CHAPTER

11 Coordinate Geometry

The streets in many cities are organized using a grid system. In a grid system, the streets run parallel and perpendicular to one another, forming rectangular city blocks. You will use the Distance Formula to find the number of blocks between different locations in a city in which the streets are laid out in a grid system.

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11.1 Meeting Friends
The Distance Formula

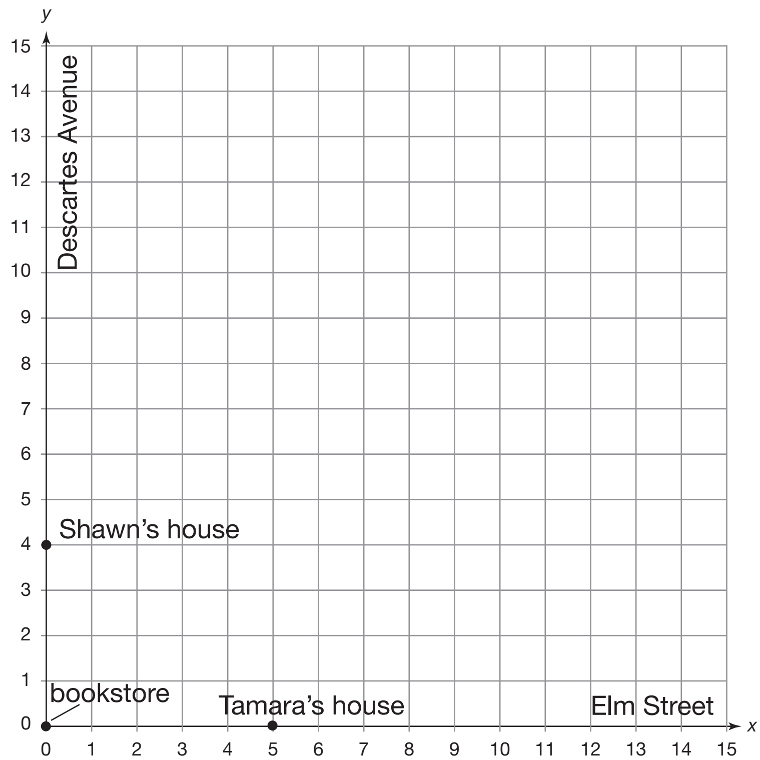
Objectives
In this lesson, you will:
- Determine the distance between two points.
- Use the Distance Formula.

Key Term
- Distance Formula

Two friends, Shawn and Tamara, live in a city in which the streets are laid out in a grid system.

Problem 1 Meeting at the Bookstore

Shawn lives on Descartes Avenue and Tamara lives on Elm Street as shown. The two friends often meet at the bookstore. Each grid square represents one city block.



A. How far in blocks does Shawn walk to get to the bookstore?

B. How far in blocks does Tamara walk to get to the bookstore?

C. Tamara wants to meet Shawn at his house so that they can go to a baseball game together. Tamara can either walk from her house to the bookstore and then to Shawn's house, or she can walk directly to Shawn's house. Which distance is shorter? Explain your reasoning.

D. Determine the distance in blocks Tamara would walk if she traveled from her house to the bookstore and then to Shawn's house. Show all your work.

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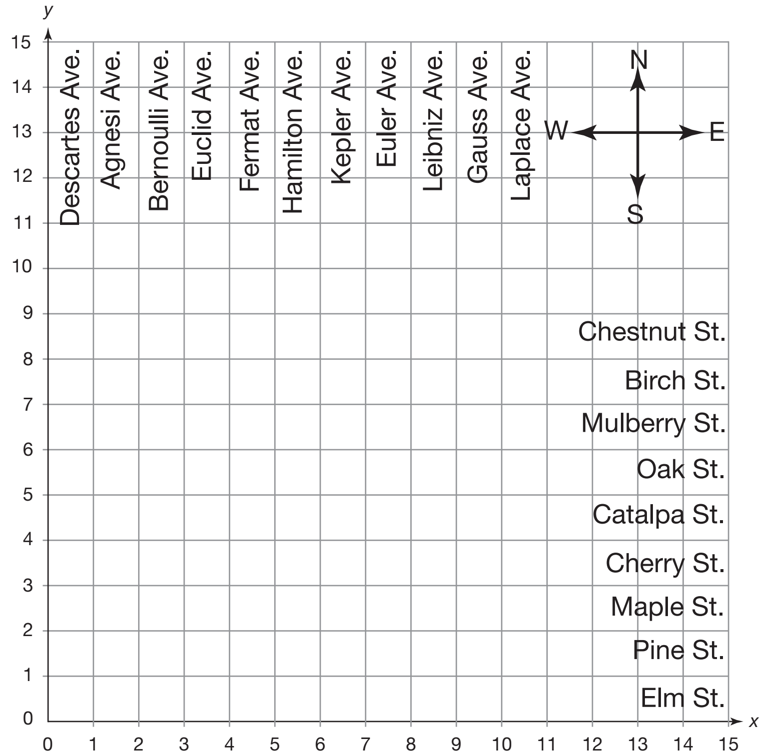
E. Determine the distance in blocks Tamara would walk if she traveled in a straight line from her house to Shawn's house. Show all your work. Round your answer to the nearest tenth of a block.

How did you calculate this distance?

Investigate Problem 1

1. Don, a friend of Shawn and Tamara, lives three blocks east of Descartes Avenue and five blocks north of Elm Street. Freda, another friend, lives seven blocks east of Descartes Avenue and two blocks north of Elm Street. Plot the location of Don's house and Freda's house on the grid. Label each location.



Name the streets that Don lives on.

Name the streets that Freda lives on.

2. Another friend, Bert, lives at the intersection of the avenue that Don lives on and the street that Freda lives on. Plot the location of Bert's house on the grid in Question 1. Then describe the location of Bert's house with respect to Descartes Avenue and Elm Street.

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3. Label the coordinates of the location of each house on the grid in Question 1. How do the coordinates of the location of Bert's house compare to the coordinates of the locations of Don's house and Freda's house?

4. Don and Bert often study French together. Use the house coordinates to write and evaluate an expression that represents the distance between Don's and Bert's houses.

How far in blocks does Don have to walk to get to Bert's house?

5. Bert and Freda often study chemistry together. Use the house coordinates to write an expression that represents the distance between Bert's and Freda's houses.

How far in blocks does Bert have to walk to get to Freda's house?

6. All three friends meet at Don's house to study geometry. Freda walks to Bert's house, and then they walk together to Don's house. Use the coordinates to write and evaluate an expression that represents the distance from Freda's house to Bert's house and from Bert's house to Don's house.

How far in blocks does Freda walk?

7. Draw the direct path from Don's house to Freda's house on the grid in Question 1. If Freda walks to Don's house on this path, how far in blocks does she walk? Explain how you determined your answer.

8. Complete the summary of the steps that you took to determine the distance between Freda's house and Don's house. Let *d* be the direct distance between Don's house and Freda's house.

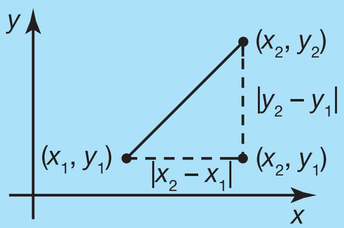
Distance between Bert's house and Freda's house		Distance between Don's house and Bert's house		Distance between Don's house and Freda's house
$(\square - \square)^2$	$+$	$(\square - \square)^2$	$=$	$\square$
$\square^2$	$+$	$\square^2$	$=$	$\square$
$\square$	$+$	$\square$	$=$	$\square$
			$\square =$	$\square$
			$\square =$	$\square$

9. **Just the Math: The Distance Formula** You used the Pythagorean Theorem to calculate the distance between two points in the plane. Your method can be written as the *Distance Formula*.

Distance Formula

If (x_1, y_1) and (x_2, y_2) are two points in the coordinate plane, then the distance d between (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



We indicate that distance is positive by using the absolute value symbol.

Do you think that it matters which point you identify as (x_1, y_1) and which point you identify as (x_2, y_2) when you use the Distance Formula? Explain your reasoning.

10. Calculate the distance between each pair of points. Round your answer to the nearest tenth if necessary. Show all your work.

a. (1, 2) and (3, 7)

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b. (−6, 4) and (2, −8)

c. (−5, 2) and (−6, 10)

d. (−1, −2) and (−3, −7)

11. The distance between (*x*, 2) and (0, 6) is five units. Use the Distance Formula to determine the value of *x*. Show all your work.

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Be prepared to share your methods and solutions.

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11.2 Treasure Hunt
The Midpoint Formula

Objective
In this lesson, you will:
- Use the Midpoint Formula.

Key Terms
- midpoint
- Midpoint Formula

A student teacher is designing a treasure hunt for the children in a kindergarten class. The goal of the treasure hunt is to have the children learn direction (right, left, forward, and backward) and start to learn about distance (near, far, in between). The treasure hunt will be on the school playground.

Problem 1 Plotting the Treasure Hunt

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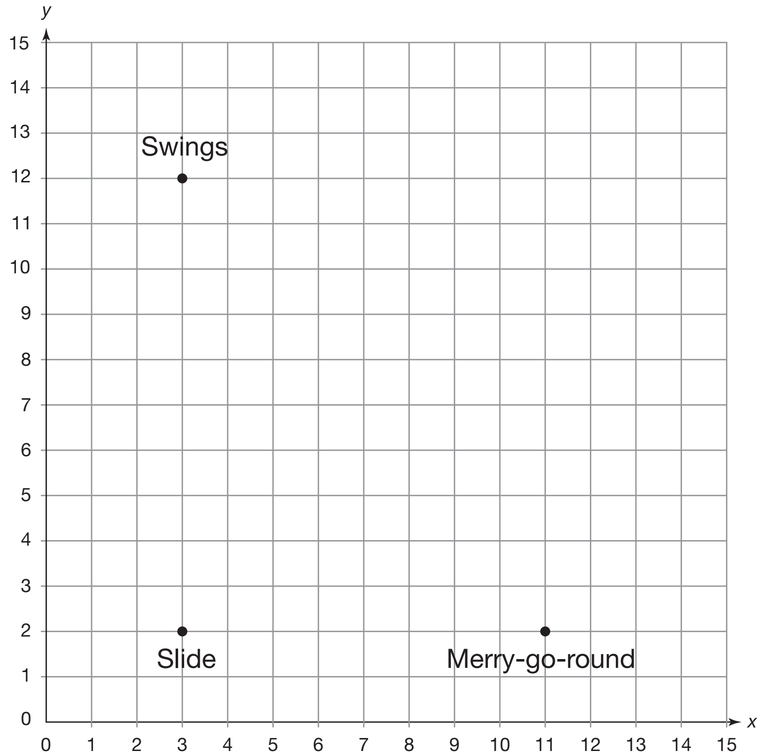
The student teacher has drawn a model of the playground on a grid as shown. The student teacher is using this model to decide where to place the items in the treasure hunt and to determine how to write the treasure hunt instructions. Each grid square represents a square that is one foot long and one foot wide.



A. What are the coordinates of the merry-go-round, the slide, and the swings?

B. Determine the distance in feet between the merry-go-round and the slide. Show all your work.

C. The teacher wants to place a small pile of beads in the grass halfway between the merry-go-round and the slide. How far in feet from the merry-go-round should the beads be placed? How far in feet from the slide should the beads be placed?

D. What should the coordinates of the pile of beads be? Explain how you determined your answer. Plot and label the pile of beads on the previous grid.

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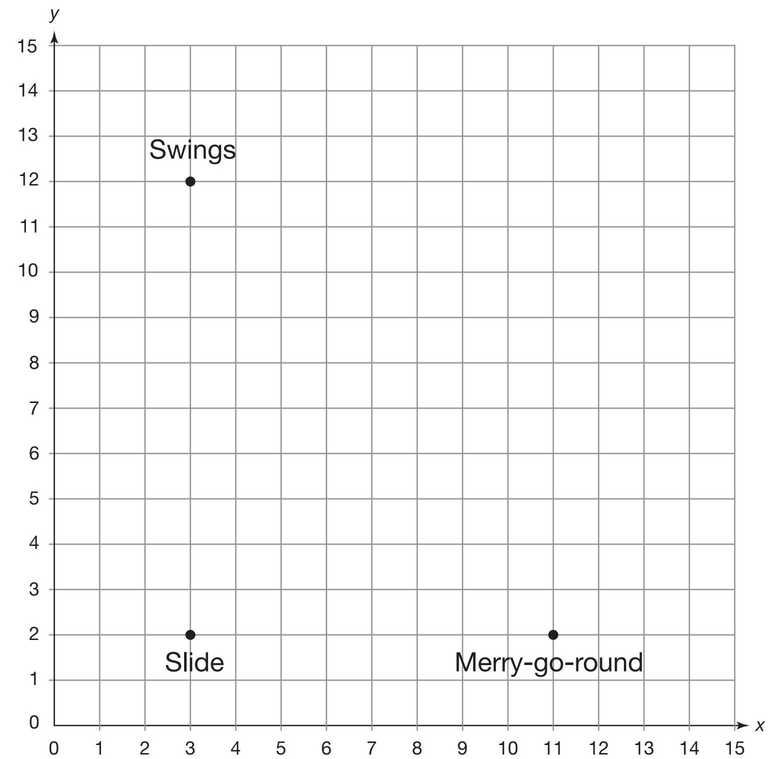
How do the coordinates of the pile of beads compare to the coordinates of the slide and merry-go-round?

E. The teacher also wants to place a pile of kazoos in the grass halfway between the slide and the swings. What should the coordinates of the pile of kazoos be? Show all your work and explain how you determined your answer. Plot and label the pile of kazoos on the grid on the previous page.

How do the coordinates of the pile of kazoos compare to the coordinates of the slide and swings?

Investigate Problem 1

1. The teacher wants to place a pile of buttons in the grass halfway between the swings and the merry-go-round. What do you think the coordinates of the pile of buttons will be? Explain your reasoning. Plot and label the pile of buttons on the following grid.



2. How far in feet from the swings and the merry-go-round will the pile of buttons be? Show all your work and explain how you determined your answer. Round your answer to the nearest tenth if necessary.

Use the Distance Formula to determine whether your answer in Question 1 is correct by calculating the distance between the buttons and the swings. Show all your work.

Would it have mattered if you verified your answer by calculating the distance between the buttons and the merry-go-round? Explain your reasoning.

3. **Just the Math: Midpoint** The coordinates of the points that you determined in part (D), part (E), and Question 1 are **midpoints,** or points that are exactly halfway between two given points. The work that you did in Problem 1 can be summarized in the *Midpoint Formula.*

Midpoint Formula

If (x_1, y_1) and (x_2, y_2) are two points in the coordinate plane, then the midpoint of the line segment that joins these two points is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Use the Midpoint Formula to verify your answer to Question 1.

4. Determine the midpoint of each line segment that has the given points as its endpoints. Show all your work.

a. (0, 5) and (4, 3)

b. (8, 2) and (6, 0)

c. (−3, 1) and (9, −7)

d. (−10, 7) and (−4, −7)

Be prepared to share your methods and solutions.

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11.3 Parking Lot Design

Parallel and Perpendicular Lines in the Coordinate Plane

Objectives

In this lesson, you will:

- Determine whether lines are parallel.
- Determine the equations of lines parallel to given lines.
- Determine whether lines are perpendicular.
- Determine the equations of lines perpendicular to given lines.
- Determine equations of horizontal and vertical lines.
- Calculate the distance between a line and a point not on the line.

Key Terms

- slope
- point-slope form
- slope-intercept form
- *y*-intercepts
- parallel lines
- perpendicular
- reciprocal
- negative reciprocal
- horizontal line
- vertical line

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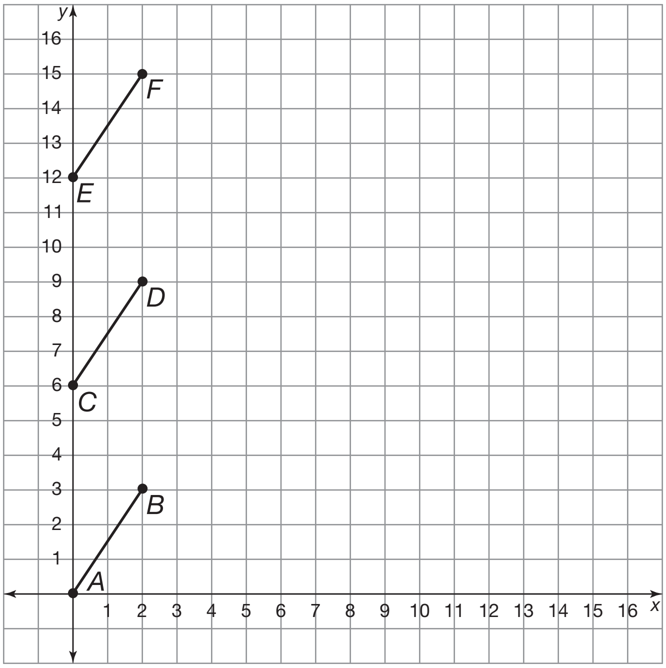
Large parking lots, such as those located in a shopping center or at a mall, have line segments painted to mark the locations where vehicles are supposed to park. The layout of these line segments must be considered carefully so that there is enough room for the vehicles to move and park in the lot without the vehicles being damaged.

Problem 1 Parking Spaces

Some line segments that form parking spaces in a parking lot are shown on the grid. One grid square represents a square that is one meter long and one meter wide.

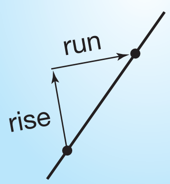


A. What do you notice about the line segments that form the parking spaces?

Take Note

Remember that the **slope** of a line is the ratio of the rise to the run:

slope $= \frac{\text{rise}}{\text{run}}$.



B. What is the vertical distance between $\overline{AB}$ and $\overline{CD}$ and between $\overline{CD}$ and $\overline{EF}$?

C. Carefully extend $\overline{AB}$ into line p, extend $\overline{CD}$ into line q, and extend $\overline{EF}$ into line r.

D. Use the graph to identify the slope of each line. What do you notice?

E. Use the point-slope form to write the equations of lines *p*, *q*, and *r*. Then write the equations in slope-intercept form.

Take Note

Remember that the **point-slope form** of the equation of the line that passes through (x_1, y_1) and has slope *m* is $y - y_1 = m(x - x_1)$.

The **slope-intercept form** of the equation of the line that has slope *m* and *y*-intercept *b* is $y = mx + b$.

What do you notice about the ***y*-intercepts** of these lines?

What do the *y*-intercepts tell you about the relationship between these lines?

F. If you were to draw a line segment above $\overline{EF}$ to form another parking space, what would be the equation of the line that coincides with this line segment? Determine your answer without graphing the line. Explain how you determined your answer.

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Investigate Problem 1

1. What can you conclude about the slopes of **parallel lines** in the coordinate plane?

   What can you conclude about the *y*-intercepts of parallel lines in the coordinate plane?

2. Write equations for three lines that are parallel to the line given by $y = -2x + 4$. Explain how you determined your answers.

3. Write an equation for the line that is parallel to the line given by $y = 5x + 3$ and passes through the point (4, 0). Show all your work and explain how you determined your answer.

4. Without graphing the equations, determine whether the lines given by $y - 2x = 5$ and $2x - y = 4$ are parallel. Show all your work.

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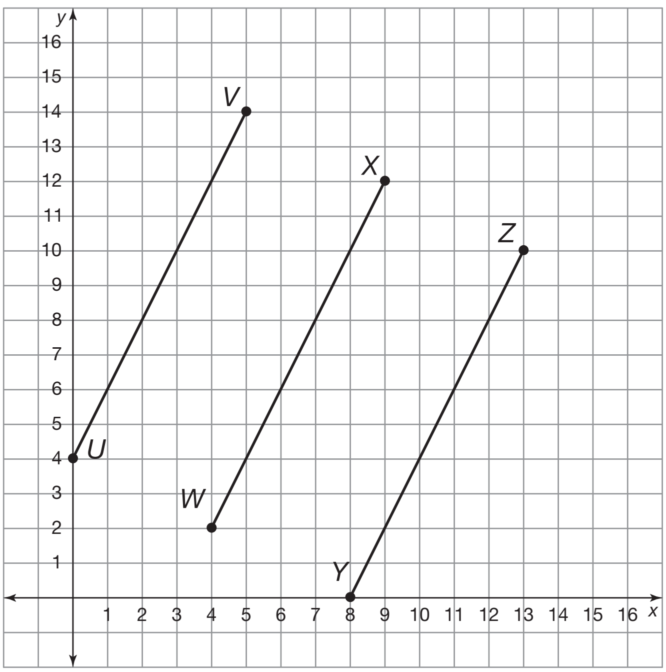
Problem 2 More Parking Spaces

Another arrangement of line segments that form parking spaces in a truck parking lot is shown on the grid. One grid square represents a square that is one meter long and one meter wide.



A. Use a protractor to determine the measures of $\angle VUM$, $\angle XWY$, and $\angle ZYW$. What similarity do you notice about the angles?

When lines or line segments intersect at right angles, we say that the lines or line segments are **perpendicular.** For instance, $\overline{UV}$ is perpendicular to $\overline{UW}$. In symbols, we can write this as $\overline{UV} \perp \overline{UW}$, where $\perp$ means "is perpendicular to."

B. Carefully extend $\overline{UY}$ into line *p*, extend $\overline{UV}$ into line *q*, extend $\overline{WX}$ into line *r*, and extend $\overline{YZ}$ into line *s*.

C. How do these lines relate to each other?

Complete each statement by using $\parallel$ or $\perp$.

line *p* ____ line *r*

line *q* ____ line *s*

D. Without actually determining the slopes, how will the slopes of the lines compare? Explain your reasoning.

E. What do you think must be true about the signs of the slopes of two lines that are perpendicular?

F. Use the graph to determine the slopes of lines *p*, *q*, *r*, and *s*.

G. How does the slope of line *p* compare to the slopes of lines *q*, *r*, and *s*?

H. What is the product of the slopes of two of your perpendicular lines?

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Investigate Problem 2

1. What can you conclude about the product of the slopes of perpendicular lines in the coordinate plane?

When the product of two numbers is 1, the numbers are **reciprocals** of one another. When the product of two numbers is -1, the numbers are **negative reciprocals** of one another. So the slopes of perpendicular lines are negative reciprocals of each other.

2. Determine the negative reciprocal of each number.

5　　　　-2　　　　$\frac{1}{3}$

3. Do you think that the y-intercepts of perpendicular lines tell you anything about the relationship between the perpendicular lines? Explain your reasoning.

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4. Write equations for three lines that are perpendicular to the line given by $y = -2x + 4$. Explain how you determined your answers.

5. Write an equation for the line that is perpendicular to the line given by $y = 5x + 3$ and passes through the point (4, 0). Show all your work and explain how you determined your answer.

6. Without graphing the equations, determine whether the lines given by $y + 2x = 5$ and $2x - y = 4$ are perpendicular. Show all your work.

7. Complete each statement.

When two lines are parallel, their slopes are ______________.

When two lines are perpendicular, their slopes are ________________________ ________________.

8. Suppose that you have a line and you choose one point on the line. How many lines perpendicular to the given line can you draw through the given point?

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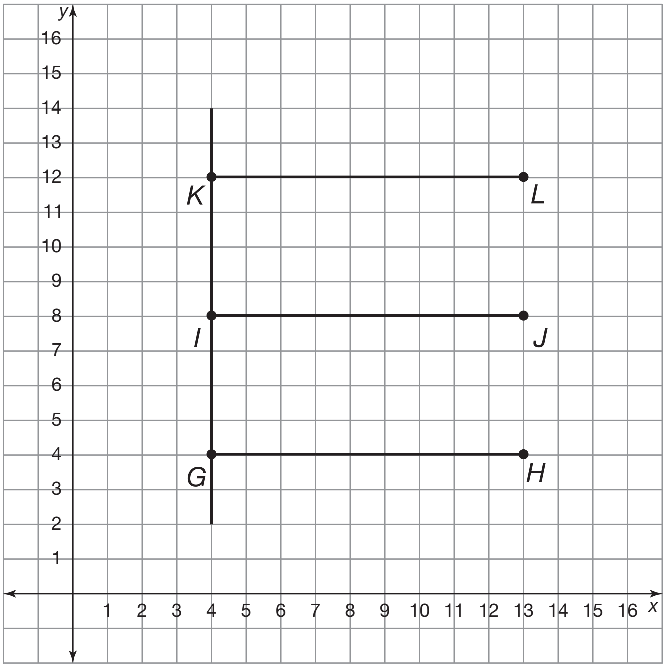
9. Suppose that you have a line and you choose one point that is not on the line. How many lines can you draw through the given point that are perpendicular to the given line? How many lines can you draw through the given point that are parallel to the given line?

Problem 3 A Very Simple Parking Lot

One final truck parking lot is shown. One grid square represents a square that is one meter long and one meter wide.



A. What type of angles are formed by the intersection of the parking lot line segments? How do you know?

B. Carefully extend $\overline{GK}$ into line p, extend $\overline{GH}$ into line q, extend $\overline{IJ}$ into line r, and extend $\overline{KL}$ into line s.

C. Choose any three points on line q and list their coordinates.

Choose any three points on line r and list their coordinates.

Choose any three points on line s and list their coordinates.

What do you notice about the x- and y-coordinates of these points?

What do you think should be the equations of lines *q*, *r*, and *s*? Explain your reasoning.

D. Choose any three points on line *p* and list their coordinates.

What do you notice about the *x*- and *y*-coordinates of these points?

What do you think should be the equation of line *p*? Explain your reasoning.

Investigate Problem 3

1. Just the Math: Horizontal and Vertical Lines In Problem 3, you wrote the equations of horizontal and vertical lines. A **horizontal line** has an equation of the form $y = a$, where a is any real number. A **vertical line** has an equation of the form $x = b$, where b is any real number.

Consider your horizontal lines in Problem 3. For any horizontal line, if *x* increases by one unit, by how many units does *y* change?

What is the slope of any horizontal line? Explain your reasoning.

Consider your vertical line in Problem 3. Suppose that *y* increases by one unit. By how many units does *x* change?

What is the rise divided by the run? Does this make any sense? Explain.

Because division by zero is undefined, we say that a vertical line has an undefined slope.

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2. Consider the statements about parallel and perpendicular lines in Question 7 of Problem 2. Are these statements true for horizontal and vertical lines? Explain.

Complete the following statements.

________ vertical lines are parallel.

________ horizontal lines are parallel.

Write a statement that describes the relationship between a vertical line and a horizontal line.

3. Write equations for a horizontal line and a vertical line that pass through the point (2, −1).

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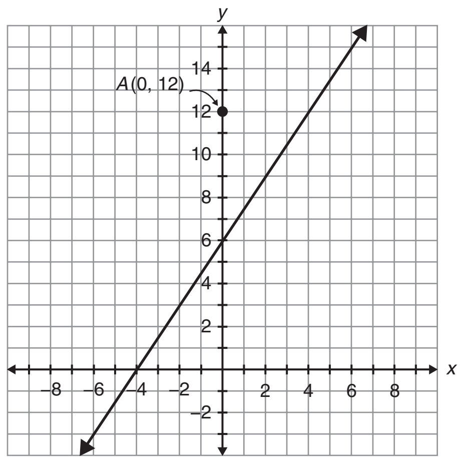
4. Write an equation of the line that is perpendicular to the line given by $x = 5$ and passes through the point (1, 0).

Write an equation of the line that is perpendicular to the line given by $y = -2$ and passes through the point (5, 6).

Problem 4 Distances Between Lines and Points

1. Sketch a line and a point not on the line. Describe the shortest distance between the point and the line.

2. The equation of the line shown in the grid is $f(x) = \frac{3}{2}x + 6$. Draw the shortest segment between the line and the point $A(0, 12)$. Label the point where the segment intersects $f(x)$ as point B.



3. What information is needed to calculate the length of segment AB using the distance formula? Explain.

4. How can you calculate the intersection point of segment AB and the line $f(x) = \frac{3}{2}x + 6$ algebraically?

5. Write an equation for segment AB.

6. Calculate the point of intersection of segment *AB* and the line $f(x) = \frac{3}{2}x + 6$.

7. Calculate the length of segment *AB*.

8. What is the distance from the point (0, 12) to the line $f(x) = \frac{3}{2}x + 6$?

9. Draw a line parallel to the line $f(x) = \frac{3}{2}x + 6$ that passes through the point (0, 12). Identify another point on this parallel line.

10. Calculate the distance from the point in Question 9 to the line $f(x) = \frac{3}{2}x + 6$.

11. Compare your answers to Question 8 and Question 10. Is this a coincidence? Explain.

12. Calculate the distance from the origin to the line $f(x) = \frac{3}{2}x + 6$.

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13. Predict the distance from the point (2, 3) to the line $f(x) = \frac{3}{2}x + 6$? Explain.

Be prepared to share your methods and solutions.

11.4 Building a Henge
Triangles in the Coordinate Plane

Objectives

In this lesson, you will:

- Study triangles inscribed in circles.
- Determine the relationship between the midpoint of the hypotenuse of a right triangle and the vertices of the triangle.
- Determine coordinates of midsegments of triangles.
- Discover properties of midsegments.
- Classify triangles in the coordinate plane.
- Construct an equilateral triangle.
- Construct points of concurrency.
- Solve for points of concurrency algebraically.

Key Terms

- inscribed triangle
- midsegment
- centroid
- points of concurrency
- circumcenter
- othocenter
- incenter
- equilateral triangle
- scalene triangle

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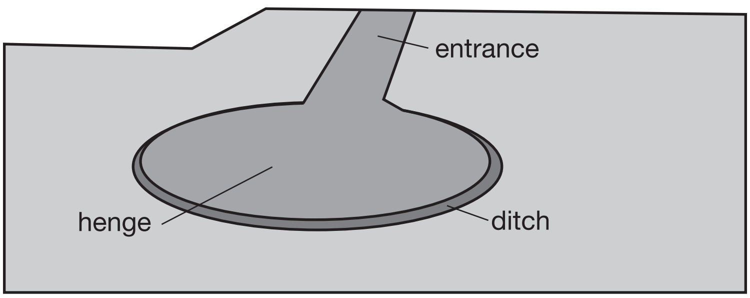
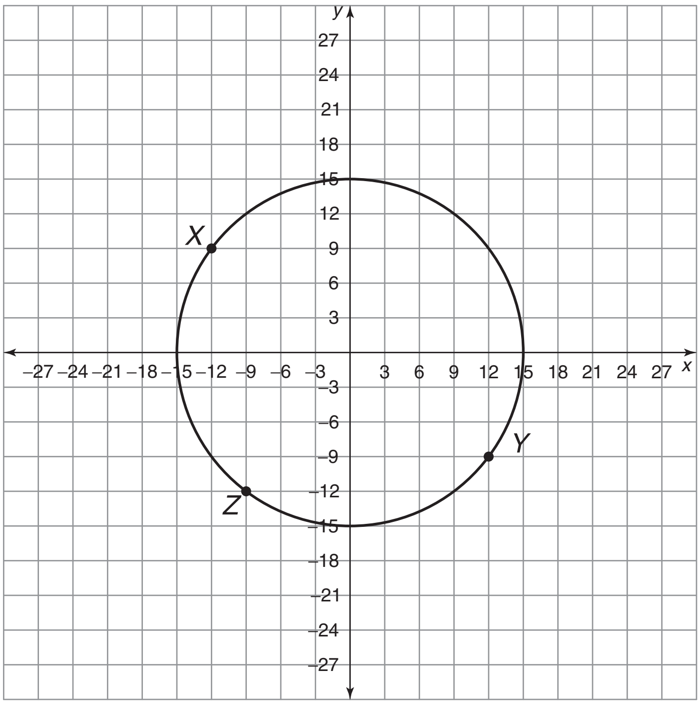
In England, you can find flat, roughly circular areas of land that are enclosed by ditches, which are surrounded by piles of earth. These areas were created in ancient times and are called *henges.* The inner circular area of a henge can be accessed by entrances that were created through the surrounding ditches and piles of earth.



Problem 1 Build Your Own Henge

The circle on the coordinate plane represents the ditch of a henge. Each grid square represents a square that is three meters long and three meters wide. The points *X*, *Y*, and *Z* on this circle represent entrances to the inner area of the henge. Note that points *X* and *Y* are directly across the circle from one another.



A. Choose three new points on the circle that will be the entrances to the inner area of the henge. Choose your points so that two of the points are the endpoints of a diameter. Label your points as points *A, B,* and *C* along with their coordinates on the coordinate plane.

B. Connect points *A*, *B*, and *C* with line segments to form a triangle. Your triangle is an *inscribed triangle*. An **inscribed triangle** is a triangle whose vertices lie on a circle.

C. Determine the slope of each side of your triangle. Show all your work.

Take Note

To determine the *slope* of a line passing through the points (x_1, y_1) and (x_2, y_2), use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

D. Compare the slopes of the sides of the triangles. What do you notice?

E. Classify $\triangle ABC$ by its angles.

F. Repeat part (A) through part (E) for three different points. Name these points *D*, *E*, and *F*. Show all your work.

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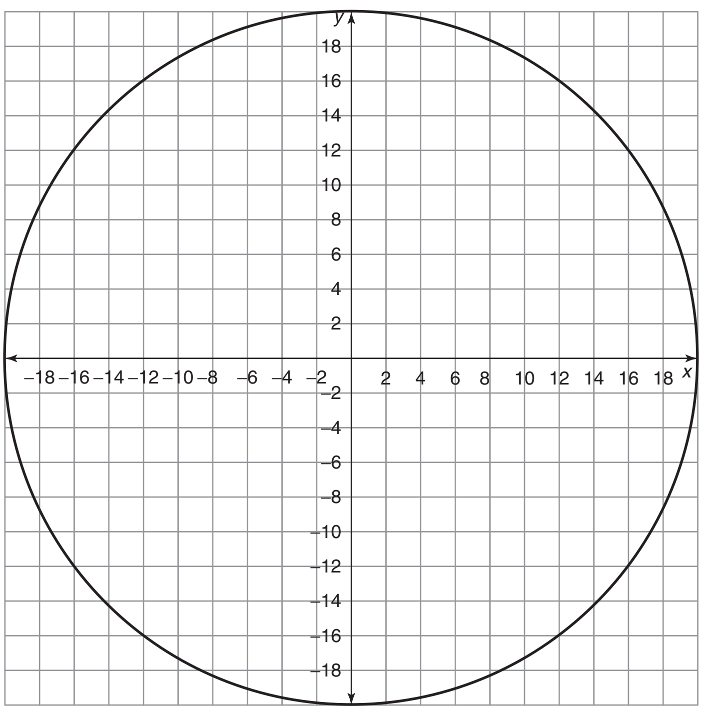
Investigate Problem 1

1. Write a conditional statement that describes the type of triangle that is created when the triangle is inscribed in a circle so that one side of the triangle is a diameter.

2. Consider a triangle inscribed in a circle so that one side of the triangle is a diameter. Classify the side that is the diameter. Is this true for every triangle that is constructed in this way? Explain your reasoning.

3. Can you draw an inscribed right triangle in which none of the sides are a diameter? If so, name the vertices of this triangle.

4. The circle on the coordinate plane presents the ditch of a different henge. Each grid square represents a square that is two meters long and two meters wide. Choose three points on the circle that will be entrances to the inner area of the henge so that the points form the vertices of a right triangle. Label your points as points *X, Y,* and *Z* along with their coordinates on the coordinate plane.



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Take Note

If (x_1, y_1) and (x_2, y_2), are two points in the coordinate plane, then the *midpoint* of the line segment that joins these two points is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Determine the coordinates of the midpoint of the hypotenuse. Label the midpoint on the graph as point *M*. Show all your work.

Calculate the distance from point *M* to each of the vertices of the right triangle. Show all your work. Simplify, but do not evaluate any radicals.

What do you notice?

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5. Determine the midpoints of the other sides of your triangle in Question 4. Show all your work. Then label these midpoints on the graph as points *N* and *P* and connect all three midpoints with line segments. These line segments are the **midsegments** of the triangle.

Determine the slopes of the sides of your triangle in Question 4 and the slopes of the midsegments of the triangle. Show all your work.

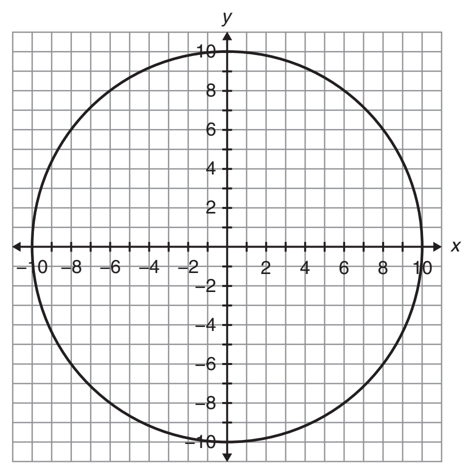
What do you notice?

6. Determine the lengths of the sides of your triangle in Question 4 and the lengths of the midsegments. Show all your work. Simplify, but do not evaluate any radicals.

7. How does the length of a midsegment of a triangle compare to the length of the side of the triangle that the midsegment does not intersect? Show all your work.

Problem 2 Isosceles Triangles in the Coordinate Plane

1. The circle on the coordinate plane shown represents the ditch of a henge. Each grid square represents a square that is one meter long and one meter wide. The points $(-6, -8)$, $(6, -8)$, and $(0, 10)$ represent entrances to the inner area of the henge. Label these three points.



2. Form a triangle by connecting the three points. Classify the triangle based on the lengths of its sides.

3. How can you verify the classification from Question 2 using algebra?

4. Verify the classification from Question 2 using algebra.

Problem 3 Centroids in the Coordinate Plane

1. The **centroid** of a triangle is a **point of concurrency** formed by the intersection of the three medians of a triangle. How can you locate the centroid of the triangle from Problem 2 using geometric tools?

2. Use geometric tools to locate the centriod.

3. How can you locate the centroid of the triangle from Problem 2 using algebra?

4. Use algebra to locate the centroid.

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5. Compare the coordinates of the centroid using geometric tools in Question 2 and algebra in Question 4.

Problem 4 Circumcenters in the Coordinate Plane

1. The **circumcenter** of a triangle is a point of concurrency formed by the intersection of the three perpendicular bisectors of a triangle. How can you locate the circumcenter of the triangle from Problem 2 using geometric tools?

2. Use geometric tools to locate the circumcenter.

3. How can you locate the circumcenter of the triangle from Problem 2 using algebra?

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4. Use algebra to locate the circumcenter.

5. Compare the coordinates of the circumcenter using geometric tools in Question 2 and algebra in Question 4.

Problem 5 Points of Concurrency of Isosceles Triangles

1. Label the centroid and circumcenter of the triangle on the grid in Problem 2 Question 1.

2. The **orthocenter** of a triangle is a point of concurrency formed by the intersection of the three altitudes of a triangle. The **incenter** of a triangle is a point of concurrency formed by the intersection of the three angle bisectors of a triangle. Use geometric tools to locate the orthocenter and the incenter of the triangle from Problem 2. Label each on the grid. What do you notice about the four points of concurrency of an isosceles triangle?

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Problem 6 Equilateral Triangles and Points of Concurrency

1. Construct an **equilateral triangle** on the grid shown by completing the following steps.

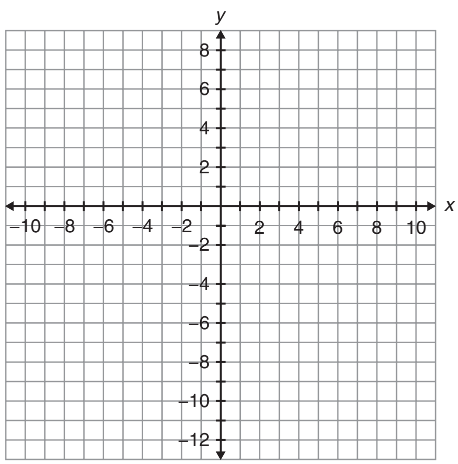
   a. Plot points $A(-10, -10)$ and $B(10, -10)$. Draw segment AB, one side of the equilateral triangle.

   b. Open the compass to a width equal to the distance between point A and point B.

   c. Place the point of the compass on point A. Draw an arc above segment AB.

   d. Keeping the compass at the same width. Place the point of the compass on point B. Draw an arc above segment AB.

   e. Label the intersection of the two arcs as point C. Draw segments AC and BC.

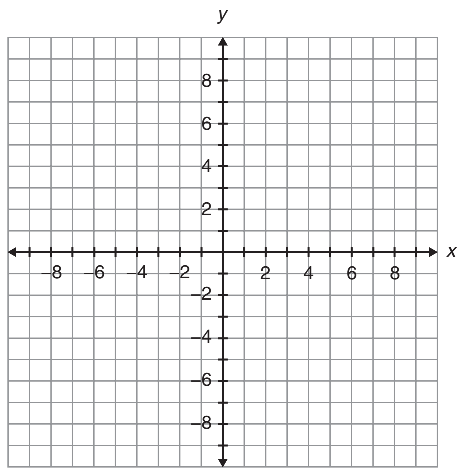


2. What are the coordinates of point C?

3. Construct the incenter, circumcenter, centroid, and othocenter. What do you notice about the four points of concurrency?

Problem 7 Scalene Triangles in the Coordinate Plane

1. Draw a large **scalene triangle** on the grid shown by plotting three points labeled *D*, *E*, and *F* and connecting the points.



2. Identify the coordinates of the vertices of the scalene triangle.

3. Show that the triangle is scalene using geometric tools.

4. Show that the triangle is scalene using algebra.

5. Construct the incenter, circumcenter, centroid, and orthocenter of triangle *DEF*. Which of these points of concurrency are collinear?

Be prepared to share your methods and solutions.

11.5 Planning a Subdivision
Quadrilaterals in the Coordinate Plane

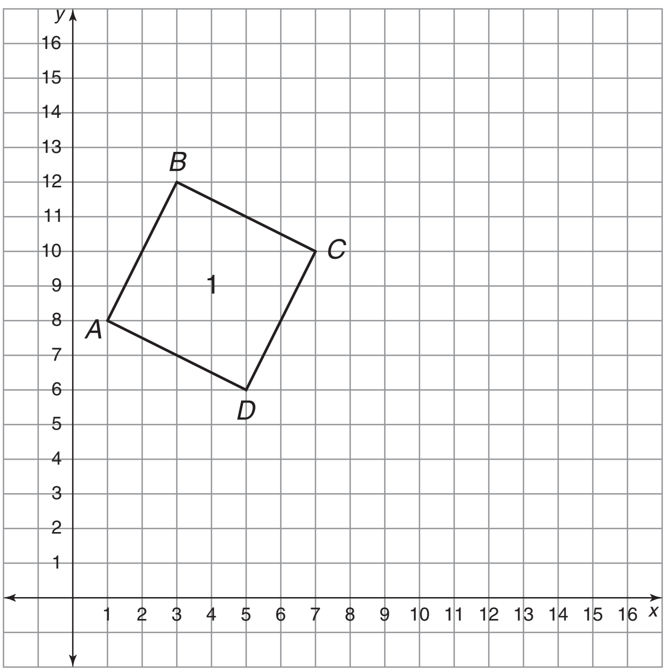
Objectives
In this lesson, you will:
- Classify quadrilaterals in the plane.
- Classify properties of quadrilaterals in the plane.

Key Terms
- rectangle
- parallelogram

A land planner is laying out different plots, or parcels, of land for a new housing subdivision. The parcels of land will be shaped like quadrilaterals.

Problem 1 The Lay of the Land

Parcel 1 is shown on the grid below. Each grid square has an area of one acre.



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A. What kind of quadrilateral do you think parcel 1 is?

B. Find the slopes of each side of the parcel. How many pairs of opposite sides, if any, are parallel? Explain how you found your answer.

Are any of the sides perpendicular? Explain how you know.

Classify the quadrilateral with the information you have so far.

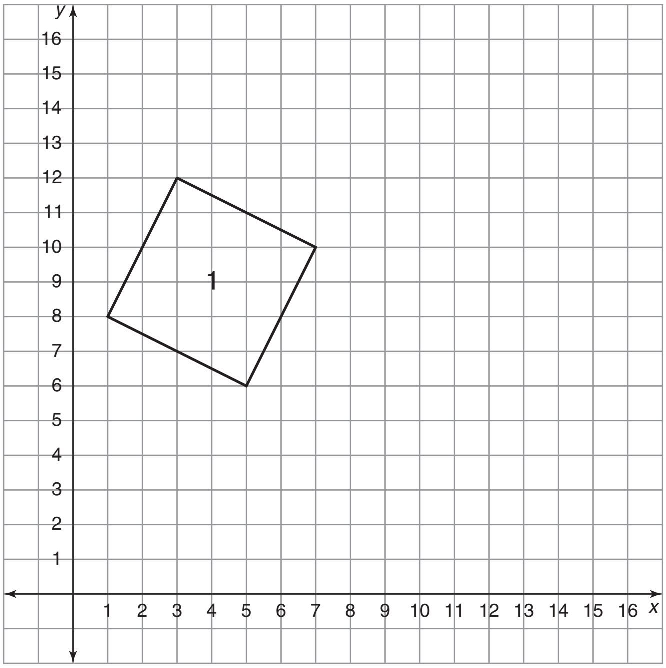
C. Find the lengths of the sides that form parcel 1. Show all your work. Are any of the side lengths congruent? If so, describe the sides that are congruent.

> **Take Note**
>
> Remember that the Distance Formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. You can use this formula to find the lengths of the sides that form parcel 1.

Can you classify parcel 1 further? If so, classify the quadrilateral.

Investigate Problem 1

1. The coordinates of the endpoints of parcel 2 are *E*(5, 1), *F*(5, 6), *G*(9, 6), and *H*(9, 1). Graph parcel 2 on the grid below. Classify this quadrilateral in as many ways as is possible. Explain how you found your answer.



2. Should the diagonals of parcel 2 be congruent? Find the lengths of the diagonals to verify your answer.

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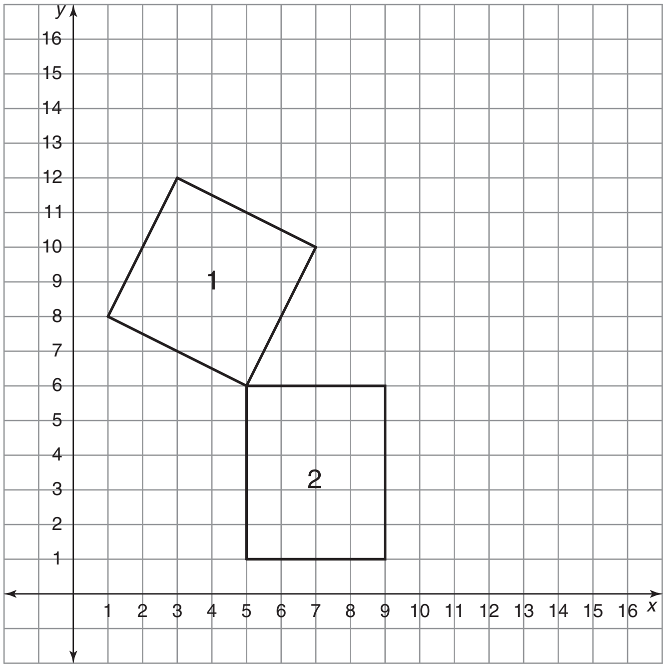
3. The coordinates of the endpoints of parcel 3 are $I(5, 6)$, $J(7, 10)$, $K(11, 10)$, and $L(9, 6)$. Graph parcel 3 on the grid below. Classify this quadrilateral in as many ways as is possible. Explain how you found your answer.



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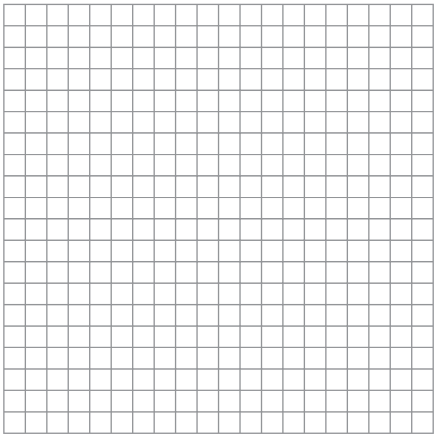
Problem 2

1. A land developer is creating parcels of land in the shape of parallelograms. Plot the points $Q(-3, -5)$, $R(2, -5)$, and $S(3, -3)$ on the grid shown.



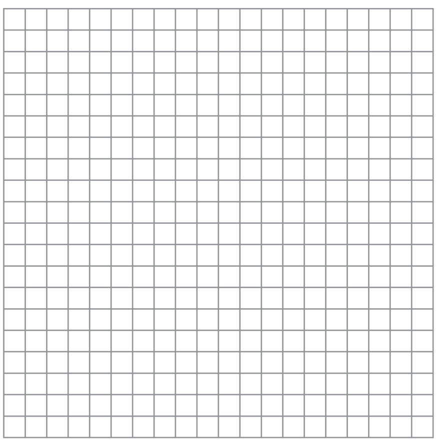
2. Locate a fourth point T such that the points Q, R, S, and T form a parallelogram. What are the coordinates of point T?

3. How can you verify that the quadrilateral is a parallelogram using algebra?

4. Use an algebraic strategy to verify the quadrilateral is a parallelogram.

5. Each square grid has an area of one acre. Calculate the area of the parcel of land defined by the parallelogram in Question 4.

6. Plot the points $Q(-3, -5)$, $R(2, -5)$, and $S(3, -3)$ on the grid shown.



7. Locate a fourth point *T* that is different than the point in Question 2 and connect the four points to form a parallelogram. What are the coordinates of point *T*?

8. Use an algebraic strategy to verify the quadrilateral is a parallelogram.

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9. Each square grid has an area of one acre. Calculate the area of the parcel of land defined by the parallelogram in Question 8. Remember that the height of the parallelogram is a perpendicular line segment that connects a vertex to the opposite side.

10. Did you calculate the same area in Questions 5 and 9? Why you think this result occurred?

11. Molly claims there is another possible location for point *T* such that the points *Q*, *R*, *S*, and *T* form a parallelogram. Is she correct? Explain.

Be prepared to share your methods and solutions.

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