?

2. Record your results in the following table.

Trial Number	Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Form a Triangle?

3. Use the table to estimate the probability that a triangle can be formed.

4. Examine your conclusion from Problem 2 Question 7. Do the calculator results agree with or contradict your original conclusion?

5. If the calculator results agree with your original conclusion, then explain why. If the calculator results contradict your original conclusion, then revise your original conclusion so that it is accurate.

6. Let's return to Sarah from Problem 2. Based on what you have learned about side lengths of triangles, state the rule Sarah must know to determine whether three segments can form a triangle.

Be prepared to share your solutions and methods.

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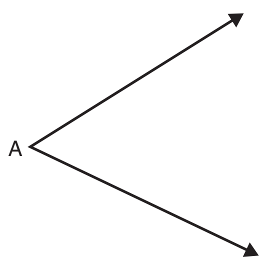
5.3 Properties of Triangles
Points of Concurrency

Objectives
In this lesson you will:
- Construct an angle bisector.
- Construct a perpendicular bisector.
- Construct the incenter of a triangle.
- Construct the circumcenter of a triangle.
- Construct the centroid of a triangle.
- Construct the orthocenter of a triangle.
- Compare the points of concurrency of triangles.

Key Terms
- bisect an angle
- angle bisector
- concurrent lines
- point of concurrency
- incenter
- bisect a segment
- segment bisector
- perpendicular bisector
- circumcenter
- median
- centroid
- altitude
- orthocenter

Problem 1 Constructing Angle Bisectors using a Compass and Straightedge

1. Perform the following steps using the angle shown.



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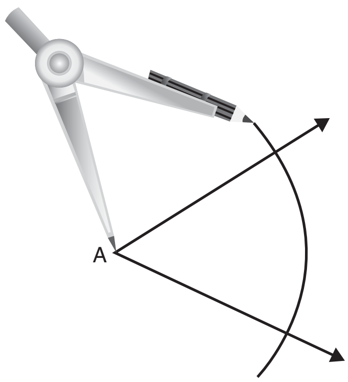
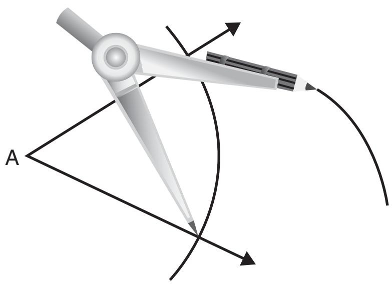
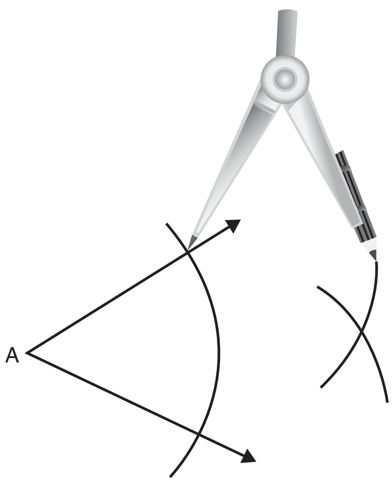
a. Place the point of the compass at the vertex of the angle, point *A*. Draw an arc that intersects each side of the angle.



b. Place the point of the compass on the point where the arc intersects one side of the angle. Draw an arc in the interior of the angle.



c. Place the point of the compass on the point where the arc intersects the other side of the angle. Draw an arc in the interior of the angle.



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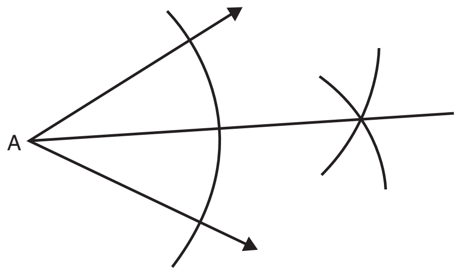
d. Use a straightedge to draw a line from point *A* to the intersection of the arcs from parts (b) and (c).



2. Use a protractor to measure the original angle from Question 1.

3. Use a protractor to measure the two angles formed from the original angle. What do you notice about the measures of these two angles and the measure of the original angle?

Question 1 asked you to **bisect an angle,** or divide the angle into two smaller angles of equal measure. An **angle bisector** is a line, segment, or ray that divides an angle into two smaller angles of equal measure.

4. An angle has a measure of 120°. What is the measure of each angle that is formed by the angle bisector? Explain.

5. An angle is bisected so that each smaller angle has a measure of 38°. What is the measure of the angle that was bisected? Explain.

Problem 2 Constructing Angle Bisectors using Patty Paper

1. Draw an angle on a piece of patty paper. Label angle *ABC*.
2. Construct the angle bisector by performing the following steps.
   a. Fold the patty paper so that *BA* and *BC* are exactly on top of each other.
   b. Unfold the patty paper and draw a ray from point *B* along the crease of the fold.
3. Use a protractor to measure the original angle from Question 1.
4. Use a protractor to measure the two angles formed from the original angle. What do you notice about the measures of these two angles and the measure of the original angle?

Problem 3 Constructing Incenters

1. Draw a right triangle. Then use a compass and straightedge to construct the angle bisector for each angle of the triangle.
2. Draw an acute triangle on patty paper. Then construct the angle bisector for each angle of the triangle.

3. Draw an obtuse triangle. Then use a compass and straightedge to construct the angle bisector for each angle of the triangle.

4. What do you notice about the angle bisectors in each triangle?

Concurrent lines are three or more lines that intersect at a common point. The point at which the lines intersect is the **point of concurrency.**

The **incenter** of a triangle is the point at which the three angle bisectors intersect.

Take Note

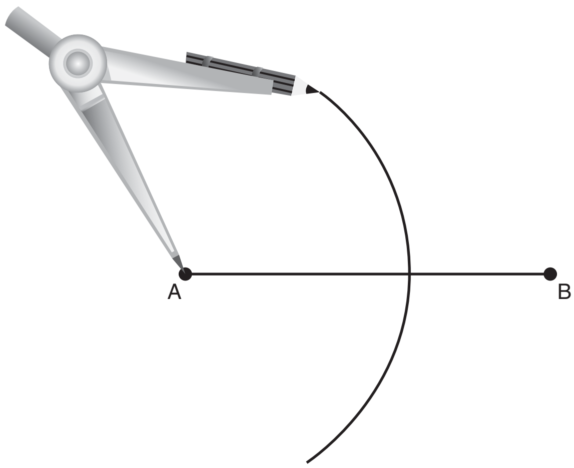
The distance from a point to a line is the perpendicular distance from the point to the line.

5. For each triangle in Questions 1 through 3, measure the distance from the incenter to each side of the triangle. What do you notice?

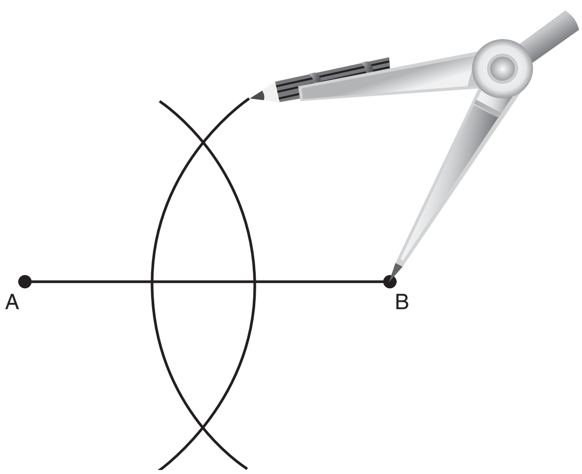
Problem 4 Constructing Perpendicular Bisectors using a Compass and Straightedge

1. Perform the following steps using the segment shown.

a. Place the point of the compass on an endpoint of the line, point *A*. Open the compass so that it is more than half of the length of the segment. Draw an arc that intersects the line segment and extends above and below the segment.



b. Keep the compass at the same width. Place the point of the compass on the other endpoint of the line, point *B*. Draw an arc that intersects the line segment and extends above and below the segment, intersecting the other arc at two points.



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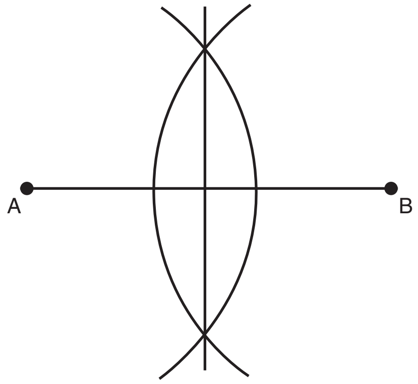
c. Use a straightedge to draw a line that connects the intersections of the arcs from parts (a) and (b).



2. Use a ruler to measure the length of the original segment from Question 1.

3. Use a ruler to measure the two segments formed from the original segment. What do you notice about the lengths of these two segments and the length of the original segment?

Question 1 asked you to **bisect a segment,** or divide the segment into two smaller segments of equal length. A **segment bisector** is a line, segment, or ray that divides a segment into two smaller segments of equal length.

4. What is a name for the point on the line segment where the segment bisector intersects the line segment?

5. Use a protractor to measure each angle formed by the intersection of the segment bisector and the line segment. What do you notice?

A **perpendicular bisector** is a segment bisector that is also perpendicular to, or forms a right angle with, the line segment.

6. Draw a segment bisector that is not a perpendicular bisector.

7. A 24-inch line segment is bisected. What is the length of each line segment that is formed by the segment bisector? Explain.

8. A segment is bisected so that each smaller segment measures 5 millimeters. What is the length of the segment that was bisected? Explain.

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Problem 5 Constructing Perpendicular Bisectors using Patty Paper

1. Draw a segment on a piece of patty paper. Label the segment *AB*.

2. Construct the perpendicular bisector by performing the following steps.

a. Fold the patty paper so that point *A* and point *B* are exactly on top of each other.

b. Unfold the patty paper and draw a line segment along the crease of the fold.

3. Use a ruler to measure the length of the original segment from Question 1.

4. Use a ruler to measure the length of the two segments formed from the original segment. What do you notice about the lengths of these two segments and the length of the original segment?

5. Use a protractor to measure each angle formed by the intersection of the segment bisector and the line segment. What do you notice?

Problem 6 Constructing Circumcenters

1. Draw a right triangle. Then use a compass and straightedge to construct the perpendicular bisector for each side of the triangle.

2. Draw an acute triangle on patty paper. Then construct the perpendicular bisector for each side of the triangle.

3. Draw an obtuse triangle. Then use a compass and straightedge to construct the perpendicular bisector for each side of the triangle.

4. What do you notice about the perpendicular bisectors in each triangle?

The **circumcenter** of a triangle is the point at which the three perpendicular bisectors intersect.

5. For each triangle in Questions 1 to 3, measure the distance from the circumcenter to each vertex of the triangle. What do you notice?

Problem 7 Constructing Centroids

A **median** of a triangle is a line segment that connects a vertex to the midpoint of the side opposite the vertex.

1. Which construction could you use to construct a median? Explain.

2. Draw a right triangle. Then use a compass and straightedge to construct the three medians of the triangle.

3. Draw an acute triangle on patty paper. Then construct the three medians of the triangle.

4. Draw an obtuse triangle. Then use a compass and straightedge to construct the three medians of the triangle.

5. What do you notice about the medians in each triangle?

The **centroid** of a triangle is the point at which the three medians intersect.

6. For each triangle in Questions 2 to 4, measure the distance from the vertex to the centroid. Then measure the length of the median. What do you notice?

Problem 8 Constructing a Perpendicular Segment from a Point not on a Line

1. Perform the following steps using the segment shown.

   a. Place the point of the compass on point *C*. Open the compass so that it is wide enough to cut an arc that intersects the line segment at two points. Draw this arc and label these points of intersection *D* and *E*.

   b. Construct the perpendicular bisector of segment *DE*.

2. Use a protractor to measure each angle formed by the intersection of the two segments. What do you notice?

3. Use a ruler to measure the two segments formed by the original segment and the bisector. Is the line you drew the perpendicular bisector of segment *AB*? Explain.

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Problem 9 Constructing Orthocenters

An **altitude** of a triangle is a perpendicular line segment that is drawn from a vertex to the opposite side.

1. Draw a right triangle. Then use a compass and straightedge to construct the three altitudes of the triangle.

2. Draw an acute triangle on patty paper. Then construct the three altitudes of the triangle.

3. Draw an obtuse triangle. Then use a compass and straightedge to construct the three altitudes of the triangle.

4. What do you notice about the altitudes in each triangle?

The **orthocenter** of a triangle is the point at which the three altitudes intersect.

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Problem 10

1. Look at the constructions from Problems 2, 6, 7, and 9. Describe whether the incenter, circumcenter, centroid, and orthocenter lie in the interior, in the exterior, or on the perimeter of each type of triangle.

   a. Acute

   b. Obtuse

   c. Right

2. Can a triangle have all four points of concurrency (incenter, circumcenter, centroid, and orthocenter) as the same point? If so, draw and describe this triangle. If not, explain why.

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Be prepared to share your solutions and methods.

5.4 Properties of Triangles
Direct and Indirect Proof

Objectives
In this lesson you will:
- Prove theorems using direct proofs.
- Prove theorems using indirect proofs.

Key Terms
- Triangle Exterior Angle Theorem
- two-column proof
- negation of the conclusion
- inequality property
- Exterior Angle Inequality Theorem

Previously you learned that direct proof and indirect proof are two methods for proving theorems. In this lesson, you will prove theorems about triangles.

Problem 1

The **Triangle Exterior Angle Theorem** states:

> *The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.*

In this problem, you will prove the Triangle Exterior Angle Theorem using a **two-column proof.** A two-column proof is a way of writing a proof such that each step is listed in one column and the reason for each step is listed in the other column.

1. Describe each rule, definition, postulate, or theorem.

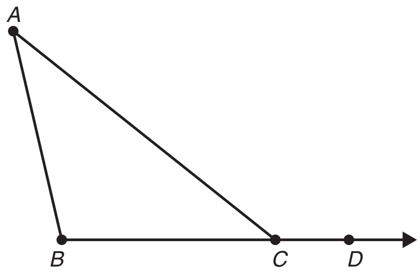
a. Subtraction Property of Equality

b. Triangle Sum Theorem

c. Linear Pair Postulate

d. Definition of Linear Pair

2. The reasons for the proof are provided. Write each step of the proof. The final step should be what you are trying to prove.



Given: Triangle *ABC* with exterior $\angle ACD$

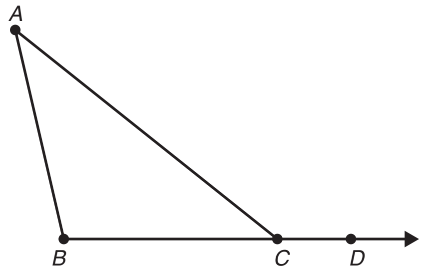
Prove: $m\angle A + m\angle B = m\angle ACD$

Statements	Reasons
	1. Given
	2. Triangle Sum Theorem
	3. Linear Pair Postulate
	4. Definition of Linear Pair
	5. Substitution using equations from steps 2 and 4
	6. Subtraction Property of Equality

Problem 2

A proof by contradiction is one type of indirect proof. A proof by contradiction begins with a **negation of the conclusion,** which means that you assume the opposite of the conclusion. When a contradiction is developed, then the conclusion must be true.

1. Prove the Triangle Exterior Angle Theorem using a proof by contradiction. Each step of the proof is provided. Write a reason for each step.



Given: Triangle *ABC* with exterior $\angle ACD$

Prove: $m\angle A + m\angle B = m\angle ACD$

Statements	Reasons
1. Triangle *ABC* with exterior $\angle ACD$	
2. $m\angle A + m\angle B \neq m\angle ACD$	
3. $m\angle A + m\angle B + m\angle BCA \neq m\angle ACD + m\angle BCA$	
4. $m\angle A + m\angle B + m\angle BCA = 180°$	
5. $\angle BCA$ and $\angle ACD$ are a linear pair	
6. $m\angle BCA + m\angle ACD = 180°$	
7. $180° \neq 180°$	

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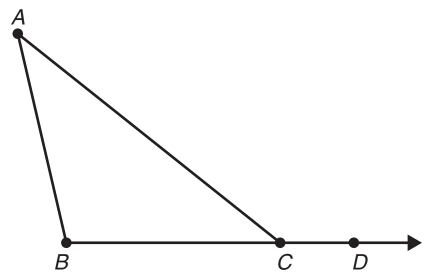
Problem 3

The **Exterior Angle Inequality Theorem** states:

An exterior angle of a triangle is greater than either of the remote interior angles of the triangle.

The direct proof requires two parts. The first part will prove the theorem for the first remote interior angle. The second part will prove the theorem for the second remote interior angle.

1. The reasons for the first part of the proof are provided. Write each step of the proof.



Given: Triangle ABC with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle A$

Statements	Reasons
	1. Given
	2. Triangle Sum Theorem
	3. Linear Pair Postulate
	4. Definition of Linear Pair
	5. Substitution using equations from steps 2 and 4
	6. Subtraction Property of Equality
	7. Definition of Angle Measure
	8. Inequality Property: If $a = b + c$ and $c > 0$, then $a > b$

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2. The reasons for the second part of the proof are provided. Write each step of the proof.

Given: Triangle ABC with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle B$

Statements	Reasons
	1. Given
	2. Triangle Sum Theorem
	3. Linear Pair Postulate
	4. Definition of Linear Pair
	5. Substitution using equations from steps 2 and 4
	6. Subtraction Property of Equality
	7. Definition of Angle Measure
	8. Inequality Property: If $a = b + c$ and $c > 0$, then $a > b$

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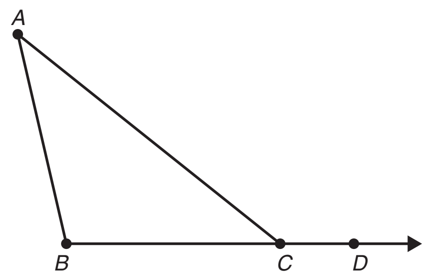
Problem 4

1. Prove the Triangle Exterior Angle Theorem using a proof by contradiction. The reasons for the first part of the proof are provided. Write each step of the proof.



Given: Triangle *ABC* with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle A$

Statements	Reasons
	1. Given
	2. Negation of Conclusion
	3. Triangle Sum Theorem
	4. Linear Pair Postulate
	5. Definition of Linear Pair
	6. Substitution using equations from steps 2 and 5
	7. Substitution using equations from steps 3 and 6
	8. Angle Subtraction
	9. Angle Subtraction
	10. Definition of Triangle

2. Prove the Triangle Exterior Angle Theorem using a proof by contradiction. The reasons for the second part of the proof are provided. Write each step of the proof.

Given: Triangle *ABC* with exterior $\angle ACD$

Prove: $m\angle ACD > m\angle B$

Statements	Reasons
	1. Given
	2. Negation of Conclusion
	3. Triangle Sum Theorem
	4. Linear Pair Postulate
	5. Definition of Linear Pair
	6. Substitution using equations from steps 2 and 5
	7. Substitution using equations from steps 3 and 6
	8. Angle Subtraction
	9. Angle Subtraction
	10. Definition of Triangle

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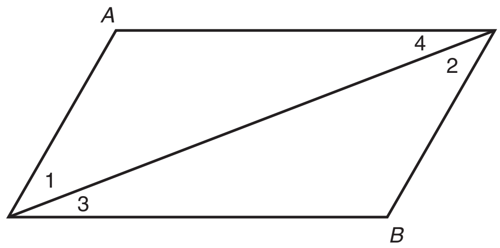
Problem 5

1. Complete the direct proof.



Given: $m\angle 1 = m\angle 2$, $m\angle 4 = m\angle 3$

Prove: $m\angle A = m\angle B$

Statements	Reasons

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2. Complete the proof from Question 1 using an indirect proof.

Given: $m\angle 1 = m\angle 2$, $m\angle 4 = m\angle 3$

Prove: $m\angle A = m\angle B$

Statements	**Reasons**

Be prepared to share your solutions and methods.

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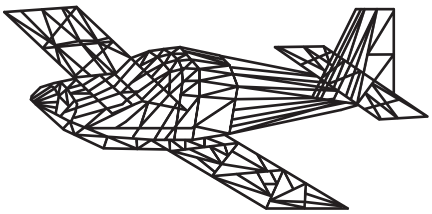
5.5 Computer Graphics
Proving Triangles Congruent: SSS and SAS

Objectives
In this lesson you will:
- Prove the Side-Side-Side (SSS) Congruence Theorem.
- Use given information to show that two triangles are congruent.

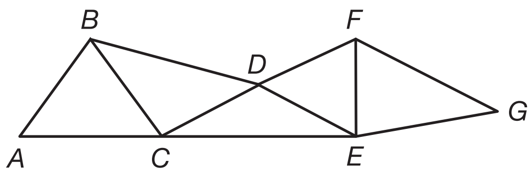
Key Terms
- Side-Side-Side Congruence Theorem
- Side-Angle-Side Congruence Theorem
- paragraph proof

In three-dimensional computer graphics, a triangle is one of the shapes that is used to create realistic graphics.



Problem 1 Constructing the Strip

To make the file size that contains the graphic smaller and to make the graphic display faster, *triangle strips* in the graphic are identified. A triangle strip is a list of triangles that share vertices. A triangle strip is shown.



A. Identify the vertices that are shared by triangles in this triangle strip.

B. For each triangle, identify the number of vertices that are shared with another triangle.

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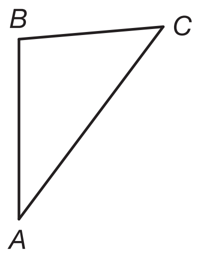
C. For each triangle, identify the number of sides that are shared with another triangle.

D. Consider this triangle. Use a compass and a straightedge to create a triangle strip of two triangles in the following way. Open your compass to the length of $\overline{AC}$. Place the compass point on point A and draw an arc to the left of $\overline{AB}$. Now open your compass to the length of $\overline{BC}$. Place the compass point on point B and draw an arc to the left of $\overline{AB}$ so that it intersects the first arc. Label the intersection of the arcs as point D. Then use your straight edge to draw $\overline{AD}$ and $\overline{BD}$.



E. Without measuring, how do the lengths of the sides of ΔABC and ΔADB compare? How do you know?

Use a ruler to verify your answer.

F. Use a protractor to measure the interior angles of the triangles. What can you conclude about the triangles?

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Investigate Problem 1

1. Suppose that you have three line segments with fixed lengths and you make a triangle with these three line segments. Is it possible to form a new triangle from the same three line segments whose corresponding angles are not congruent to angles of the original triangle? Why or why not?

2. If the corresponding sides of two triangles are congruent, what can you conclude about the corresponding angles of the triangles? Explain your reasoning.

If you know that the corresponding sides of two triangles are congruent, then the triangles are congruent. This result is called the **Side-Side-Side (SSS) Congruence Theorem.**

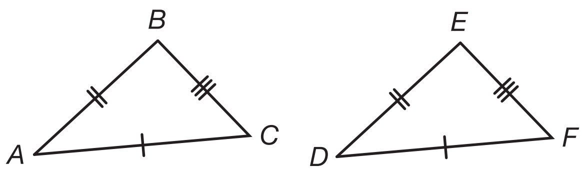
Take Note

The *SSS Similarity Postulate* states that if the corresponding sides of two triangles are proportional, then the triangles are similar.

The *SAS Similarity Postulate* states that if two pairs of corresponding sides of two triangles are proportional and the included angles are congruent, then the triangles are similar.

3. How does the SSS Congruence Theorem follow from the SSS Similarity Postulate?

Complete the two-column proof of the SSS Congruence Theorem.



Statements	Reasons
1. $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$	**1.** ____________________
2. $AC = \square$, $\square = DE$, $BC = \square$	**2.** Definition of Congruence
3. $\frac{AC}{DF} = \square$, $\frac{AB}{DE} = \square$, $\frac{BC}{EF} = \square$	**3.** Division Property of Equality
4. $\frac{\square}{DF} = \frac{AB}{\square} = \frac{BC}{\square}$	**4.** Transitive Property of Equality
5. $\Delta ABC \sim \Delta DEF$	**5.** ____________________
6. $\angle A \cong \square$, $\square = \angle E$, $\angle C \cong \square$	**6.** Definition of similar triangles
7. $\Delta ABC \cong \Delta DEF$	**7.** Definition of congruence

4. Two sides of one triangle are congruent to two sides of another triangle, and the angles formed by these sides are congruent. Draw and label a diagram of this situation. Be sure to name the vertices of the triangles.

What do you know about the ratios of the lengths of the corresponding congruent sides?

Are the triangles similar? Why or why not?

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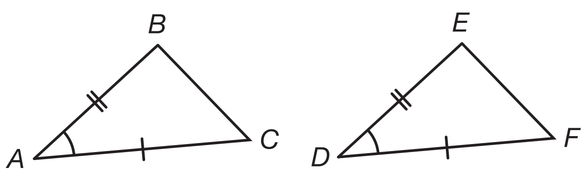
Are the corresponding angles of the triangles congruent? Why or why not?

Write ratios that compare the lengths of the corresponding sides of the triangles. Are these ratios equal? Why or why not?

Are the triangles congruent? Why or why not?

5. Write a congruence theorem that follows from the SAS Similarity Postulate.

Use this figure to complete the statement of this theorem, the **Side-Angle-Side (SAS) Congruence Theorem,** in symbols.



If $\boxed{} \cong \boxed{}$, $\boxed{} \cong \boxed{}$, and $\boxed{} \cong \boxed{}$,

then $\boxed{}$.

You can use an argument similar to the one in Question 3 to prove the SAS Congruence Theorem.

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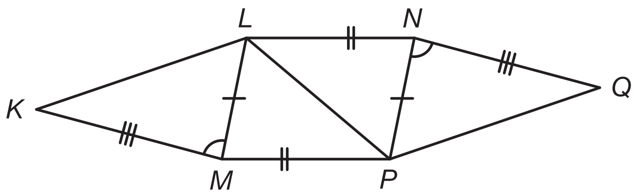
6. A triangle strip is shown. Use a **paragraph proof** to show that ΔKLM is congruent to ΔQPN. Then use a paragraph proof to show that ΔMLP is congruent to ΔNPL.



Be prepared to share your solutions and methods.

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5.6 Wind Triangles
Proving Triangles Congruent: ASA and AAS

Objectives

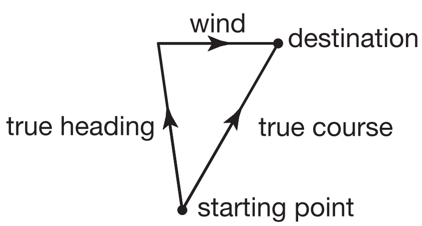
In this lesson you will:

- Use given information to show that two triangles are congruent.
- Determine whether there is enough information given to determine whether two triangles are congruent.

Key Terms

- Angle-Side-Angle Congruence Postulate
- Angle-Angle-Side Congruence Theorem

When airplanes fly from one destination to another, their exact course and speed are determined by the speed of the wind in the air and the direction of the wind. The calculation of the course involves a triangle called a *wind triangle*. In the wind triangle shown, the true course is the straight line from the starting point to the destination. The true heading is the course that must be flown to account for the wind in order to arrive at the destination. The last leg of the triangle shows the direction of the wind. The arrows indicate direction.



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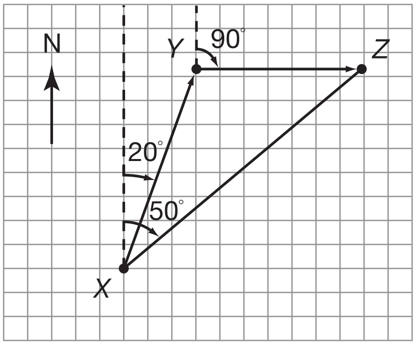
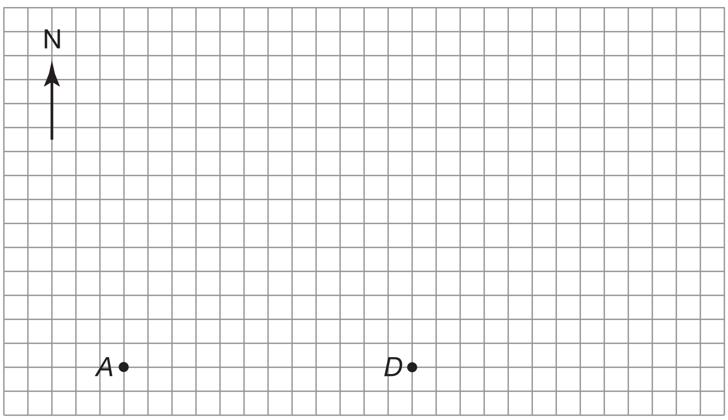
Problem 1 Constructing Wind Triangles

In this grid, one square represents a square that is 10 knots by 10 knots. A *knot* is a unit that is used to measure speed in nautical miles per hour. The velocity vector from *X* to *Y* represents a true heading of 020°. The velocity vector from *X* to *Z* represents a true course of 050°. The velocity vector from *Y* to *Z* represents a wind direction of 090°. Notice that we use three digits to write the angle measurements.



A. An airplane starts out at *A*, and its true heading is 030°. This means that the true heading is 30° clockwise of true north, shown in the upper left corner in the grid. Place the center of your protractor along the vertical line that passes through *A*. Line up the bottom of the protractor with *A*. On the grid mark 30° clockwise from true north.



The plane is traveling at a speed of 100 knots. Open your compass to measure 100 knots on the grid. Then place the point of your compass at *A* and draw an arc that is above *A*. Then draw a velocity vector from *A* to the arc that passes through your 30° mark. This velocity vector is your true heading. Mark the unknown endpoint as *B*.

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B. The true course is 040°. Place the center of your protractor along the vertical line that passes through *A*. Line up the bottom of the protractor with *A*. On the grid mark 40° clockwise from true north and draw a velocity vector that starts at *A* and passes through your mark.

C. The wind direction is 080°. Place the center of your protractor along the vertical line that passes through *B*. Line up the bottom of the protractor with *B*. On the grid mark 80° clockwise from true north and draw a line segment that starts at *B* and passes through your mark and meets the velocity vector from part (B). Label the last vertex as *C*.

D. Determine the measure of $\angle BAC$ without using your protractor. Show all your work and label this angle on the triangle.

E. Label the length of $\overline{AB}$ on the triangle.

F. Determine the measure of $\angle CBA$. Show all your work and label this angle on the triangle. Hint: Draw a diagram of the triangle and draw vertical velocity vectors through *A* and *B*. Then use what you know about the measures of the angles formed by parallel lines and a transversal.

G. Another airplane starts out at point *D*. Its true heading is 040° and its speed is 100 knots. The true course of this plane is 050°. The wind direction is 090°. Follow the steps you used to draw ΔABC to draw the wind triangle for this airplane on the grid in part A. Label the other vertices of this triangle as *E* and *F*.

H. Compare the side lengths and interior angle measures of ΔABC and ΔDEF. What do you notice?

I. What information about both triangles' sides and angles did you need to know in order to draw the triangles?

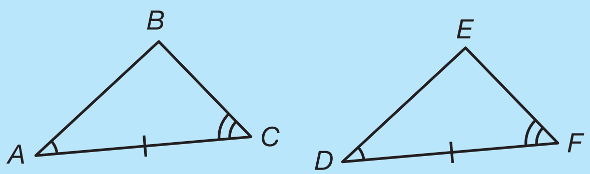
How does this information in the triangles compare?

Investigate Problem 1

1. The result of Problem 1 is called the *Angle-Side-Angle (ASA) Congruence Postulate*.

Angle-Side-Angle (ASA) Congruence Postulate

If two angles of one triangle are congruent to two angles of another triangle and the included sides are congruent, then the triangles are congruent.



If $\angle A \cong \angle D$, $\angle C \cong \angle F$, and $\overline{AC} \cong \overline{DF}$, then $\Delta ABC \cong \Delta DEF$.

Which of the similarity postulates do you think that this postulate is related to? Explain your reasoning.

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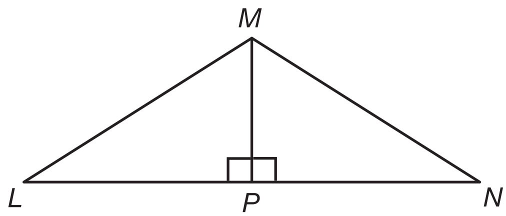
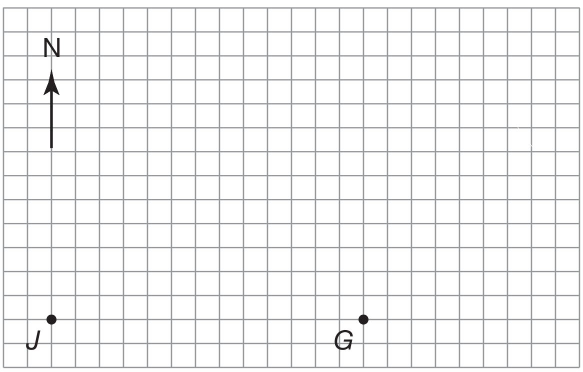
2. Use the given information to show that $\Delta LMP \cong \Delta NMP$. Explain your reasoning.



$\overline{MP}$ is the angle bisector of $\angle LMN$.

Problem 2 Different Sides and Different Angles



A. An airplane is starting from G. Its true heading is 015° and its speed is 80 knots. Draw the true heading and label its length on the grid. Label the unknown endpoint as H.

B. The true course is 040°. Draw a velocity vector on the grid that represents the true course. Label the measure of $\angle G$ on the grid.

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C. The angle between the wind direction and the true course is 40°. Can you use this information to determine the location of the last vertex of the wind triangle? If so, determine the location of the vertex on the grid and label the point as *I*. Label the angle measure you used to determine *I* on the grid. Explain your method.

D. Another airplane is starting from *J*. Its true heading is 035° and its speed is 80 knots. Draw the true heading and label its length on the grid and label the unknown endpoint as *K*.

E. The angle between this airplane's true heading and true course is the same as that of the airplane in part (A) through part (C). What is the true course?

Draw a line segment on the grid that represents the true course. Label the measure of $\angle J$ on the grid.

F. The wind direction is 100°. Use the location of *K* to determine the location of the last vertex on the grid and label this point as *L*.

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Investigate Problem 2

1. Compare the side lengths and interior angle measures of ΔGHI and ΔJKL. What do you notice?

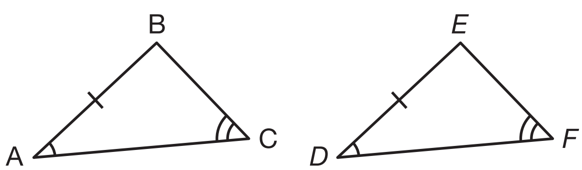
2. What information did you have about the triangles in order to draw them?

3. If two angles of one triangle are congruent to two angles of another triangle, what other piece of information do you need to know in order to conclude that the triangles are congruent? Explain your reasoning.

4. The ASA Congruence Postulate allows you to develop the *Angle-Angle-Side (AAS) Congruence Theorem*.

Angle-Angle-Side (AAS) Congruence Theorem
If two angles of one triangle are congruent to two angles of another triangle and two corresponding non-included sides are congruent, then the triangles are congruent.

Complete the statement of this theorem in symbols.



If $\angle A \cong$ ☐, ☐ $\cong \angle F$, and $\overline{AB} \cong$ ☐,

then ☐.

This theorem can be proved by using the Angle-Angle-Side (AAS) Congruence Theorem.

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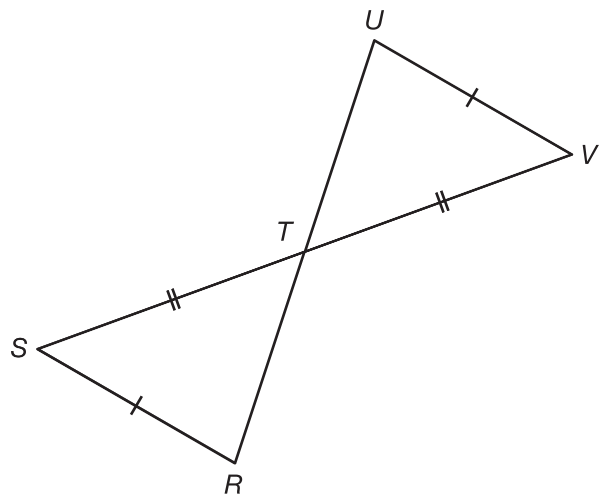
5. Can you use the information given in the figure to show that $\Delta STR \cong \Delta VTU$ If so, explain how. If not, explain why not.



Be prepared to share your solutions and methods.

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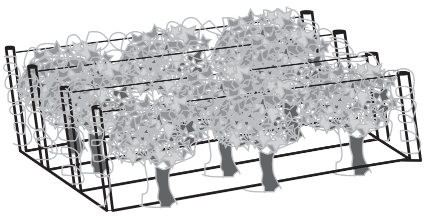
5.7 Planting Grape Vines
Proving Triangles Congruent: HL

Objectives
In this lesson you will:
- Use given information to show that two triangles are congruent.
- Prove the Hypotenuse-Leg (HL) Congruence Theorem.

Key Term
- Hypotenuse-Leg Congruence Theorem

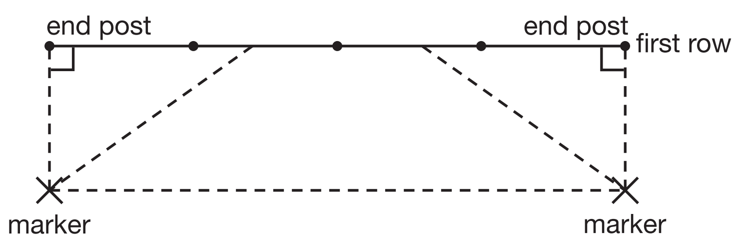
At a vineyard, grape vines are planted in straight rows that are parallel to each other. Vertical posts are erected along a row, and horizontal wires are attached to these posts so that the vines can grow along these wires.



Problem 1 Locating the Row

After the first row is in place, the people that are erecting the posts can make sure that the rows are straight and are parallel to each other by using right triangles.

Right triangles are used to locate the positions of markers that will be used to ensure that the end posts of each row lie in straight lines. These markers are also the locations of the end posts of a row. This figure shows an overhead view of the vineyard.



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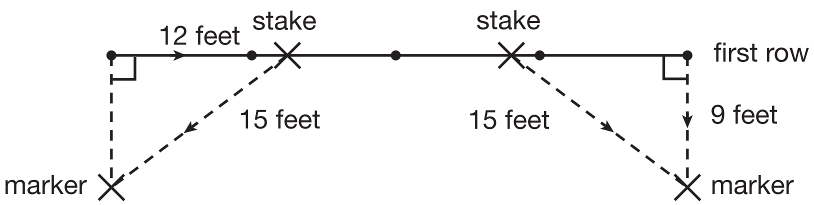
A. In order for the first row and the row formed by the markers to be parallel, what must be true about the distances between the markers and the end posts?

B. In order for the first row and the row formed by the markers to be parallel, what must be true about the triangles used to determine the locations of the markers?

Investigate Problem 1

1. Two groups of construction workers needed to lay out two ninety degree angles on either end of a line between two posts. Each group was given three lengths of rope, 9 feet, 12 feet, and 15 feet long. One group laid the 12-foot rope on the line between the posts, then took the 9-foot piece from the left post and the 15-foot length from the other end of the 12-foot rope and walked them out until they met to form a triangle, as shown. The other group walked the 9-foot length of the rope from the right post straight toward the second row, making sure that the rope formed a 90-degree angle with the first row. They then took the 12-foot length from the post and the 15-foot length from the marker and walked them out to form a triangle as shown.



Are they right triangles? Are the triangles congruent? Justify your answers.

2. Two groups of construction workers needed to lay out two ninety degree angles on either end of a line between two posts. Each group was given three lengths of rope, 18 feet, 24 feet, and 30 feet long. One group laid the 24-foot rope on the line between the posts, then took the 18-foot piece from the left post and the 30-foot length from the other end of the 24-foot rope and walked them out until they met to form a triangle. The other group laid the 18-foot length of the rope from the right post straight toward the second row, making sure that the rope formed a 90-degree angle with the first row. They then took the 24-foot length from the post and the 30-foot length from the marker and walked them out to form a triangle.

Draw a diagram of the situation. Are they right triangles? Are the triangles congruent? Justify your answers.

3. Do you think that you need to determine the lengths of all three sides of two right triangles in order to determine that the triangles are congruent? Explain your reasoning.

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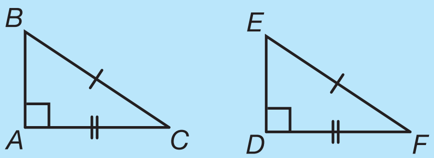
4. If you know that two triangles are right triangles and the corresponding legs are congruent, how do you know that the triangles are congruent? Explain your reasoning.

5. If you know that two triangles are right triangles and a pair of legs is congruent and the hypotenuses are congruent, how do you know that the triangles are congruent? Explain your reasoning.

6. Complete this two-column proof to prove the *Hypotenuse-Leg (HL) Congruence Theorem*.

Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.



If $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$, then $\Delta ABC \cong \Delta DEF$.

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Statements	Reasons
1. ΔABC and ΔDEF are ______________.	**1.** Given
2. $\overline{AC} = \square$ and $\overline{BC} = \square$	**2.** Given
3. $\boxed{}$	**3.** Definition of congruence
4. $AB^2 + \square^2 = BC^2$; $\square^2 + DF^2 = EF^2$	**4.** Pythagorean Theorem
5. $AB^2 + DF^2 = EF^2$	**5.** ____________ Property of Equality
6. $AB^2 + DF^2 = DE^2 + DF^2$	**6.** ____________ Property of Equality
7. $\square^2 = DE^2$	**7.** Subtraction Property of Equality
8. $\square = DE$	**8.** Property of Square Roots
9. $\overline{AB} \cong \square$	**9.** Definition of congruence
10. $\Delta ABC \cong \Delta DEF$	**10.** _____ Congruence Theorem

Be prepared to share your solutions and methods.

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