CHAPTER

9 Function Transformations and Symmetry

The first well-documented postal system was in ancient Rome, where mail was carried by horse-drawn carriages and ox-drawn wagons. The US Postal Service delivers approximately 200 billion letters and 1 billion packages each year. Most businesses use both public and private mail systems to ship products to their customers. You will use graphs of equations to determine the effect of shipping costs on the total cost of purchasing machine tools.

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9.1 Shifting Away
Vertical and Horizontal Translations

Objectives
In this lesson, you will:
- Translate graphs of functions vertically and horizontally.
- Write equations for translated functions.

Key Terms
- vertical translation
- horizontal translation

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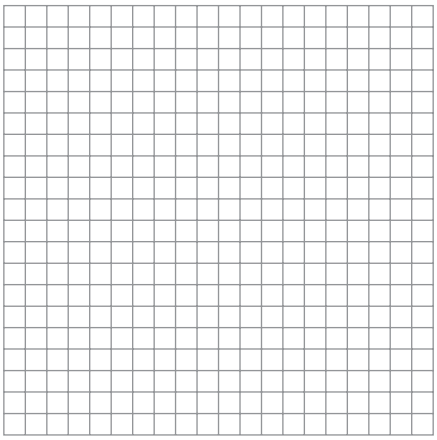
Problem 1 Moving On Up!

1. Graph each linear function on the grid shown.

a. $f(x) = x$

b. $y = x + 5$

c. $y = x - 5$

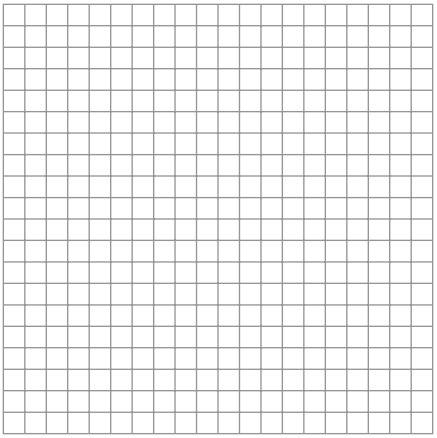


2. Graph each absolute value function on the grid shown.

a. $y = |x|$

b. $y = |x| + 5$

c. $f(x) = |x| - 5$

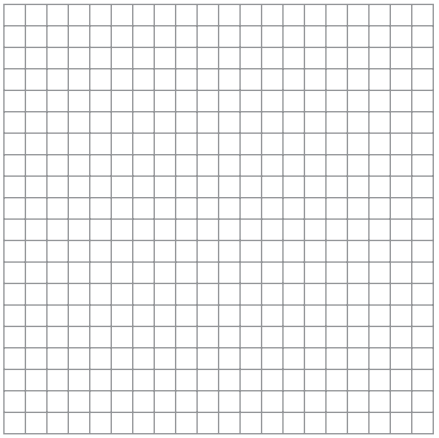


3. Graph each quadratic function on the grid shown.

a. $y = x^2$

b. $y = x^2 + 5$

c. $f(x) = x^2 - 5$



4. In Questions 1 through 3, part (a) is a basic function. What operation is performed to each basic function to result in the equations in parts (b) and (c)?

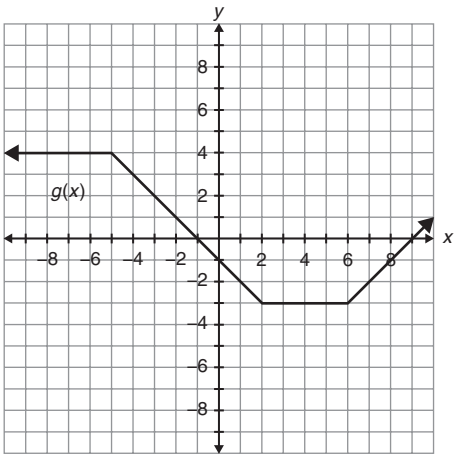
5. Describe how the graph of $f(x) + k$ is formed from the graph of $f(x)$.

A **vertical translation** of a graph is a shift of the entire graph up or down.

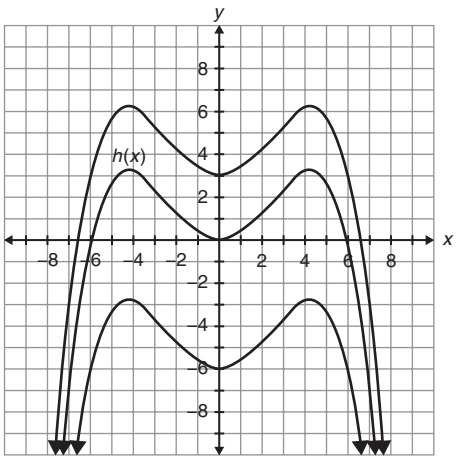
6. The graph of a function $g(x)$ is shown. Sketch the graph of

a. $g(x) + 2$

b. $g(x) - 4$



7. The graph of the function $h(x)$ is shown. Write a function in terms of $h(x)$ for each vertical translation shown.



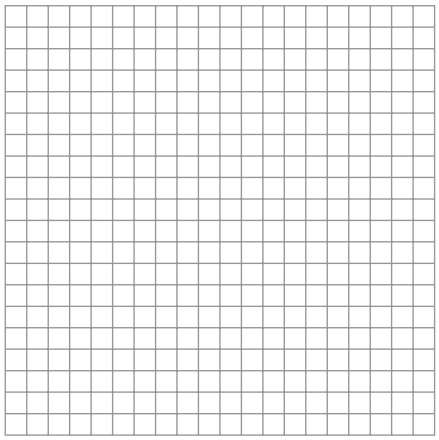
Problem 2 To the Left, to the Right!

1. Graph each quadratic function on the grid shown.

a. $f(x) = x^2$

b. $y = (x + 5)^2$

c. $y = (x - 5)^2$

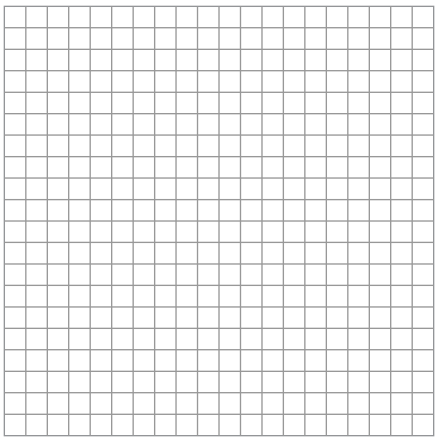


2. Graph each absolute value function on the grid shown.

a. $y = |x|$

b. $y = |x + 5|$

c. $f(x) = |x - 5|$

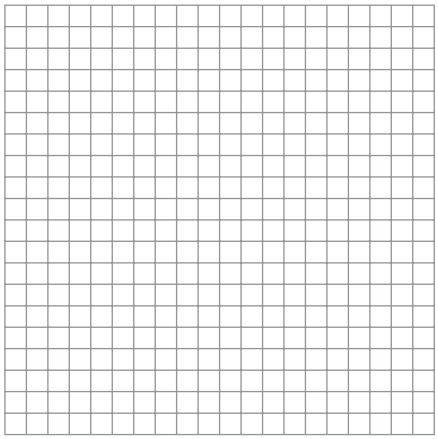


3. Graph each square root function on the grid shown.

a. $y = \sqrt{x}$

b. $y = \sqrt{x + 5}$

c. $f(x) = \sqrt{x - 5}$



4. In Questions 1 through 3, part (a) is a basic function. What operation is performed on each basic function to result in the equations in parts (b) and (c)?

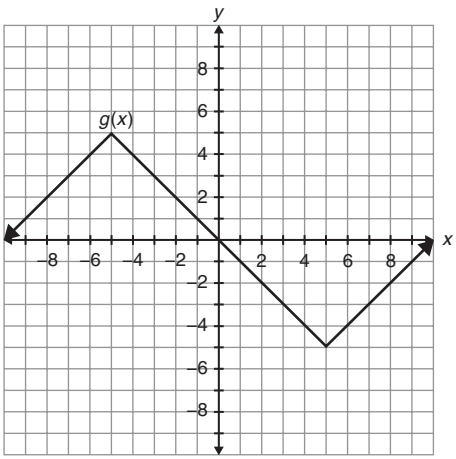
5. Describe how the graph of $f(x - h)$ is formed from the graph of $f(x)$.

A **horizontal translation** of a graph is a shift of the entire graph to the left or to the right.

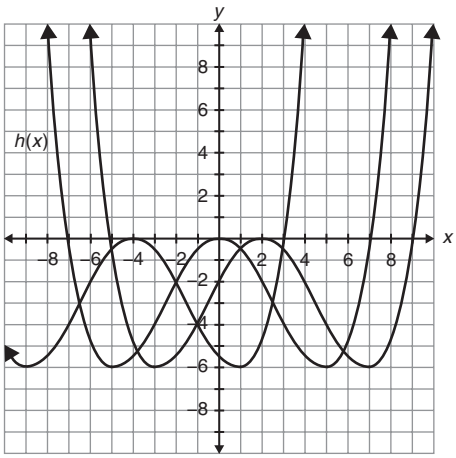
6. The graph of a function $g(x)$ is shown. Sketch the graph of

a. $g(x - 2)$

b. $g(x + 3)$



7. The graph of the function $h(x)$ is shown. Write a function in terms of $h(x)$ for each horizontal translation shown.



Be prepared to share your methods and solutions.

9.2 Expanding, Contracting, and Mirroring
Dilations and Reflections

Objectives
In this lesson, you will:
- Dilate graphs of functions.
- Reflect graphs of functions.
- Write equations for dilated and rotated functions.

Key Terms
- dilation
- reflection
- line of reflection

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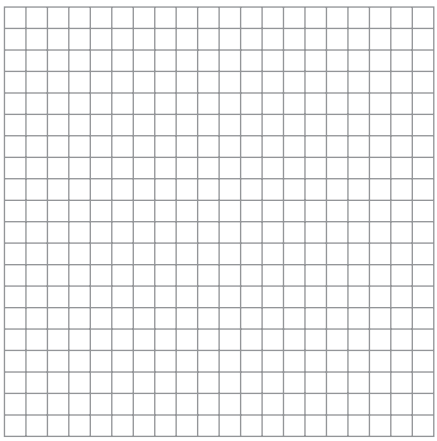
Problem 1 Dilating

1. Graph each absolute value function on the grid shown.

a. $f(x) = |x|$

b. $g(x) = 2|x|$

c. $h(x) = 5|x|$



2. Complete the table to calculate the rate of change from 0 to 1 for each function in Question 1.

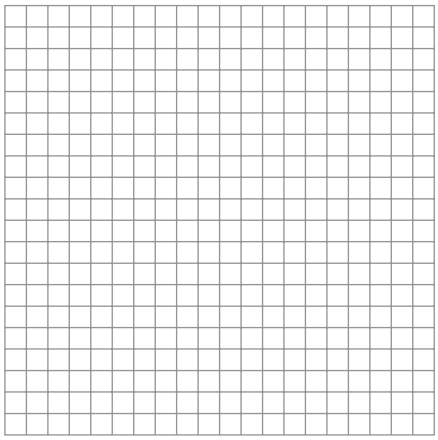
Function	Value at $x = 0$	Value at $x = 1$	Rate of Change
$f(x) = \lvert x \rvert$	$f(0) =$	$f(1) =$	$\frac{\Delta f(x)}{\Delta x} =$
$g(x) = 2\lvert x \rvert$	$g(0) =$	$g(1) =$	$\frac{\Delta g(x)}{\Delta x} =$
$h(x) = 5\lvert x \rvert$	$h(0) =$	$h(1) =$	$\frac{\Delta h(x)}{\Delta x} =$

3. Graph each quadratic function on the grid shown.

a. $f(x) = x^2$

b. $g(x) = 2x^2$

c. $h(x) = 3x^2$



4. Complete the table to calculate the average rate of change from 0 to 2 for each function in Question 3.

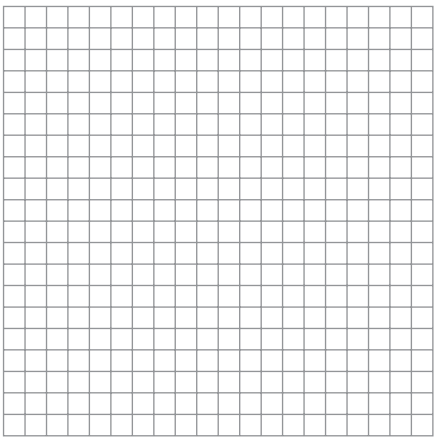
Function	Value at $x = 0$	Value at $x = 2$	Average Rate of Change
$f(x) = x^2$	$f(0) =$	$f(2) =$	$\frac{\Delta f(x)}{\Delta x} =$
$g(x) = 2x^2$	$g(0) =$	$g(2) =$	$\frac{\Delta g(x)}{\Delta x} =$
$h(x) = 3x^2$	$h(0) =$	$h(2) =$	$\frac{\Delta h(x)}{\Delta x} =$

5. Graph each square root function on the grid shown.

a. $f(x) = \sqrt{x}$

b. $g(x) = 0.5\sqrt{x}$

c. $h(x) = 0.25\sqrt{x}$



6. Complete the table to calculate the average rate of change from 0 to 4 for each function in Question 5.

Function	Value at $x = 0$	Value at $x = 4$	Average Rate of Change
$f(x) = \sqrt{x}$	$f(0) =$	$f(4) =$	$\frac{\Delta f(x)}{\Delta x} =$
$g(x) = 0.5\sqrt{x}$	$g(0) =$	$g(4) =$	$\frac{\Delta g(x)}{\Delta x} =$
$h(x) = 0.25\sqrt{x}$	$h(0) =$	$h(4) =$	$\frac{\Delta h(x)}{\Delta x} =$

7. In Questions 1, 3, and 5, part (a) is a basic function. What operation is performed on each basic function to result in the equations in parts (b) and (c)?

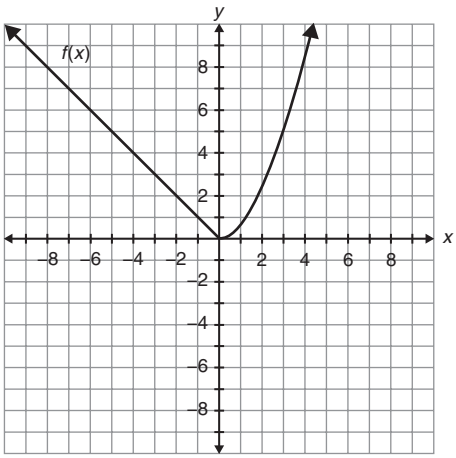
8. Describe how the graph of $a \cdot f(x)$ is formed from the graph of $f(x)$.

A **dilation** of a graph is an increase or decrease of the rate of change by a constant amount.

9. The graph of the function $f(x)$ is shown. Sketch the graph of

a. $2f(x)$

b. $0.5f(x)$

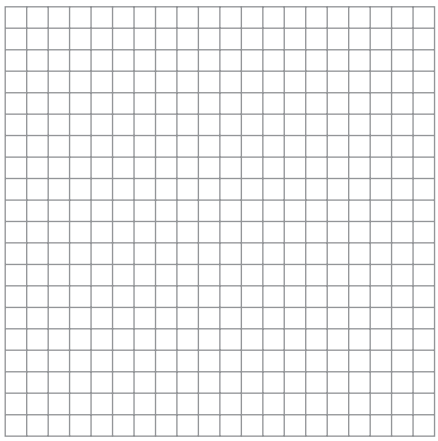


Problem 2 Reflecting

1. Graph each linear function on the grid shown.

a. $f(x) = x + 5$

b. $g(x) = -x + 5$



2. Evaluate the functions for each value.

a. $f(2) =$ $g(-2) =$

b. $f(-2) =$ $g(2) =$

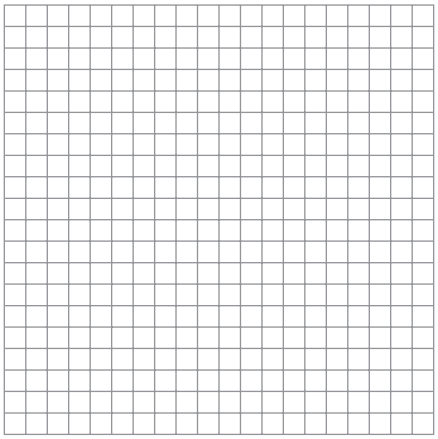
c. $f(5) =$ $g(-5) =$

d. $f(-5) =$ $g(5) =$

3. Graph each square root function on the grid shown.

a. $f(x) = \sqrt{x}$

b. $g(x) = \sqrt{-x}$



4. Evaluate the functions for each value.

a. $f(1) =$ $g(-1) =$

b. $f(4) =$ $g(-4) =$

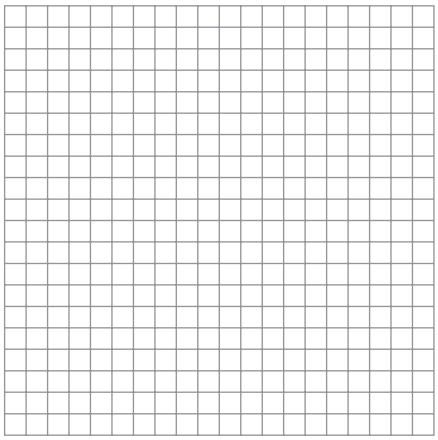
c. $f(9) =$ $g(-9) =$

d. $f(16) =$ $g(-16) =$

5. Graph each cubic function on the grid shown.

a. $f(x) = x^3$

b. $g(x) = (-x)^3 = -x^3$



6. Evaluate the functions for each value.

a. $f(1) =$ $g(-1) =$

b. $f(2) =$ $g(-2) =$

c. $f(-3) =$ $g(3) =$

d. $f(-2) =$ $g(2) =$

7. In Questions 1, 3, and 5, can you write $g(x)$ in terms of $f(x)$? Explain.

8. In Questions 1, 3, and 5, what do you notice about the graphs of $f(x)$ and $g(x)$?

9. If the ordered pair (x, y) is on the graph of $f(x)$, then what point must be on the graph of $f(-x)$?

A **reflection** of a graph is the mirror image of the graph about a line. The line that the graph is reflected about is the **line of reflection.**

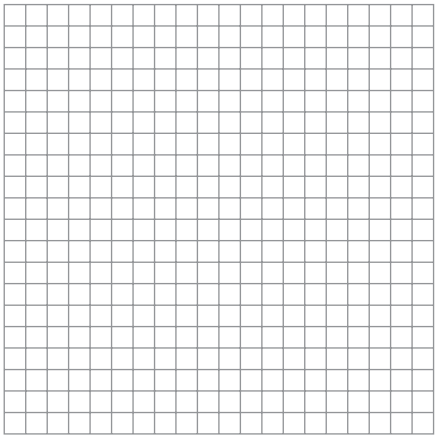
10. What is the line of reflection for the graphs in Questions 1, 3, and 5?

Problem 3 More Reflecting

1. Graph each linear function on the grid shown.

a. $f(x) = x + 5$

b. $g(x) = -x - 5$



2. Evaluate the functions for each value.

a. $f(2) =$ $g(2) =$

b. $f(-2) =$ $g(-2) =$

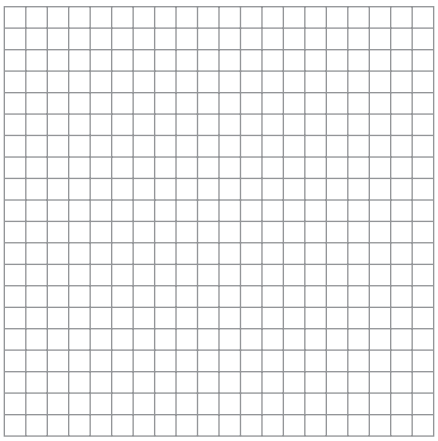
c. $f(5) =$ $g(5) =$

d. $f(-5) =$ $g(-5) =$

3. Graph each square root function on the grid shown.

a. $f(x) = \sqrt{x}$

b. $g(x) = -\sqrt{x}$



4. Evaluate the functions for each value.

   a. $f(1) =$ $g(1) =$

   b. $f(4) =$ $g(4) =$

   c. $f(9) =$ $g(9) =$

   d. $f(16) =$ $g(16) =$

5. In Questions 1 and 3, can you write $g(x)$ in terms of $f(x)$? Explain.

6. In Questions 1 and 3, what do you notice about the graphs of $f(x)$ and $g(x)$?

7. If the ordered pair (x, y) is on the graph of $f(x)$, then what point must be on the graph of $f(-x)$?

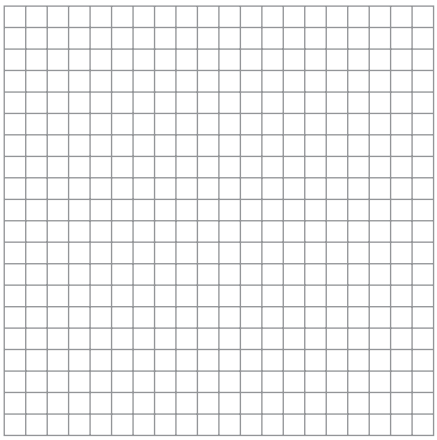
8. What is the line of reflection for the graphs in Questions 1 and 3?

Problem 4 Reflecting Twice

1. Graph the following square root functions using technology.

   a. $f(x) = \sqrt{x}$

   b. $g(x) = -\sqrt{-x}$



2. Evaluate the functions for each value.

a. $f(1) =$ $g(-1) =$

b. $f(4) =$ $g(-4) =$

c. $f(9) =$ $g(-9) =$

d. $f(16) =$ $g(-16) =$

3. Can you write $g(x)$ in terms of $f(x)$? Explain.

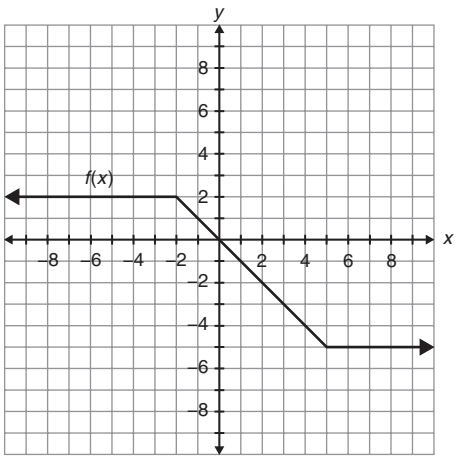
4. What do you notice about the graphs of $f(x)$ and $g(x)$?

5. If the ordered pair (x, y) is on the graph of $f(x)$, then what point must be on the graph of $g(x)$?

6. The graph of $f(x)$ is shown. Sketch the graph of

a. $g(x) = f(-x)$

b. $h(x) = -f(x)$



Be prepared to share your methods and solutions.

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9.3 Mirroring!
Symmetry and Odd/Even

Objectives
In this lesson, you will:
- Describe the symmetry of functions.
- Classify functions as odd and even.

Key Terms
- even function
- odd function

9

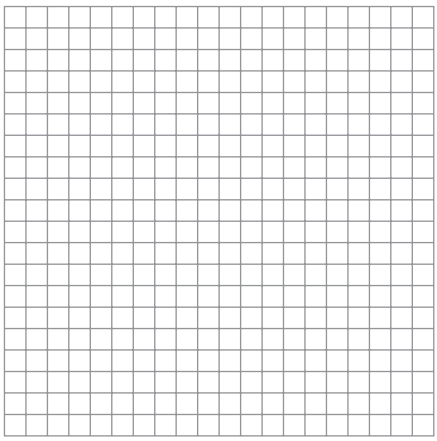
Problem 1 Symmetry

1. Graph each quadratic function on the grid shown.

a. $f(x) = x^2$

b. $g(x) = x^2 - 6x + 8$

c. $h(x) = x^2 + 6x + 8$



2. For each function in Question 1, identify the vertex, *y*-intercept, *x*-intercept(s), and line of symmetry.

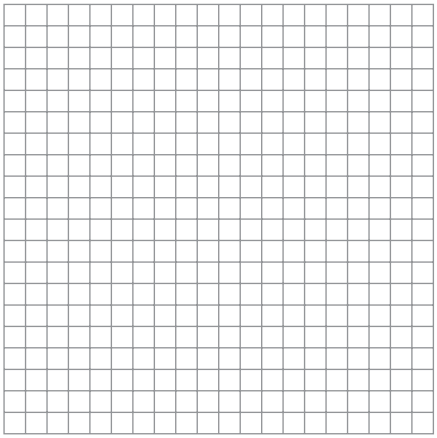
Function	Vertex	*y*-Intercept	*x*-Intercept(s)	Line of Symmetry
$f(x) = x^2$				
$g(x) = x^2 - 6x + 8$				
$h(x) = x^2 + 6x + 8$				

3. Graph each absolute value function on the grid shown.

a. $f(x) = |x|$

b. $g(x) = |x + 2| - 3$

c. $h(x) = |x - 5| + 3$



4. For each function in Question 3, identify the vertex, *y*-intercept, *x*-intercept(s), and line of symmetry.

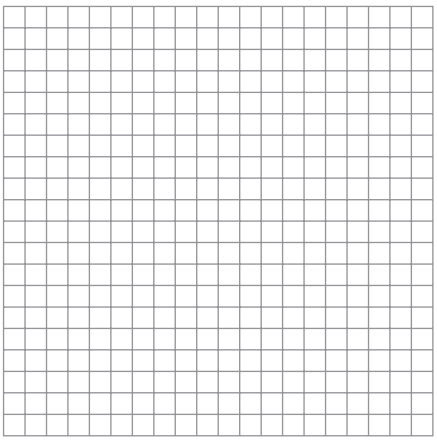
Function	Vertex	*y*-Intercept	*x*-Intercept(s)	Line of Symmetry
$f(x) = \|x\|$				
$g(x) = \|x + 2\| - 3$				
$h(x) = \|x - 5\| + 3$				

5. Graph each cubic function on the grid shown.

a. $f(x) = x^3$

b. $g(x) = x^3 - 1$

c. $h(x) = x^3 - 4x$



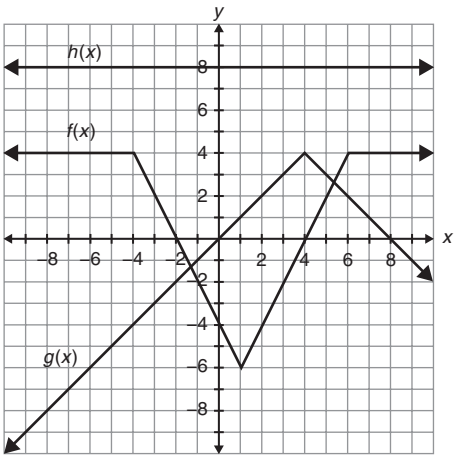
6. For each function in Question 5, identify the *y*-intercept, *x*-intercept(s), and line of symmetry.

Function	Vertex	*y*-Intercept	*x*-Intercept(s)	Line of Symmetry
$f(x) = x^3$				
$g(x) = x^3 - 1$				
$h(x) = x^3 - 4x$				

7. What do you notice about the vertex and the line of symmetry for each function in Questions 1 through 6?

8. What do you notice about the distance between the *x*-intercepts and the line of symmetry for each function in Questions 1–6?

9. Identify the equation of the line of symmetry, if it exists, for each graph shown.



a. $f(x)$

b. $g(x)$

c. $h(x)$

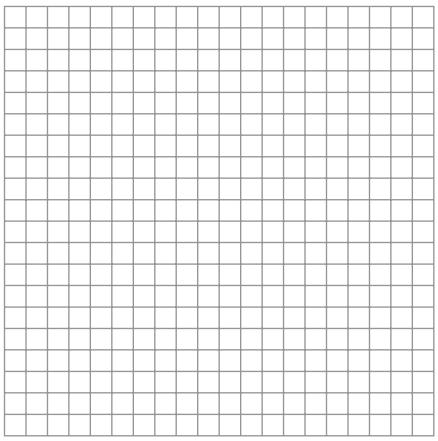
10. Describe the result of reflecting a symmetric graph about its line of symmetry.

Problem 2 Odd and Even Functions

1. Graph each function on the grid shown.

a. $f(x) = x^2$

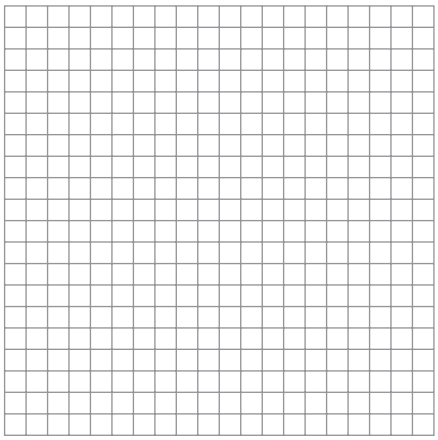
b. $g(x) = x^3 - 4x$



2. Graph each function on the grid shown.

a. $h(x) = x^4 - 5x^2 + 4$

b. $k(x) = x^5 - 5x^3 + 4x$



3. Which functions in Questions 1 and 2 are symmetric about the y-axis?

4. Describe the result of reflecting the graphs of the functions from Question 3 about the *y*-axis.

5. Evaluate the functions used in Questions 1 and 2 for each value.

   a. $f(1) =$

   $f(-1) =$

   b. $f(4) =$

   $f(-4) =$

   c. $f(6) =$

   $f(-6) =$

   d. $f(x) =$

   $f(-x) =$

   e. $h(1) =$

   $h(-1) =$

   f. $h(2) =$

   $h(-2) =$

   g. $h(5) =$

   $h(-5) =$

   h. $h(x) =$

   $h(-x) =$

6. If a function is symmetric about the *y*-axis, what is true about $f(x)$ and $f(-x)$?

An **even function** is a function such that $f(x) = f(-x)$ for all values of x. Even functions are symmetric with respect to the *y*-axis.

7. Describe the result of reflecting the graphs of $g(x)$ and $k(x)$ about the *y*-axis and then about the *x*-axis.

8. Evaluate the functions used in Questions 1 and 2 for each value.

   a. $g(1) =$

   $g(-1) =$

   b. $g(3) =$

   $g(-3) =$

   c. $g(2) =$

   $g(-2) =$

   d. $g(x) =$

   $g(-x) =$

   e. $k(1) =$

   $k(-1) =$

   f. $k(3) =$

   $h(-3) =$

   g. $k(2) =$

   $k(-2) =$

   h. $k(x) =$

   $k(-x) =$

9. What do you notice about $g(x)$ and $g(-x)$? About $k(x)$ and $k(-x)$?

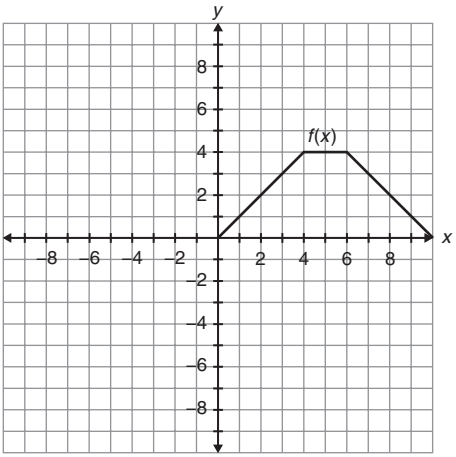
An **odd function** is a function such that $f(x) = -f(-x)$ for all values of x.

10. Classify each function in Questions 1 and 2 as even or odd.

11. The portion of the graph of $f(x)$ to the right of the y-axis is shown. Sketch the portion of the graph to the left of the x-axis if

a. The function is an even function.

b. The function is an odd function.



Be prepared to share your methods and solutions.

9.4 Machine Parts
Solving Equations Graphically

Objective
In this lesson, you will:
- Solve equations algebraically and graphically.

Key Terms
- point of intersection
- consistent
- inconsistent
- identity

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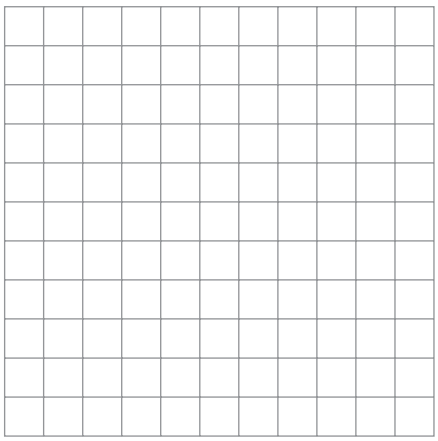
Problem 1 Machine Parts

A company sells a machine part for $15 with a fixed shipping charge of $25 for any number of parts. Another company sells the same part for $16 with a shipping charge of $1.50 per part.

1. Define variables for the independent and dependent quantities. Then write equations for the total cost of an order from each company.

2. Graph the equations from Question 1 on the grid shown.



3. Does it make sense to connect the points of each graph? Explain.

4. Identify the slope and *y*-intercept for each function. What do the slope and *y*-intercept mean in terms of the problem?

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5. An order consists of 10 machine parts. Calculate the cost from each company. Which company provides a lower total cost? Explain.

6. An order consists of less than 10 machine parts. Which company provides a lower total cost? Explain.

7. An order consists of more than 10 machine parts. Which company provides a lower total cost? Explain.

A **point of intersection** of two graphs is a point that both graphs have in common. The ordered pair of the point of intersection satisfies the equations representing the graphs.

The point of intersection can be calculated using tables, graphs, or equations.

8. Complete the table shown.

Number of Machine Parts	Cost from Company A	Cost from Company B
Parts		
x		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

9. Use the table of values to determine the point of intersection. How is the intersection point shown in the table?

10. Can you always use a table to determine a point of intersection? Explain.

11. Use the graph from Question 2 to determine the point of intersection. How is the intersection point shown on the graph?

12. Can you always use a graph to determine a point of intersection? Explain.

13. Use the equations to solve for the point of intersection algebraically.

14. Can you always use the equations to determine a point of intersection? Explain.

Problem 2 Solving Equations Graphically

The methods that you used in Problem 1 to determine the point of intersection can also be used to solve a single equation. To solve a single equation, define a function equal to the left side of the equation and a function equal to the right side of the equation. The x-coordinate of the point of intersection is the solution to the equation.

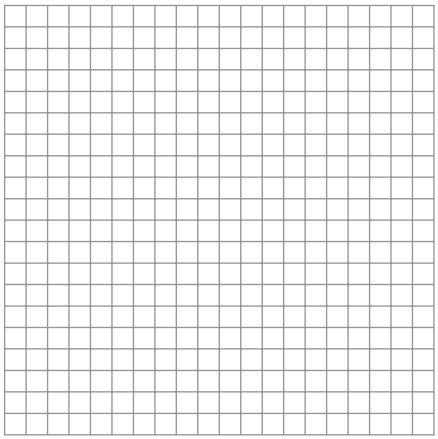
1. Consider the quadratic equation $x^2 = 5x - 4$. Define the function $f(x)$ equal to the left side of the equation and $g(x)$ equal to the right side of the equation.

   $f(x) =$ $g(x) =$

2. Use a table of values to determine the point(s) of intersection of $f(x)$ and $g(x)$. What is the solution to the equation $x^2 = 5x - 4$?

x		
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		

3. Use a graph to determine the point(s) of intersection of $f(x)$ and $g(x)$. What is the solution to the equation $x^2 = 5x - 4$?



4. Compare the solutions using a table of values and the solutions using a graph. What do you notice?

5. Solve the quadratic equation $x^2 = 5x + 3$ using a table of values and a graph.

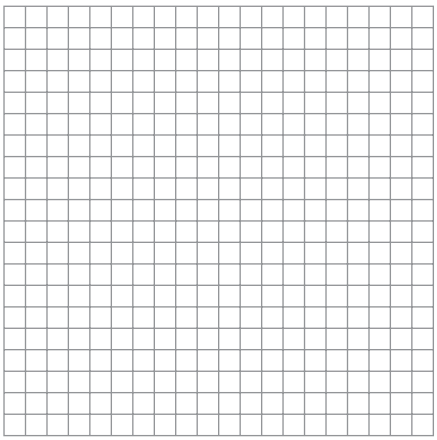
a. Define the function $f(x)$ equal to the left side of the equation and $g(x)$ equal to the right side of the equation.

$f(x) =$ $g(x) =$

b. Table of values:

x		
−2		
−1.5		
−1		
−0.5		
0		
0.5		
1		
1.5		
2		
2.5		
3		
3.5		
4		
4.5		
5		
5.5		
6		

c. Graph:



6. Did you determine exact solutions to $x^2 = 5x + 3$? Explain.

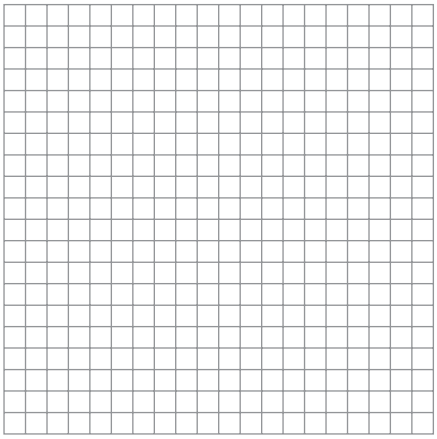
Two equations are **consistent** if the graphs of the two equations have at least one point of intersection. Two equations are **inconsistent** if the graphs of the two equations do not have a point of intersection.

7. Solve the cubic equation $x^3 + 2x^2 - x - 2 = (x + 1)(x - 1)(x + 2)$ using a graph.

a. Define the function $f(x)$ equal to the left side of the equation and $g(x)$ equal to the right side of the equation.

$f(x) =$ $g(x) =$

b. Graph:



An **identity** is an equation that is true for all values of x. An identity has an infinite number of solutions. The graphs of the left and right sides of an identity are identical.

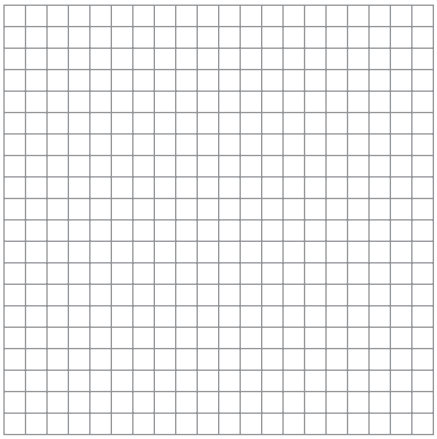
Problem 3 Solving Equations Graphically using Technology

In Problem 2 Question 5, you were not able to determine exact solutions to the equation $x^2 = 5x + 3$. Technology, such as a graphing calculator, can be useful to determine more accurate solutions.

1. Consider the quadratic equation $2^x = x^2 - 4$. Define the function $f(x)$ equal to the left side of the equation and $g(x)$ equal to the right side of the equation.

$f(x) =$ $g(x) =$

2. Graph $f(x)$ and $g(x)$ on the grid shown.



3. How many points of intersection exist?

4. Calculate an intersection point using a graphing calculator by performing the following.

a. Press the Y= button. Enter the functions as Y_1 and Y_2.

b. Graph the functions using appropriate bounds.

c. Press the 2ND button and the TRACE button. You will see the CALC menu.

d. Select 5: intersection.

e. You will be prompted for the first function. Use the up and down buttons to toggle between functions. Select the first function.

f. Use the left and right buttons to trace along the curve. Move the cursor to an intersection point and press ENTER.

g. You will be prompted for the second function. Use the up and down buttons to toggle between functions. Select the second function.

h. Use the left and right buttons to trace along the curve. Move the cursor to an intersection point and press ENTER.

i. You will be prompted for a guess. Use the left and right buttons to trace along the curve. Move the cursor close to an intersection point and press ENTER.

j. The coordinates of the intersection point will be displayed.

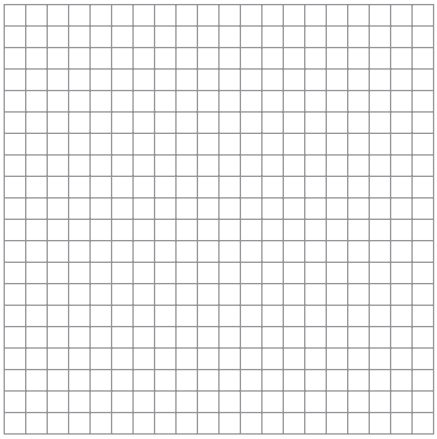
5. What is the point of intersection of $f(x)$ and $g(x)$? What is the solution to the equation $2^x = x^2 - 4$?

6. Solve the equation $x^2 - 6x + 8 = \sqrt{x}$ using a graphing calculator.

a. Define the function $f(x)$ equal to the left side of the equation and $g(x)$ equal to the right side of the equation.

$f(x) =$ $g(x) =$

b. Graph $f(x)$ and $g(x)$ on the grid shown.



c. Solution(s):

Be prepared to share your methods and solutions.

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