3.1 Graph Cubic Functions

Georgia Performance Standard(s)

MM1A1b, MM1A1c MM1A1d, MM1A1h

Your Notes

Goal • Graph and analyze cubic functions.

VOCABULARY

Cubic function

Odd function

Even function

End behavior

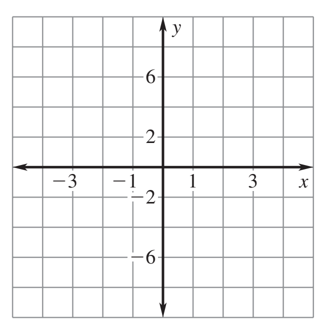
Example 1 ***Graph $y = x^3 + c$***

Graph $y = x^3 - 1$. Compare the graph with the graph of $y = x^3$.

Solution

Make a table of values for $y = x^3 - 1$.

x	-2	-1	0	1	2
y	_____	_____	_____	___	___



Plot points from the table and connect them with a __________ ________. The degrees of both functions are ______ and leading coefficients are __________, so the graphs have _____ ____________________. The graph of $y = x^3 - 1$ is a vertical _____________ (of ___ unit ________) of the graph of $y = x^3$.

Your Notes

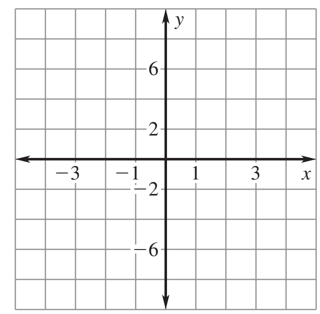
Example 2 ***Graph $y = ax^3$***

Graph $y = -3x^3$. Compare the graph with the graph of $y = x^3$.

Solution

Make a table of values for $y = -3x^3$.

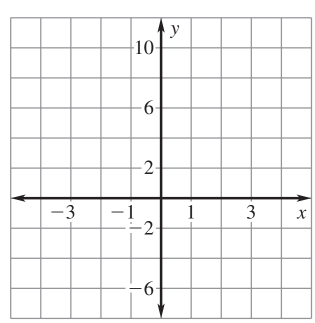
x	$-\frac{4}{3}$	-1	0	1	$\frac{4}{3}$
y	____	___	___	_____	______



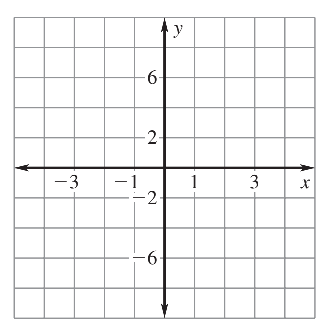
Plot points from the table and connect them with a __________ ________. The degrees of both functions are ______ but the leading coefficients ______________ the same sign, so the graphs have ________________________. The graph of $y = -3x^3$ is ___________ than the graph of $y = x^3$. This is because the graph of $y = -3x^3$ is a vertical _________ (by a factor of ___) with a ________________________ of the graph of $y = x^3$. The graphs could also be viewed as being reflected in the y-axis.

✔ ***Checkpoint*** **Graph the function. Compare the graph with the graph of $y = x^3$.**

1. $y = x^3 + 3$



2. $y = 2x^3$



Your Notes

Example 3 *Analyze cubic functions*

Consider the cubic function $f(x) = \frac{1}{3}x^3 - x$.

a. Tell whether the function is *even*, *odd*, or *neither*. Does the graph of the function have symmetry?

b. Identify the intervals of increase and decrease of the graph of the function.

Solution

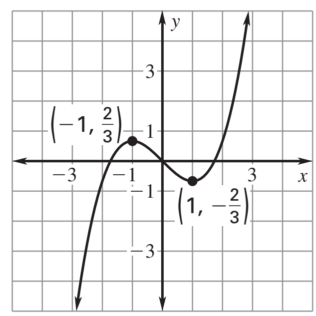
a. The function is ______ because

$$f(-x) = \frac{1}{3}(\_\_\_\_)^3 - (\_\_\_\_)$$

$$= \_\_\_\_ x^3 + \_\_\_$$

$$= \_\_\_\_\_\_$$

Therefore, the graph is symmetric about the ________.



b. You can see from the graph that the function is increasing on the interval ________, decreasing on the interval ____________, and increasing on the interval _______. You can use a graphing calculator to verify the turning points.

Checkpoint Complete the following exercise.

3. Is the function $f(x) = -2x^3 + 3x^2$ *even*, *odd*, or *neither*? Does the graph of the function have symmetry? What are the intervals of increase and decrease?

Homework

Name ______________________________ Date ______________

LESSON 3.1

Practice

Describe **the end behavior of the graph of the function.**

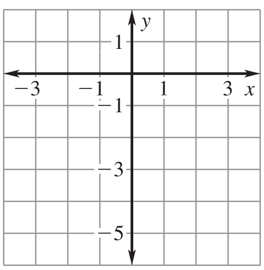
1. $f(x) = 2x^3 - 7$

2. $f(x) = -x^3 + 3x$

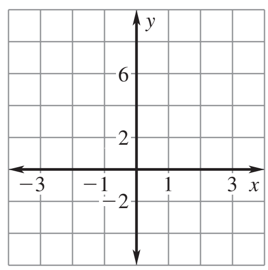
3. $f(x) = -\frac{2}{3}x^3 - 2x^2$

Graph the function. ***Compare*** **the graph with the graph of** $y = x^3$**.**

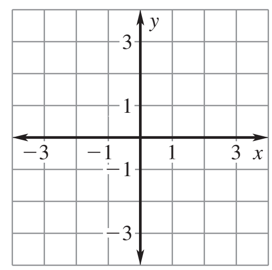
4. $y = x^3 - 3$



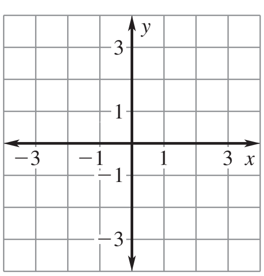
5. $y = x^3 + 4$



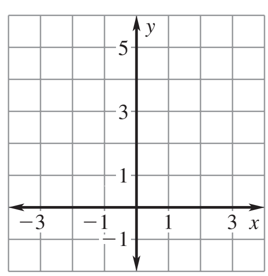
6. $y = -x^3$



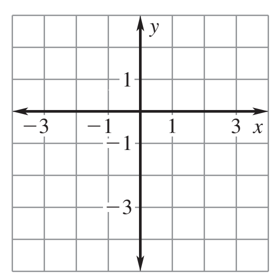
7. $y = 3x^3$



8. $y = -x^3 + 2$



9. $y = \frac{1}{2}x^3 - 1$



Name ______________________________ Date ______________

LESSON 3.1
Practice *continued*

Tell whether the function is *even, odd,* or *neither*.

10. $f(x) = 5x^3$

11. $f(x) = x^2 - 5$

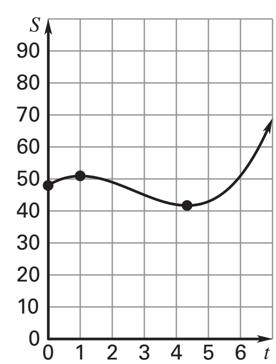
12. $f(x) = x^3 - 2x^2$

13. $f(x) = -x^3 + x + 8$

14. $f(x) = x^4 - 3x^2$

15. $f(x) = x^3 + 8x$

16. **Driving** The speed S (in miles per hour) of a car between 0 and 7 minutes after entering a highway can be modeled by the function $S = 0.5t^3 - 4t^2 + 6.5t + 48$, where t is the number of minutes since the car entered the highway. The graph of S is shown at the right.



a. Find the speed after 7 minutes.

b. Between what times did the speed increase?

c. Between what times did the speed decrease?

3.2 Use Special Products to Factor Cubics

Georgia Performance Standard(s)

MM1A2f

Your Notes

Goal • Factor cubics using special product patterns.

SPECIAL PRODUCT PATTERNS

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

Example 1 *Use special product patterns*

Factor the expression.

a. $x^3 + 12x^2 + 48x + 64$

b. $x^3 - 15x^2 + 75x - 125$

Solution

a. $x^3 + 12x^2 + 48x + 64$

$= x^3 +$ _____(___) + ____(____) + ____

= __________

b. $x^3 - 15x^2 + 75x - 125$

$= x^3 -$ _____(___) + ____(____) − ____

= __________

Example 2 *Factor out a monomial first*

Factor $-2x^3 + 18x^2 - 54x + 54$.

Solution

$-2x^3 + 18x^2 - 54x + 54$

= _____(_______________________)

= _____(___________________________)

= ____________

Your Notes

Example 3 ***Factor cubics with multiple variables***

Factor the expression.

a. $x^3y^3 - 12x^2y^2 + 48xy - 64$

$= \_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_ - \_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

b. $x^3 + 9x^2y + 27xy^2 + 27y^3$

$= \_\_\_\_ - \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$

✔ ***Checkpoint*** **Factor the expression.**

1. $x^3 - 18x^2 + 108x - 216$

2. $8x^3 + 24x^2 + 24x + 8$

3. $125a^3 - 75a^2b + 15ab^2 - b^3$

Homework

Name ____________________ Date ____________

LESSON 3.2 Practice

Factor the expression.

1. $x^3 - 3x^2 + 3x - 1$

2. $x^3 - 24x^2 + 192x - 512$

3. $x^3 + 21x^2 + 147x + 343$

4. $64x^3 + 48x^2 + 12x + 1$

5. $27x^3 - 54x^2 + 36x - 8$

6. $8x^3 + 60x^2 + 150x + 125$

7. $40x^3 + 60x^2 + 30x + 5$

8. $-x^3 + 6x^2 - 12x + 8$

9. **Multiple Choice** For what value of k can the expression $x^3 + kx^2 + 27x + 27$ be factored using a special product pattern?

A. 1 **B.** 3 **C.** 9 **D.** 27

Name ______________________________ Date ______________

LESSON 3.2

Practice *continued*

In Exercises 10–13, match the polynomial with the appropriate factorization.

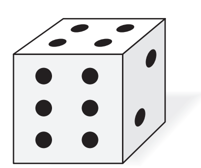
10. $x^3 + 15x^2y + 75xy^2 + 125y^3$ **A.** $(x + 5y)^3$

11. $x^3 - 15x^2y + 75xy^2 - 125y^3$ **B.** $(5x + y)^3$

12. $125x^3 + 75x^2y + 15xy^2 + y^3$ **C.** $(x - 5y)^3$

13. $125x^3 - 75x^2y + 15xy^2 - y^3$ **D.** $(5x - y)^3$

14. **Volume** The diagram at the right shows a number cube and an expression for its volume. Find a binomial the represents a side length of the number cube.



Volume:
$216x^3 - 108x^2 + 18x - 1$

3.3 Graph Square Root Functions

Georgia Performance Standard(s)

MM1A1b, MM1A1c, MM1A1d

Your Notes

Goal • Graph square root functions.

VOCABULARY

Radical expression

Radical function

Square root function

Parent square root function

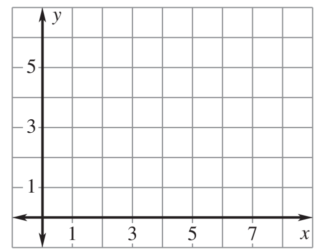
Your Notes

Example 1 *Graph $y = a\sqrt{x}$ where $|a| > 1$*

Graph the function $y = 2\sqrt{x}$ and identify its domain and range. Compare the graph with the graph of $y = \sqrt{x}$.

Solution

Step 1 **Make** a table. Because the square root of a negative number is __________, x must be nonnegative. So, the domain is ______.



x	0	1	2	3
y	___	___	____	____

Step 2 **Plot** the points.

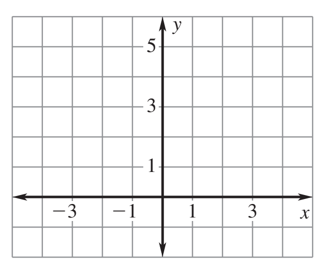
Step 3 **Draw** a _____________ through the points. From either the table or the graph, you can see the range of the function is ______.

Step 4 **Compare** the graph with the graph of $y = \sqrt{x}$. The graph of $y = 2\sqrt{x}$ is a vertical ________ (by a factor of ___) of the graph $y = \sqrt{x}$.

Example 2 *Graph $y = a\sqrt{-x}$*

Graph the function $y = \frac{4}{5}\sqrt{-x}$ and identify its domain and range. Compare the graph with the graph of $y = \sqrt{x}$.

To graph the function, make a table, plot the points, and draw a smooth curve through the points. The domain is ______.



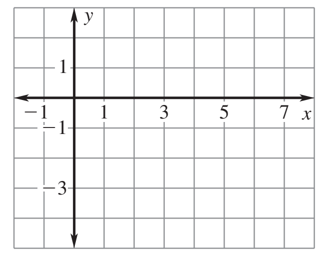
x	-4	-3	-2	-1	0
y	____	____	____	____	___

The range is ______. The graph of $y = \frac{4}{5}\sqrt{-x}$ is a vertical ________ (by a factor of ___) with a __________ ____________ of the graph of $y = \sqrt{x}$.

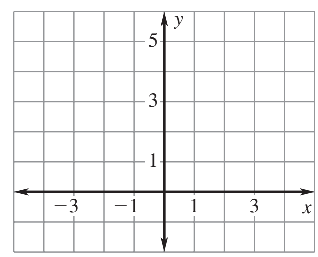
Your Notes

✔ ***Checkpoint*** **Graph the function and identify its domain and range. Compare the graph with the graph of $y = \sqrt{x}$.**

1. $y = -3\sqrt{x}$



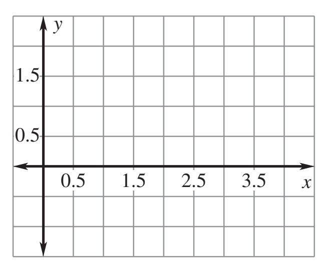
2. $y = 2\sqrt{-x}$



Example 3 ***Graph $y = a\sqrt{x}$ where $|a| < 1$***

Graph the function $y = -\frac{1}{3}\sqrt{x}$ and identify its domain and range. Compare the graph with the graph of $y = \sqrt{x}$.

To graph the function, make a table, plot the points, and draw a smooth curve through the points. The domain is _______.



x	0	1	2	3	4
y	___	_______	_______	_______	_______

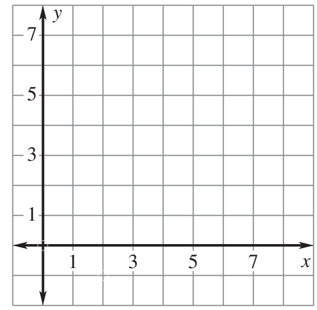
The range is _______. The graph of $y = -\frac{1}{3}\sqrt{x}$ is a vertical ________ (by a factor of ___) with a ________________ _______ of the graph of $y = \sqrt{x}$.

Your Notes

Example 4 ***Graph a function in the form $y = a\sqrt{x - h} + k$***

Graph the function $y = 3\sqrt{x - 2} - 1$.

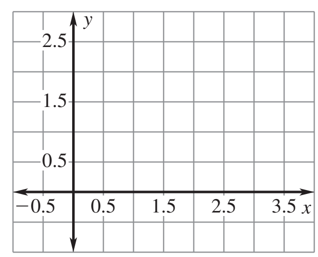
Step 1 **Sketch** the graph of $y = 3\sqrt{x}$.



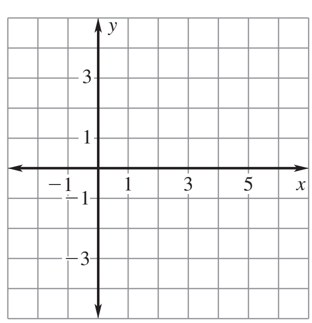
Step 2 **Shift** the graph $|h|$ units horizontally and $|k|$ units vertically. Notice that the function can be written as $y = 3\sqrt{x - 2} + (\_\_\_\_\_)$ So, $h =$ ___ and $k =$ _____. Shift the graph ______________ and ______________.

✔ ***Checkpoint*** **Complete the following exercises.**

3. Graph the function $y = \frac{1}{2}\sqrt{x}$ and identify its domain and range. *Compare* the graph with the graph of $y = \sqrt{x}$.



4. Graph the function $y = 2\sqrt{x + 1} - 3$.



Homework

Name ________________________________ Date ________________

LESSON 3.3 Practice

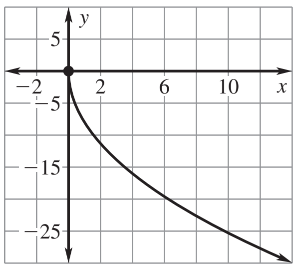
Match the function with its graph.

1. $y = 8\sqrt{x}$

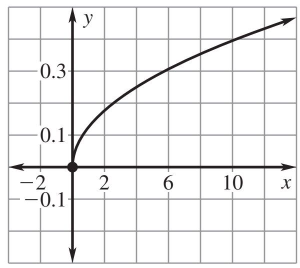
2. $y = -8\sqrt{x}$

3. $y = \frac{1}{8}\sqrt{x}$

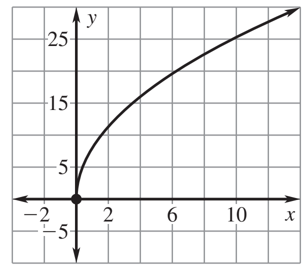
A.



B.

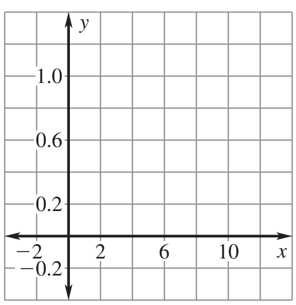


C.

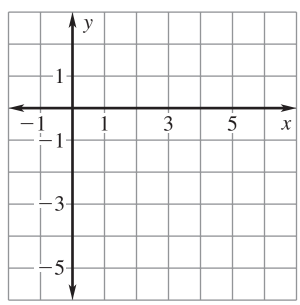


Graph the function and identify its domain and range. *Compare* the graph with the graph of $y = \sqrt{x}$.

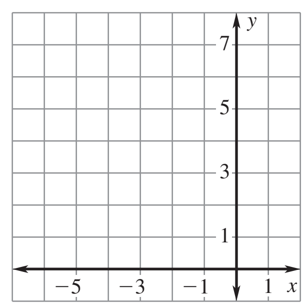
4. $y = 0.4\sqrt{x}$



5. $y = -2\sqrt{x}$



6. $y = 3\sqrt{-x}$



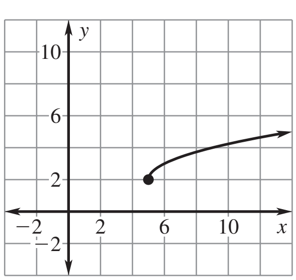
Match the function with its graph.

7. $y = \sqrt{x + 5} - 2$

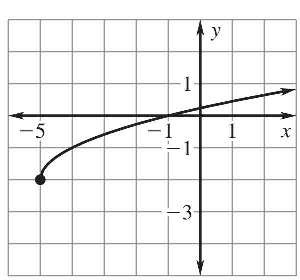
8. $y = \sqrt{x + 5} + 2$

9. $y = \sqrt{x - 5} + 2$

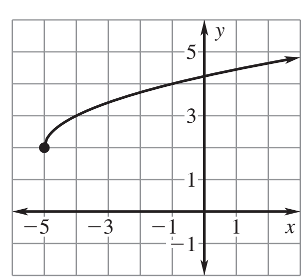
A.



B.



C.



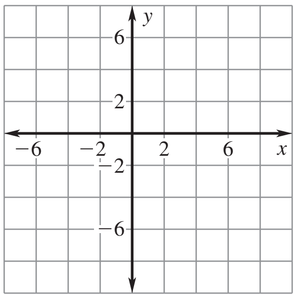
Name ____________________ Date ____________

LESSON 3.3

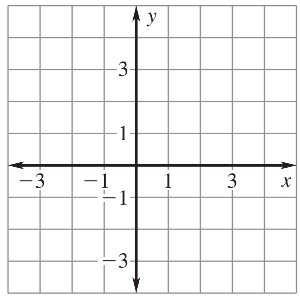
Practice *continued*

Graph the function and identify its domain and range. *Compare* the graph with the graph of $y = \sqrt{x}$.

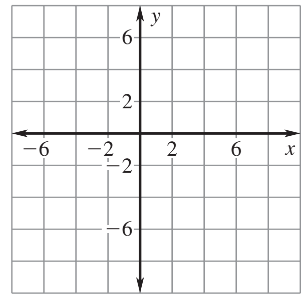
10. $y = \sqrt{x} - 5$



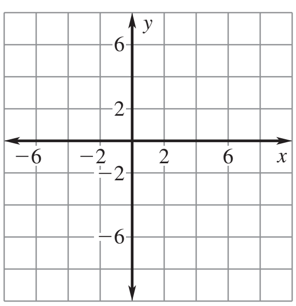
11. $y = \sqrt{x} + 3$



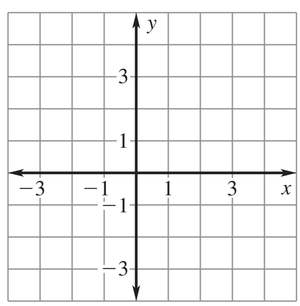
12. $y = \sqrt{x} - 6$



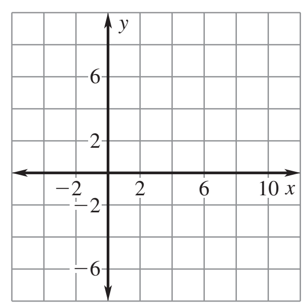
13. $y = \sqrt{x - 2}$



14. $y = \sqrt{x + 3}$

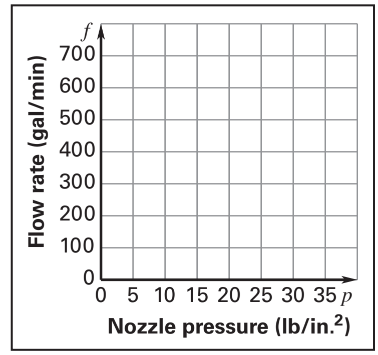


15. $y = \sqrt{x - 5}$



LESSON 3.3 Practice *continued*

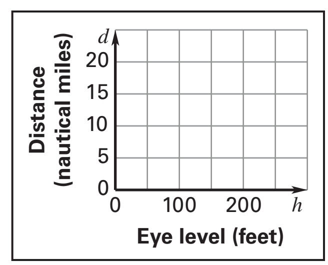
16. Fire Hoses For a fire hose with a nozzle that has a diameter of 2 inches, the flow rate f (in gallons per minute) can be modeled by $f = 120\sqrt{p}$ where p is the nozzle pressure in pounds per square inch.



a. Graph the function and identify its domain and range.

b. If the flow rate is 720 gallons per minute, what is the nozzle pressure?

17. Horizon The distance d (in nautical miles) that a person can see to the horizon is given by the formula $d = 1.17\sqrt{h}$ where h is the person's eye level in feet.



a. Graph the function and identify its domain and range.

b. A person can see 20 nautical miles to the horizon. What is the person's eye level? Round your answer to the nearest foot.

3.4 Simplify Radical Expressions

Georgia Performance Standard(s)

MM1A2a, MM1A2b

Your Notes

Goal • Simplify radical expressions.

VOCABULARY

Simplest form of a radical expression

Rationalizing the denominator

Radical conjugates

PRODUCT PROPERTY OF RADICALS

Words The square root of a product equals the ________ of the ____________ of the factors.

Algebra $\sqrt{ab}$ = ____ • ____ where $a \geq 0$ and $b \geq 0$

Example $\sqrt{9x}$ = ____ • ____ = ____

Example 1 ***Use the product property of radicals***

Simplify $\sqrt{12x^2}$.

Solution

$\sqrt{12x^2}$ = $\sqrt{\_\_\_ \cdot \_\_\_ \cdot \_\_\_}$ **Factor using perfect square factors.**

= ____ • ____ • ____ ________________ **of radicals**

= ______ **Simplify.**

Your Notes

Example 2 ***Multiply radicals***

a. $\sqrt{8} \cdot \sqrt{2} = \sqrt{\_\_\_\ \ \_\_\_}$

$= \_\_\_\_\_$

$= \_\_\_$

b. $\sqrt{5x^3y} \cdot 2\sqrt{x} = \_\_\_\sqrt{\_\_\_\_\_ \cdot \_\_\_}$

$= \_\_\_\sqrt{\_\_\_\_\_}$

$= \_\_\_ \cdot \_\_\_\_ \cdot \_\_\_\_\_ \cdot \_\_\_\_$

$= \_\_\_\_\_\_\_\_$

QUOTIENT PROPERTY OF RADICALS

Words The square root of a quotient equals the __________ of the ______________ of the numerator and denominator.

Algebra $\sqrt{\frac{a}{b}} = \frac{\square}{\square}$ where $a \geq 0$ and $b > 0$

Example $\sqrt{\frac{4}{9}} = \frac{\square}{\square} = \_\_\_$

Example 3 ***Use the quotient property of radicals***

a. $\sqrt{\frac{11}{49}} = \frac{\square}{\square}$ **Quotient property of radicals**

$= \frac{\square}{\square}$ **Simplify.**

b. $\sqrt{\frac{t^2}{36}} = \frac{\square}{\square}$ **Quotient property of radicals**

$= \_\_\_$ **Simplify.**

Your Notes

Checkpoint Simplify the expression.

1. $\sqrt{16z^4}$	2. $4\sqrt{mn} \cdot \sqrt{5m}$	3. $\sqrt{\frac{15}{25}}$

Example 4 *Rationalize the denominator*

a. $\frac{1}{\sqrt{7r}} = \frac{1}{\sqrt{7r}} \cdot \frac{\sqrt{7r}}{\sqrt{7r}}$ Multply by ______.

$= \frac{\square}{\square}$ Product property of radicals

$= \frac{\square}{\square}$ Product property of radicals

$=$ ______ Simplify.

b. $\frac{1}{5 + \sqrt{2}} = \frac{1}{5 + \sqrt{2}} \cdot \frac{\square}{\square}$ Multiply by ______.

$= \frac{\square}{\square}$ Multiply.

$=$ ______ Simplify.

Your Notes

Example 5 *Add and subtract radicals*

a. $7\sqrt{5} + 4\sqrt{45}$

$= \_\_\_\_\_\_\_\_\_\_\_\_$ **Factor using perfect square factor.**

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ **Product property of radicals**

$= \_\_\_\_\_\_\_\_\_\_$ **Simplify.**

$= \_\_\_\_\_$ **Simplify.**

b. $2\sqrt{2} - \sqrt{18}$

$= \_\_\_\_\_\_\_\_\_\_\_$ **Factor using perfect square factor.**

$= \_\_\_\_\_\_\_\_\_\_\_\_$ **Product property of radicals**

$= \_\_\_\_\_\_\_\_\_$ **Simplify.**

$= \_\_\_\_\_$ **Simplify.**

✔ ***Checkpoint*** **Simplify the expression.**

4. $\frac{2}{\sqrt{5y}}$	**5.** $3\sqrt{11} + 2\sqrt{44}$

Homework

Name ______________________________ Date ______________

LESSON 3.4

Practice

Match the radical with the simplified expression.

1. $\sqrt{150}$ **2.** $\sqrt{90}$ **3.** $\sqrt{60}$

A. $3\sqrt{10}$ **B.** $2\sqrt{15}$ **C.** $5\sqrt{6}$

Simplify the expression.

4. $\sqrt{99}$ **5.** $\sqrt{28}$ **6.** $\sqrt{54}$

7. $\sqrt{50}$ **8.** $\sqrt{27a}$ **9.** $\sqrt{16x^2}$

10. $\sqrt{100n^3}$ **11.** $\sqrt{125p^3}$ **12.** $\sqrt{3} \cdot \sqrt{15}$

Name the value of 1 that you would multiply the radical expression by to rationalize the denominator.

13. $\frac{1}{\sqrt{23}}$ **14.** $\frac{3}{1 + \sqrt{10}}$ **15.** $\frac{1}{\sqrt{5x}}$

Simplify the expression by rationalizing the denominator.

16. $\frac{1}{\sqrt{5}}$ **17.** $\frac{1}{\sqrt{17}}$ **18.** $\frac{7}{2 - \sqrt{3}}$

Name ______________________ Date ______________

LESSON 3.4

Practice *continued*

Simplify the expression.

19. $3\sqrt{5} + 4\sqrt{5}$

20. $10\sqrt{2} - 3\sqrt{2}$

21. $\sqrt{7} - 4\sqrt{7}$

22. $4\sqrt{18} + \sqrt{18}$

23. $5\sqrt{8} - 4\sqrt{8}$

24. $\sqrt{12} + 3\sqrt{3}$

25. $\sqrt{2}(1 + \sqrt{2})$

26. $\sqrt{3}(\sqrt{3} - 2)$

27. $\sqrt{3}(1 + \sqrt{12})$

28. **Electricity** The voltage V (in volts) required for a circuit is given by $V = \sqrt{PR}$ where P is the power (in watts) and R is the resistance (in ohms). Find the volts needed to light a 60-watt light bulb with a resistance of 110 ohms. Round your answer to the nearest tenth.

29. **Drum Heads** The radius r (in inches) of a circle with an area A (in square inches) is given by the function $r = \sqrt{\frac{A}{\pi}}$.

a. The drum head on a conga drum has an area of 16π square inches. Find the diameter of the drum head.

b. The drum head on a bongo has an area of 9π square inches. Find the diameter of the drum head.

3.5 Solve Radical Equations

Georgia Performance Standard(s)

MM1A3b

Your Notes

Goal • Solve radical equations.

VOCABULARY

Radical equation

Extraneous solution

Example 1 ***Solve a radical equation***

Solve $3\sqrt{x+1} - 15 = -6$.

Solution

$3\sqrt{x+1} - 15 = -6$	**Write original equation.**
$3\sqrt{x+1} = \_\_\_$	**Add ____ to each side.**
$\sqrt{x+1} = \_\_\_$	**Divide each side by ___.**
$\_\_\_\_\_\_\_\_ = \_\_\_\_$	**Square each side.**
$\_\_\_\_\_\_ = \_\_\_$	**Simplify.**
$x = \_\_\_$	**Subtract ___ from each side.**

The solution is ___.

Check the solution by substituting it in the original equation.

Checkpoint **Complete the following exercise.**

1. Solve $\sqrt{4x - 19} - 2 = 5$.

Your Notes

Example 2 ***Solve an equation with an extraneous solution***

Solve $x = \sqrt{2x + 15}$.

Solution

$x = \sqrt{2x + 15}$	**Write original equation.**
____ = ____________	**Square each side.**
____ = __________	**Simplify.**
_______________ = 0	**Write in standard form.**
(_______)(_______) = 0	**Factor.**
(_______) = 0 *or* (_______) = 0	
x = ___ *or* x = _____	

CHECK Check ___ and _____ in the original equation.

x = ___:

___ $\stackrel{?}{=}$ $\sqrt{2(\_\_\_) + 15}$

5 = ___ ✓

x = _____:

_____ $\stackrel{?}{=}$ $\sqrt{2(\_\_\_\_\_) + 15}$

-3 = ___ ✗

Because _____ does not check in the original equation, it is an ______________________. The only solution of the equation is ___.

✔ ***Checkpoint*** **Solve the equation.**

2. $\sqrt{30 - x} = x$	**3.** $\sqrt{7 + 6x} = x$

Your Notes

> To solve a radical equation that contains two radical expressions, be sure that each side of the equation has only one radical expression before squaring each side.

Example 3 *Solve an equation with radicals on both sides*

Solve $\sqrt{3x - 3} = \sqrt{2x + 8}$.

Solution

$\sqrt{3x - 3} = \sqrt{2x + 8}$	**Write original equation.**
________ = ________	**Square each side.**
______ = ______	**Simplify.**
_____ = __	**Subtract ___ from each side.**
x = ___	**Add __ to each side.**

The solution is ___. Check the solution.

Checkpoint Solve the equation.

4. $\sqrt{5x - 4} = \sqrt{3x + 20}$	**5.** $\sqrt{13 - x} = \sqrt{3x - 15}$

Homework

Name ______________________ Date ____________

LESSON 3.5 Practice

Tell whether the given value is a solution of the equation.

1. $\sqrt{2x + 5} = 3; 2$

2. $\sqrt{3x - 1} = 4; -5$

3. $\sqrt{7x + 3} = 10; 1$

4. $\sqrt{2x + 10} = 4; -3$

5. $\sqrt{1 - 4x} = 5; -6$

6. $\sqrt{6 + 3x} = 12; -2$

Isolate the radical expression on one side of the equation. Do not solve the equation.

7. $7\sqrt{x} - 21 = 0$

8. $-2\sqrt{x} + 8 = 0$

9. $3\sqrt{x} + 5 = 14$

10. $\sqrt{x + 5} - 1 = 8$

11. $\sqrt{x - 4} - 6 = -2$

12. $\sqrt{2x + 3} - 10 = 3$

Solve the equation. Check for extraneous solutions.

13. $\sqrt{x} - 2 = 13$

14. $\sqrt{x} + 6 = 14$

15. $8\sqrt{x} - 24 = 0$

16. $6\sqrt{x} - 18 = 0$

17. $\sqrt{4x} + 3 = 15$

18. $\sqrt{2x} - 7 = 5$

19. $\sqrt{2x - 1} = 7$

20. $\sqrt{3x + 7} = 4$

21. $2\sqrt{x + 5} = 12$

Name ______________________________ Date ________________

LESSON 3.5

Practice *continued*

Simplify each side of the equation.

22. $(\sqrt{7x + 3})^2 = (\sqrt{7x - 1})^2$

23. $(\sqrt{5x - 8})^2 = (\sqrt{1 - 6x})^2$

24. $(\sqrt{9 - 2x})^2 = (5x)^2$

25. $(2x)^2 = (\sqrt{3x + 1})^2$

26. $(x + 1)^2 = (\sqrt{1 - 3x})^2$

27. $(\sqrt{4x - 3})^2 = (x - 2)^2$

Solve the equation. Check for extraneous solutions.

28. $\sqrt{2x + 5} = \sqrt{3x + 4}$

29. $\sqrt{9x - 3} = \sqrt{7x + 9}$

30. $x = \sqrt{6 - x}$

31. **Free-Falling Velocity** The velocity v of a free-falling object (in feet per second), the height h from which it falls (in feet), and the acceleration due to gravity, 32 feet per second squared, are related by the function $v = \sqrt{64h}$.

a. Find the height from which a tennis ball was dropped if it hits the ground with a velocity of 32 feet per second.

b. How much higher than the ball in part (a) was a tennis ball dropped from if it hits the ground with a velocity of 40 feet per second?

32. **Children's Museum** A new children's museum opens. For the first 12 weeks, the number of people N (in hundreds of people) that visit the museum can be modeled by the function $N = \sqrt{1000 + 300t}$ where t is the number of weeks since the opening week.

a. After how many weeks did 4000 (or 40 hundred) people visit the museum?

b. After how many weeks did 5000 (or 50 hundred) people visit the museum?

3.6 Graph Rational Functions

Georgia Performance Standard(s)

MM1A1b, MM1A1c, MM1A1d

Your Notes

Goal • Graph rational functions.

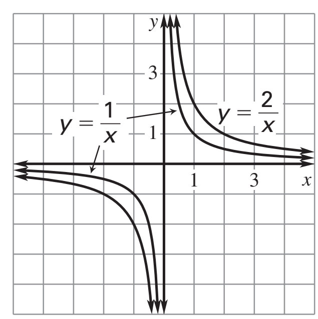
VOCABULARY

Rational function

Asymptote

Example 1 ***Compare graph of $y = \frac{a}{x}$ where $a > 0$ with graph of $y = \frac{1}{x}$***

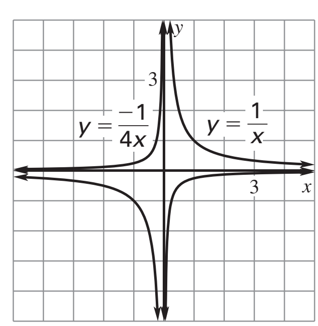
The graph of $y = \frac{2}{x}$ is a vertical ________ of the graph of $y = \frac{1}{x}$.



Example 2 ***Compare graph of $y = \frac{a}{x}$ where $a < 0$ with graph of $y = \frac{1}{x}$***

The graph of $y = \frac{-1}{4x}$ is a vertical ________ with a reflection in the ________ of the graph of $y = \frac{1}{x}$.

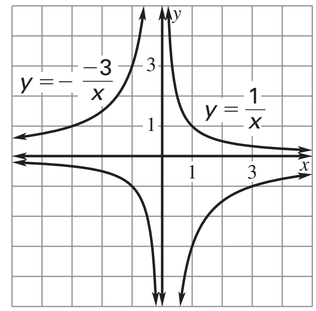
Note that the graph could also be viewed as being reflected in the ________.



Your Notes

Checkpoint **Complete the following exercise.**

1. *Compare* the graph of $y = \frac{-3}{x}$ with the graph of $y = \frac{1}{x}$.



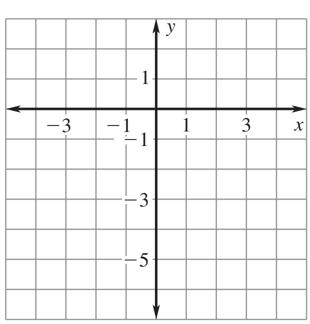
Example 3 ***Graph $y = \frac{1}{x} + k$***

Graph $y = \frac{1}{x} - 3$ and identify its domain and range. Compare the graph with the graph of $y = \frac{1}{x}$.

Solution

Graph the function using a table of values. The domain is all real numbers except ___. The range is all real numbers except _____. The graph of $y = \frac{1}{x} - 3$ is a _________ translation (of ___ units _______) of the graph of $y = \frac{1}{x}$.

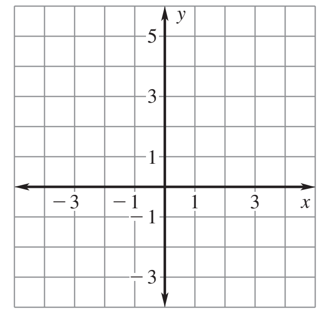
x	*y*
−2	________
−1	________
−0.5	________
0	________
0.5	________
1	________
2	________



Your Notes

✔ ***Checkpoint*** **Complete the following exercise.**

2. Graph $y = \frac{1}{x} + 2$ and identify its domain and range. *Compare* the graph with the graph of $y = \frac{1}{x}$.



Example 4 *Graph $y = \frac{1}{x - h}$*

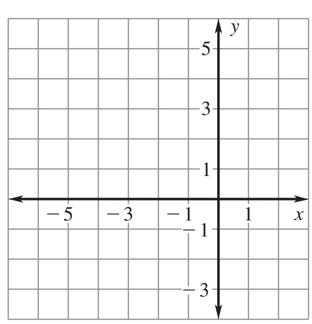
Graph $y = \frac{1}{x + 3}$ and identify its domain and range. Compare the graph with the graph of $y = \frac{1}{x}$.

Solution

Graph the function using a table of values. The domain is all real numbers except _____. The range is all real numbers except ___.

The graph of $y = \frac{1}{x + 3}$ is a ____________ translation (of ___ units ____________) of the graph of $y = \frac{1}{x}$.

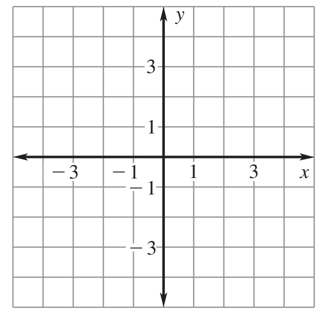
x	*y*
−5	________
−4	________
−3.5	________
−3	________
−2.5	________
−2	________
−1	________



Your Notes

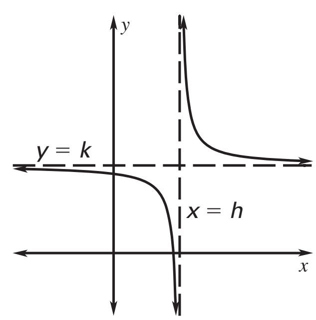
✔ ***Checkpoint*** **Complete the following exercise.**

3. Graph $y = \frac{1}{x - 1}$ and identify its domain and range. *Compare* the graph with the graph of $y = \frac{1}{x}$.



GRAPH OF $y = \frac{a}{x - h} + k$

The graph of $y = \frac{a}{x - h} + k$ has the following characteristics:



- If $|a| > 1$, the graph is a vertical ________ of the graph of $y = \frac{1}{x}$.
  If $0 < |a| < 1$, the graph is a vertical ________ of the graph of $y = \frac{1}{x}$. If $a < 0$, the graph is a reflection in the ________ of the graph of $y = \frac{1}{x}$.
- The horizontal asymptote is $y =$ ___. The vertical asymptote is $x =$ ___.

The domain of the function is all real numbers except $x =$ ___. The range is all real numbers except $y =$ ___.

Your Notes

Example 5 ***Graph $y = \frac{a}{x - h} + k$***

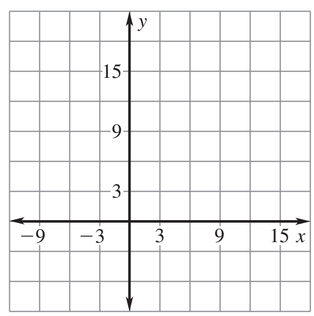
Graph $y = \frac{2}{x - 3} + 4$.

Solution

Step 1 **Identify** the asymptotes of the graph. The vertical asymptote is $x =$ ___. The horizontal asymptote is $y =$ ___.

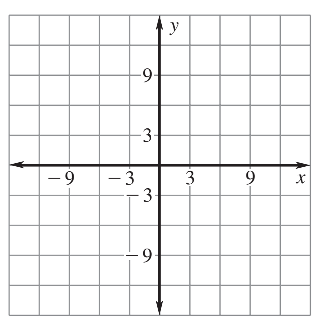
Step 2 **Plot** several points on each side of the ________ asymptote.

Step 3 **Graph** two branches that pass through the plotted points and approach the ____________.



✔ ***Checkpoint*** **Complete the following exercise.**

4. Graph $y = \frac{3}{x + 2} - 1$.



Homework

Name ____________________ Date ____________

LESSON 3.6 Practice

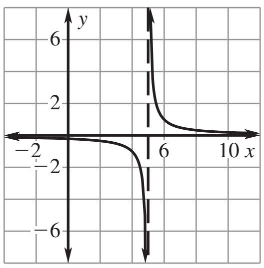
Match the function with its graph.

1. $y = \frac{1}{5x}$

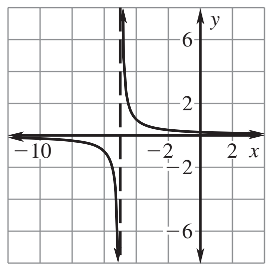
2. $y = \frac{1}{x - 5}$

3. $y = \frac{1}{x + 5}$

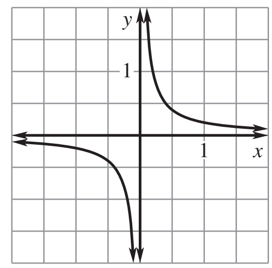
A.



B.

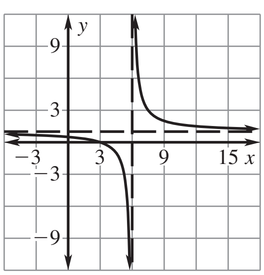


C.

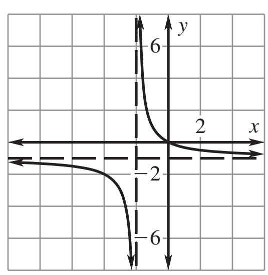


Identify the domain and range of the function from its graph.

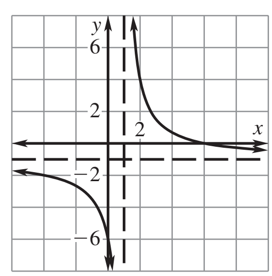
4.



5.



6.



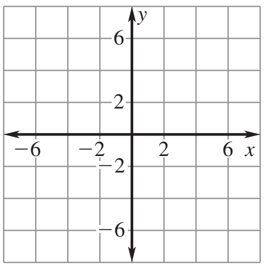
Name ______________________ Date ______________

LESSON 3.6

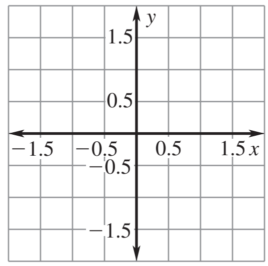
Practice *continued*

Graph the function and identify its domain and range. Then compare the graph with the graph of $y = \frac{1}{x}$.

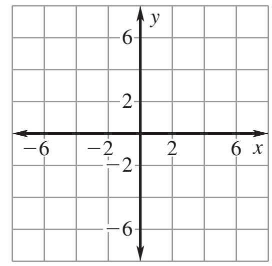
7. $y = \frac{4}{x}$



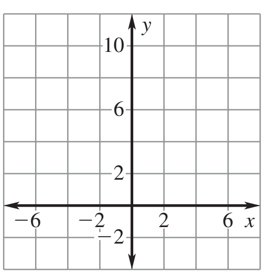
8. $y = \frac{1}{2x}$



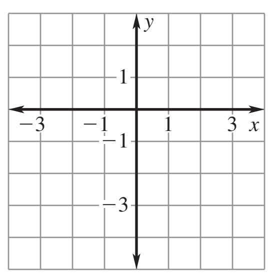
9. $y = \frac{-5}{x}$



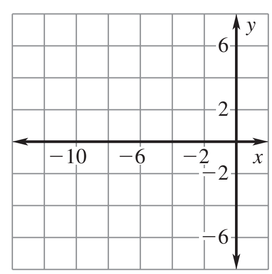
10. $y = \frac{1}{x} + 4$



11. $y = \frac{1}{x} - 1$



12. $y = \frac{1}{x + 6}$



Name ____________________ Date ____________

LESSON 3.6

Practice *continued*

Match the function with its asymptotes.

13. $y = \frac{1}{x + 3} - 2$

14. $y = \frac{1}{x - 2} + 3$

15. $y = \frac{1}{x - 3} + 2$

A. $x = 3, y = 2$

B. $x = 2, y = 3$

C. $x = -3, y = -2$

Determine the asymptotes of the graph of the function.

16. $y = \frac{-3}{x - 8}$

17. $y = \frac{-11}{x} - 14$

18. $y = \frac{6}{x - 6} + 5$

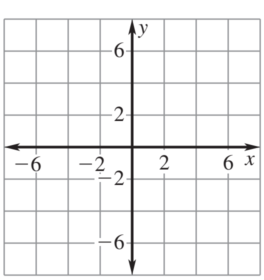
19. $y = \frac{-4}{x + 13} + 1$

20. $y = \frac{10}{x + 10} - 2$

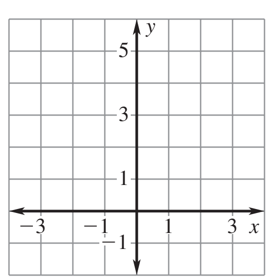
21. $y = \frac{8}{x + 5} - 7$

Graph the function.

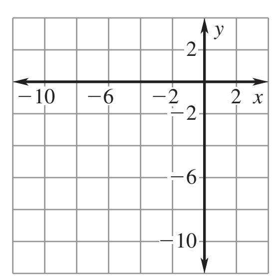
22. $y = \frac{4}{x} - 1$



23. $y = \frac{2}{x} + 2$



24. $y = \frac{1}{x + 3} - 5$



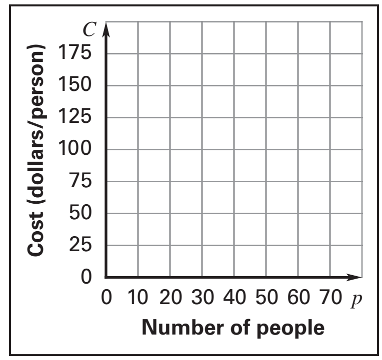
Name ______________________ Date ______________

LESSON 3.6

Practice *continued*

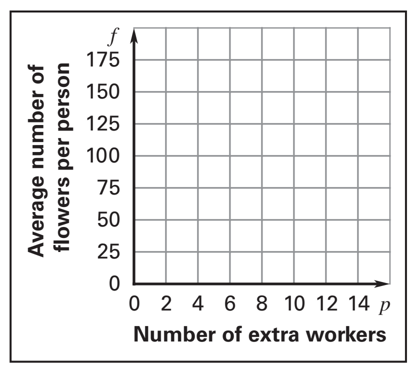
25. **Football Hall of Fame** Your football team is planning a bus trip to the Pro Football Hall of Fame. The cost for renting a bus is $500, and the cost will be divided equally among the people who are going on the trip. One admission costs $16.

a. Write an equation that gives the cost C (in dollars per person) of the trip as a function of the number p of people going on the trip.



b. Graph the equation.

26. **Prom** During prom season, a florist has orders for 400 boutonnieres and corsages. There are 3 people currently scheduled to put together the flowers. The florist hopes to call some extra workers to complete all of the orders. Write an equation that gives the average number f of boutonnieres and corsages made per person as a function of the number p of extra workers that help complete the orders. Then graph the equation.



3.7 Divide Polynomials

Georgia Performance Standard(s)

MM1A2c

Goal • Divide polynomials.

Your Notes

Example 1 ***Divide a polynomial by a monomial***

Divide $10x^3 - 25x^2 + 15x$ by $5x$.

Solution

Method 1: Write the division as a fraction.

$(10x^3 - 25x^2 + 15x) \div 5x$

$= \dfrac{\square}{\square}$ **Write as a fraction.**

$= \dfrac{\square}{\square} - \dfrac{\square}{\square} + \dfrac{\square}{\square}$ **Divide each term by ____.**

$=$ ____________ **Simplify.**

Method 2: Use long division.

Think: $10x^3 \div 5x = ?$	Think: $-25x^2 \div 5x = ?$	Think: $15x \div 5x = ?$

$\square - \square + \square$

$5x\overline{)10x^3 - 25x^2 + 15x}$

To check your answer, multiply the quotient by the divisor.

$(10x^3 - 25x^2 + 15x) \div 5x =$ ____________

✔ ***Checkpoint*** **Complete the following exercise.**

1. Divide $(12x^3 + 9x^2 - 3x)$ by $3x$.

Your Notes

Example 2 ***Divide a polynomial by a binomial***

Divide $2x^2 + 9x - 6$ by $2x + 3$.

Solution

First divide the first term of $2x^2 + 9x - 6$ by the first term of $2x + 3$.

$2x + 3\overline{)2x^2 + 9x - 6}$

Multiply ___ and ________.

Subtract ___________. Bring down _____. Divide the first term of _________ by the first term of $2x + 3$.

Multiply ___ and ________.

Subtract _________.

$(2x^2 + 9x - 6) \div (2x + 3) =$ ________________

Checkpoint **Divide.**

2. $(3x^2 - x - 14) \div (3x - 7)$

3. $(6x^2 - 13x + 11) \div (3x - 5)$

Your Notes

Example 3 *Insert missing terms*

Divide $4x^2 - 11$ by $-3 + 2x$.

$2x - 3\overline{)4x^2 + 0x - 11}$ — Rewrite polynomials. Insert missing term.

________ Multiply ____ and ________.

________ Subtract ________. Bring down ______.

________ Multiply ___ and ________.

_____ Subtract ________.

$(6x^2 - 11) \div (-3 + 2x) =$ ________________

✔ ***Checkpoint*** Divide.

4. $(6 - 2x + x^2) \div (2 + x)$

5. $(-11 + 3x^2) \div (-3 + x)$

Homework

Name ______________________ Date ______________

LESSON 3.7 Practice

Simplify the expression.

1. $\frac{18x^3}{6x}$

2. $\frac{-15x^2}{5x}$

3. $\frac{-10x}{10x}$

Divide.

4. $(9x^3 - 6x^2 + 18x) \div 3x$

5. $(14x^3 + 21x^2 - 28x) \div 7x$

6. $(16x^4 - 16x^3 - 24x^2) \div 8x$

7. $(20x^4 - 5x^2 + 10x) \div 5x$

8. $(-2x^3 + 6x^2 + 4x) \div (-2x)$

9. $(4x^3 - 16x^2 + 20x) \div (-4x)$

Match the equivalent expressions.

10. $(x^2 + 3x - 10) \div (x + 5)$

A. $x - 2$

11. $(x^2 - 3x - 10) \div (x + 5)$

B. $x + 5$

12. $(x^2 + 3x - 10) \div (x - 2)$

C. $x - 8 + \frac{30}{x + 5}$

Name ______________________ Date ____________

LESSON 3.7

Practice *continued*

Divide.

13. $(x^2 + 10x + 24) \div (x + 6)$

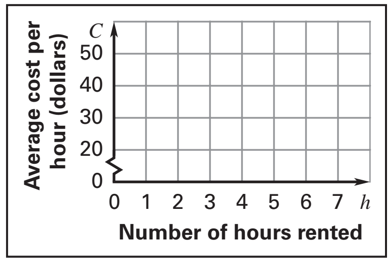
14. $(x^2 - 2x - 15) \div (x + 3)$

15. $(x^2 - 7x + 6) \div (x - 1)$

16. $(x^2 + 3x + 2) \div (x - 1)$

17. **Moped Rental** While on vacation, you decide to rent a moped to see the sights. A local rental store offers mopeds for \$20 an hour plus a \$5 gasoline fill-up fee.

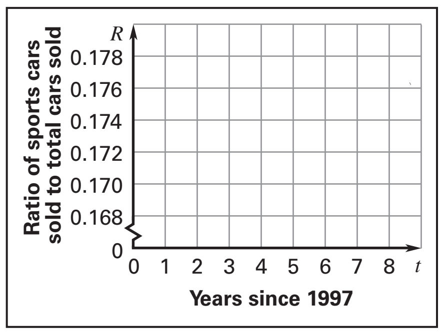
a. Write an equation that gives the average cost C per hour as a function of the number h of hours you rent the moped.



b. Rewrite the equation in the form $y = \frac{a}{x - h} + k$. Then graph the equation.

18. **Car Dealer** The number of sports cars that a dealer sold per year between 1997 and 2006 can be modeled by $S = 4t + 21$ where t is the number of years since 1997. The total number of cars sold by the dealer can be modeled by $C = 24t + 120$.

a. Use long division to find a model for the ratio R of the number of sports cars sold to the total number of cars sold.



b. Graph the model.

3.8 Simplify Rational Expressions

Georgia Performance Standard(s)

MM1A2c

Goal • Simplify rational expressions.

Your Notes

VOCABULARY

Rational expression

Excluded value

Simplest form of a rational expression

Example 1 ***Find excluded values***

Find the excluded values, if any, of the expression.

a. $\frac{x}{4x - 8}$ **b.** $\frac{3x}{x^2 - 16}$ **c.** $\frac{1}{x^2 + 2}$

Solution

a. The expression $\frac{x}{4x - 8}$ is undefined when ________ $= 0$, or $x =$ ___. The excluded value is ___.

b. The expression $\frac{3x}{x^2 - 16}$ is undefined when ________ $= 0$, or (_______)(_______) $= 0$. The solutions of the equation are _____ and ___. The excluded values are _____ and ___.

c. The expression $\frac{1}{x^2 + 2}$ is undefined when ________ $= 0$. The graph of $y = x^2 + 2$ ________________________. So, the quadratic equation has ______________. There are ____________________.

Your Notes

✔ *Checkpoint* Find the excluded values, if any, of the expression.

1. $\frac{x+6}{14x}$	**2.** $\frac{9x+1}{x^2-x-20}$

Example 2 ***Simplify expressions by dividing out monomials***

Simplify the rational expression, if possible. State the excluded values.

a. $\frac{18x}{6x^2} = \frac{\square}{\square}$ **Divide out common factors.**

$= \underline{\quad}$ **Simplify.**

The excluded value is ___.

b. $\frac{12x^2 - 6x}{24x} = \frac{\square}{\square}$ **Factor numerator and denominator.**

$= \frac{\square}{\square}$ **Divide out common factors.**

$= \underline{\quad\quad}$ **Simplify.**

The excluded value is ___.

Your Notes

✔ *Checkpoint* Simplify the rational expression, if possible. State the excluded values.

3. $\frac{7}{5x + 3}$	**4.** $\frac{5x}{5x^2 - 25}$	**5.** $\frac{6x^3}{2x + 4}$

Example 3 ***Simplify an expression by dividing out binomials***

Simplify $\frac{x^2 + x - 12}{x^2 - 5x + 6}$. State the excluded values.

Solution

$\frac{x^2 + x - 12}{x^2 - 5x + 6} = \frac{\boxed{}}{\boxed{}}$ **Factor and divide out common factor.**

$= \frac{}{\rule{3em}{0.4pt}}$ **Simplify.**

The excluded values are ___ and ___.

✔ *Checkpoint* Simplify the rational expression. State the excluded values.

6. $\frac{x^2 + 7x + 6}{x^2 + 3x - 18}$	**7.** $\frac{-(x^2 - 4)}{x^2 + 5x - 14}$

Homework

Name ______________________ Date ______________

LESSON 3.8

Practice

Find the excluded values, if any, of the expression.

1. $\frac{8x}{24}$

2. $\frac{15}{4x}$

3. $\frac{10}{x - 6}$

4. $\frac{-4}{x + 3}$

5. $\frac{1}{2x - 2}$

6. $\frac{5}{8x - 16}$

7. $\frac{8}{3x + 6}$

8. $\frac{5}{2x - 1}$

9. $\frac{-1}{3x + 2}$

Determine whether the expression is in simplest form.

10. $\frac{x - 1}{3x - 3}$

11. $\frac{x + 1}{x^2 - 1}$

12. $\frac{x + 10}{x^2 - 4}$

13. $\frac{x + 3}{x^2 - 4x}$

14. $\frac{x + 5}{x^2 + 5x}$

15. $\frac{x}{x^2 - 4x + 4}$

Name ______________________ Date ______________

LESSON 3.8

Practice *continued*

Simplify the rational expression, if possible. Find the excluded values.

16. $\frac{14}{21x}$

17. $\frac{42}{12x}$

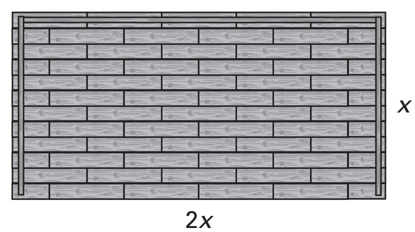
18. $\frac{2x + 4}{x + 2}$

19. $\frac{x + 5}{x - 5}$

20. $\frac{x - 6}{x^2 - 36}$

21. $\frac{10x}{x^2 - 100}$

22. **Deck** You have drawn up preliminary plans for a rectangular deck that will be attached to the back of your house. You have decided that the length of the deck should be twice the width as shown.



a. Write a rational expression for the ratio of the perimeter to the area of the deck.

b. Simplify your expression from part (a).

23. **School Enrollment** The total enrollment (in thousands) of students in public schools from kindergarten through college from 2000 to 2004 can be modeled by $E = 660t + 59{,}240$ where t is the number of years since 2000. The total enrollment (in thousands) of students in public colleges can be modeled by $C = 410t + 11{,}980$.

a. Write a model for the ratio of the number of enrollments in college to the total number of enrollments.

b. Simplify your model from part (a).

3.9 Multiply and Divide Rational Expressions

Georgia Performance Standard(s)

MM1A2e

Your Notes

Goal • Multiply and divide rational expressions.

Example 1 ***Multiply rational expressions involving polynomials***

Find the product $\frac{x}{5x^2 - 6x - 8} \cdot \frac{2x^2 - 4x}{7x^2}$.

$\frac{x}{5x^2 - 6x - 8} \cdot \frac{2x^2 - 4x}{7x^2}$

$= \frac{\square}{\square}$ **Multiply numerators and denominators.**

$= \frac{\square}{\square}$ **Factor and divide out common factors.**

$=$ ______ **Simplify.**

Example 2 ***Multiply a rational expression by a polynomial***

Find the product $\frac{4x}{x^2 - x - 12} \cdot (x - 4)$.

$\frac{4x}{x^2 - x - 12} \cdot (x - 4)$

$= \frac{4x}{x^2 - x - 12} \cdot \frac{\square}{\square}$ **Rewrite polynomial as a fraction.**

$= \frac{\square}{\square}$ **Multiply numerators and denominators.**

$= \frac{\square}{\square}$ **Factor and divide out common factor.**

$=$ ______ **Simplify.**

Your Notes

✔ *Checkpoint* Find the product.

1. $\dfrac{x^2 - 5x + 4}{3x^2 - 12x} \cdot \dfrac{2x^2 + 2}{x^2 + 6x - 7}$

2. $\dfrac{2x}{x^2 + 5x - 24} \cdot (x + 8)$

Example 3 ***Divide rational expressions involving polynomials***

Find the quotient $\dfrac{x^2 + 5x - 24}{x^2 + 9x + 8} \div \dfrac{x^2 - 9}{6x - 18}$.

Solution

$\dfrac{x^2 + 5x - 24}{x^2 + 9x + 8} \div \dfrac{x^2 - 9}{6x - 18}$

$= \dfrac{x^2 + 5x - 24}{x^2 + 9x + 8} \cdot \dfrac{\square}{\square}$ **Multiply by multiplicative inverse.**

$= \dfrac{\square}{\square}$ **Multiply numerators and denominators.**

$= \dfrac{\square}{\square}$ **Factor and divide out common factors.**

$=$ ______ **Simplify.**

Your Notes

Example 4 ***Divide a rational expression by a polynomial***

Find the quotient $\frac{x^2 - 25}{x - 3} \div (x - 5)$.

Solution

$\frac{x^2 - 25}{x - 3} \div (x - 5)$

$= \frac{x^2 - 25}{x - 3} \div \frac{\square}{\square}$ Rewrite polynomial as fraction.

$= \frac{x^2 - 25}{x - 3} \cdot \frac{\square}{\square}$ Multiply by multiplicative inverse.

$= \frac{\square}{\square}$ Multiply numerators and denominators.

$= \frac{\square}{\square}$ Factor and divide out common factor.

$=$ ______ Simplify.

✔ ***Checkpoint*** Find the quotient.

3. $\frac{x^2 + 2x - 15}{x^2 + 4x - 5} \div \frac{x^2 - 4}{7x - 14}$

4. $\frac{x^2 + 8x + 7}{x^2 - 1} \div (x + 7)$

Homework

Name ______________________ Date ______________

LESSON 3.9 Practice

Match the equivalent expressions.

1. $\frac{4x^2}{10} \cdot \frac{5}{-2x}$

2. $\frac{4x^2}{10} \div \frac{5}{-2x}$

3. $\frac{2x}{5} \cdot \frac{10}{4x^2}$

A. $\frac{1}{x}$

B. $\frac{-4x^3}{25}$

C. $-x$

Find the product.

4. $\frac{14x^2}{3} \cdot \frac{9}{2x}$

5. $\frac{7}{9x^4} \cdot \frac{3x^2}{2}$

6. $\frac{6x^2}{5} \cdot \frac{10}{12x^3}$

7. $\frac{x + 3}{4x} \cdot \frac{2x^2}{4x + 12}$

8. $\frac{3x - 6}{5x^2} \cdot \frac{10x^4}{x - 2}$

9. $\frac{x + 5}{6x^3} \cdot \frac{15x}{2x + 10}$

10. $\frac{x + 3}{x^2 - 2x} \cdot \frac{x - 2}{x^2 + 4x + 3}$

11. $\frac{5x + 5}{x + 3} \cdot \frac{x^2 + 5x + 6}{x + 1}$

12. $\frac{x + 2}{x - 3} \cdot \frac{x^2 - 4x + 3}{x^2 + 6x + 8}$

Name ____________________ Date ____________

LESSON 3.9

Practice *continued*

Find the quotient.

13. $\frac{4x^2}{5} \div \frac{8x}{15}$

14. $\frac{11}{6x} \div \frac{22}{9x^2}$

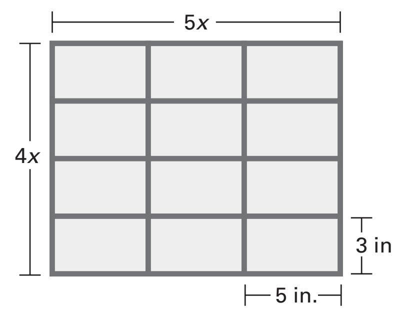
15. $\frac{x + 4}{5x} \div \frac{x + 4}{9x^2}$

16. $\frac{2x + 2}{3x^2} \div \frac{x + 1}{4}$

17. $\frac{8x - 16}{5x^2} \div \frac{4x - 8}{10x}$

18. $\frac{x + 1}{14x} \div \frac{x^2 + 3x + 2}{-7x^2}$

19. **Model Cars** You want to create a display box that will hold your model cars. You want each section of the box to be 5 inches by 3 inches and you want the box's dimensions to be related as shown. Write and simplify an expression that you can use to determine the number of sections you can have in the display box.



20. **Total Cost** The cost C (in dollars) of producing a product from 1995 to 2005 can be modeled by $C = \frac{10 + 3t}{80 - t}$ where t is the number of years since 1995. The number N (in hundreds of thousands) of units made each year from 1995 to 2005 can be modeled by $N = \frac{160 - 2t}{11 - t}$ where t is the number of years since 1995.

a. Write a model that gives the total production cost T of the product each year.

b. Approximate the total production cost in 2000.

3.10 Add and Subtract Rational Expressions

Georgia Performance Standard(s)

MM1A2e

Your Notes

Goal • Add and subtract rational expressions.

VOCABULARY

Least common denominator of rational expressions (LCD)

Example 1 ***Add and subtract with the same denominator***

Find the sum or difference.

a. $\frac{3}{8x} + \frac{4}{8x}$ **b.** $\frac{2x + 9}{x + 1} - \frac{7}{x + 1}$

Solution

a. $\frac{3}{8x} + \frac{4}{8x} = \frac{\boxed{}}{8x}$ **Add numerators.**

$= \underline{}$ **Simplify.**

b. $\frac{2x + 9}{x + 1} - \frac{7}{x + 1} = \frac{\boxed{}}{x + 1}$ **Subtract numerators.**

$= \frac{\boxed{}}{x + 1}$ **Simplify.**

$= \frac{\boxed{}}{\boxed{}}$ **Factor and divide out common factor.**

$= \underline{}$ **Simplify.**

Your Notes

✔ *Checkpoint* Find the sum or difference.

1. $\frac{x+8}{4x} + \frac{3}{4x}$	**2.** $\frac{6x-5}{x} - \frac{2x-5}{x}$

Example 2 *Find the LCD of rational expressions*

Find the LCD of the rational expressions.

a. $\frac{1}{3x^3}, \frac{5}{4x^4}$ **b.** $\frac{7}{x^2-4}, \frac{x+3}{x^2+x-2}$

Solution

a. Find the ____________________ of $3x^3$ and $4x^4$.

$3x^3 =$ ____________

$4x^4 =$ ______________

LCM = __________________ = ______

The LCD of $\frac{1}{3x^3}$ and $\frac{5}{4x^4}$ is ______.

b. Find the ____________________ of $x^2 - 4$ and $x^2 + x - 2$.

$x^2 - 4 =$ _______________

$x^2 + x - 2 =$ _______________

LCM = ______________________

The LCD of $\frac{7}{x^2-4}$ and $\frac{x+3}{x^2+x-2}$ is

___________________.

✔ *Checkpoint* Find the LCD of the rational expressions.

3. $\frac{5}{36x}, \frac{x+2}{4x^3}$	**4.** $\frac{7x}{x-8}, \frac{x-1}{x+3}$

Your Notes

Example 3 *Add expressions with different denominators*

Find the sum $\frac{1}{3x^3} + \frac{5}{4x^4}$.

Solution

$\frac{1}{3x^3} + \frac{5}{4x^4}$

$= \frac{1 \cdot \square}{3x^3 \cdot \square} + \frac{5 \cdot \square}{4x^4 \cdot \square}$ Rewrite fractions using LCD, ______.

$= \frac{\square}{\square} + \frac{\square}{\square}$ Simplify numerators and denominators.

$=$ ______ Add fractions.

Example 4 *Subtract expressions with different denominators*

Find the difference $\frac{x + 1}{x^2 + 5x + 6} - \frac{x - 4}{x^2 - 9}$.

Solution

$\frac{x + 1}{x^2 + 5x + 6} - \frac{x - 4}{x^2 - 9}$

$= \frac{x + 1}{(\square)(\square)} - \frac{x - 4}{(\square)(\square)}$

$= \frac{(x + 1)(\square)}{\square(\square)} - \frac{(x - 4)(\square)}{\square(\square)}$

$= \frac{\square}{\square}$

$= \frac{\square}{\square}$

$= \frac{\square}{\square}$

Your Notes

✔ ***Checkpoint*** **Find the sum or difference.**

5. $\frac{9}{x - 1} - \frac{15}{3x + 1}$

6. $\frac{12}{5x} + \frac{3x}{x - 4}$

7. $\frac{x - 1}{x^2 - 2x - 24} + \frac{4}{x^2 - 5x - 6}$

8. $\frac{x + 2}{x^2 + 2x - 15} - \frac{x - 6}{x^2 + 4x - 21}$

Homework

Name ______________________ Date ______________

LESSON 3.10

Practice

Find the sum or difference.

1. $\frac{1}{4x} + \frac{2}{4x}$

2. $\frac{4}{5x} + \frac{6}{5x}$

3. $\frac{8}{3x^2} - \frac{7}{3x^2}$

4. $\frac{20}{7x^3} - \frac{6}{7x^3}$

5. $\frac{x - 3}{2x} + \frac{7}{2x}$

6. $\frac{x - 10}{9x} - \frac{17}{9x}$

7. $\frac{2x + 1}{5x} + \frac{6}{5x}$

8. $\frac{x + 4}{2x^2} - \frac{x}{2x^2}$

9. $\frac{x + 6}{x - 1} + \frac{x - 2}{x - 1}$

Find the LCD of the rational expressions.

10. $\frac{2}{5x}, \frac{4}{10x}$

11. $\frac{1}{12x}, \frac{x + 1}{4x^3}$

12. $\frac{3}{x + 1}, \frac{1}{x}$

13. $\frac{5}{x - 4}, \frac{3}{x}$

14. $\frac{6x}{x + 2}, \frac{5}{x + 4}$

15. $\frac{9}{x - 3}, \frac{8x}{x + 7}$

Name ______________________ Date ______________

LESSON 3.10

Practice *continued*

Find the sum or difference.

16. $\frac{8x}{3} + \frac{1}{5x}$

17. $\frac{7x}{2} - \frac{4}{8x}$

18. $\frac{5}{4x} + \frac{7}{9x}$

19. $\frac{2}{3x^2} - \frac{8}{5x}$

20. $\frac{4}{x} + \frac{3}{x + 4}$

21. $\frac{4}{x - 2} + \frac{5}{x + 7}$

22. **Cabin Cruiser** A cabin cruiser travels 48 miles upstream (against the current) and 48 miles downstream (with the current). The speed of the current is 4 miles per hour.

a. Write an expression for the travel time of the cruiser going upstream and write an expression for the travel time of the cruiser going downstream.

b. Use your answers from part (a) to write an equation that gives the total travel time t (in hours) as a function of the boat's average speed r (in miles per hour) in still water.

c. Find the total travel time if the cabin cruiser's average speed in still water is 12 miles per hour.

23. **Driving** You drive 40 miles to visit a friend. On the drive back home, your average speed decreases by 4 miles per hour. Write an equation that gives the total driving time t (in hours) as a function of your average speed r (in miles per hour) when driving to visit your friend. Then find the total driving time if you drive to your friend's at an average speed of 52 miles per hour. Round your answer to the nearest tenth.

3.11 Solve Rational Equations

Georgia Performance Standard(s)

MM1A3d

Your Notes

Goal • Solve rational equations.

VOCABULARY

Rational equation

Example 1 ***Use the cross products property***

Solve $\frac{5}{x-1} = \frac{x}{4}$. Check your solution.

Solution

$\frac{5}{x-1} = \frac{x}{4}$ — **Write original equation.**

$20 =$ ________ — **Cross products property**

$0 =$ ______________ — **Subtract _____ from each side.**

$0 =$ (________)(________) — **Factor polynomial.**

________ $= 0$ *or* ________ $= 0$ — **Zero-product property**

$x =$ ___ *or* $x =$ _____ — **Solve for x.**

The solutions are ___ and _____.

CHECK If $x =$ ___:

$\frac{5}{\square - 1} \stackrel{?}{=} \frac{\square}{4}$

___ = ___ ✓

If $x =$ _____:

$\frac{5}{\square - 1} \stackrel{?}{=} \frac{\square}{4}$

_____ = _____ ✓

Your Notes

Checkpoint **Solve the equation. Check your solution(s).**

1. $\frac{-2}{x+9} = \frac{x}{7}$	**2.** $\frac{6}{x-4} = \frac{3}{x}$

Example 2 ***Multiply by the LCD***

Solve $\frac{x}{x+6} - \frac{1}{2} = \frac{4}{x+6}$.

Solution

$$\frac{x}{x+6} - \frac{1}{2} = \frac{4}{x+6}$$

$$\frac{x}{x+6} \cdot \boxed{} - \frac{1}{2} \cdot \boxed{} = \frac{4}{x+6} \cdot \boxed{}$$

$$\frac{\boxed{}}{\cancel{x+6}} - \frac{\boxed{}}{\cancel{2}} = \frac{\boxed{}}{\cancel{x+6}}$$

$$\underline{} = \underline{}$$

$$\underline{} = \underline{}$$

$$x = \underline{}$$

The solution is ____.

Checkpoint **Complete the following exercise.**

3. Solve $\frac{3}{x-3} - \frac{1}{x+3} = \frac{14}{x^2-9}$. Check your solution(s).

Your Notes

Example 3 ***Factor to find the LCD***

Solve $\frac{3}{x + 2} - 1 = \frac{-5}{x^2 - 3x - 10}$.

Write each denominator in factored form. The LCD is
________________.

$$\frac{3}{x + 2} - 1 = \frac{-5}{(x + 2)(x - 5)}$$

$$\frac{3 \cdot \boxed{}}{x + 2} - 1 \cdot \boxed{} = \frac{-5 \cdot \boxed{}}{(x + 2)(x - 5)}$$

$$\frac{\boxed{}}{\boxed{}} - \boxed{} = \frac{\boxed{}}{\boxed{}}$$

__________ − (________________) = _____

________________ = _____

____________ = 0

___(__________) = 0

___ = 0 *or* __________ = 0

x = ___ *or* x = ___

The solutions are ___ and ___.

✔ ***Checkpoint*** **Complete the following exercise.**

4. Solve $\frac{1}{x + 6} + 2 = \frac{x^2 - 38}{x^2 + 2x - 24}$.

Homework

Name ______________________ Date ____________

LESSON 3.11 Practice

Identify the excluded values for the rational expressions in the equation.

1. $\frac{5x}{x-6} = 0$

2. $\frac{x+4}{x+10} = \frac{1}{x+4}$

3. $\frac{x+2}{x^2-9} = \frac{1}{x-3}$

Solve the equation. Check your solution.

4. $\frac{4}{x} = \frac{x}{9}$

5. $\frac{x}{2} = \frac{32}{x}$

6. $\frac{5}{x} = \frac{4}{x-3}$

7. $\frac{10}{x+4} = \frac{12}{x}$

8. $\frac{1}{x+5} = \frac{2}{x-6}$

9. $\frac{5}{x+2} = \frac{x}{3}$

Find the LCD of the rational expressions in the equation.

10. $\frac{7}{x+4} + \frac{1}{x} = 8$

11. $\frac{4}{x-3} + 3 = \frac{1}{x}$

12. $7 - \frac{3}{x-5} = \frac{1}{x+2}$

Name ______________________ Date ______________

LESSON 3.11

Practice *continued*

Solve the equation. Check your solution.

13. $\frac{1}{3} + \frac{4}{x} = \frac{1}{x}$

14. $\frac{1}{5} - \frac{6}{5x} = \frac{1}{x}$

15. $\frac{1}{x-4} + 2 = \frac{2x}{x-4}$

16. $\frac{2x}{x-5} + 1 = \frac{5}{x-5}$

17. $\frac{x}{x+6} - 4 = \frac{-1}{x+6}$

18. $3 + \frac{x}{x-2} = \frac{3}{x-2}$

19. **Rain** It has rained 3 of the last 8 days. How many consecutive days does it have to rain in order for the percent of the number of rainy days to be raised to 75%?

20. **Field Goal Average** A field goal kicker has made 25 out of 40 attempted field goals so far this season. How many consecutive field goals must he make to increase his average to about 0.680?

3.12 Use Graphs of Functions

Georgia Performance Standard(s)

MM1A1g, MM1A1i

Your Notes

Goal • Find average rates of change and use graphs to solve equations.

VOCABULARY

Average rate of change

Example 1 ***Find an average rate of change***

Find the average rate of change of $f(x) = x^3 - 2x$ from (a) $x_1 = -1$ to $x_2 = 1$ and (b) $x_1 = 1$ to $x_2 = 2$.

a. Average rate of change of f from $x_1 = -1$ to $x_2 = 1$:

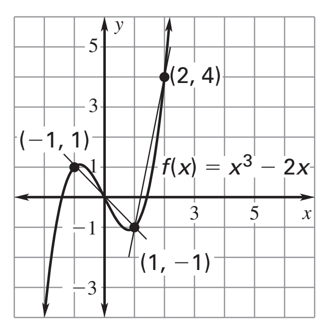
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{\square}{\square} = \_\_\_\_$$

b. Average rate of change of f from $x_1 = 1$ to $x_2 = 2$:

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(1)}{2 - 1}$$

$$= \frac{\square}{\square}$$

$$= \_\_\_$$



Checkpoint **Find the average rate of change of the function from $x_1 = -1$ to $x_2 = 0$.**

1. $g(x) = x^2 + 1$	**2.** $h(x) = x^3 - x^2$

Your Notes

Example 2 *Compare average rates of change*

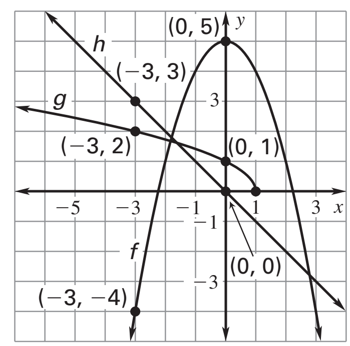
Compare the average rates of change of the functions from $x_1 = -3$ to $x_2 = 0$.

a. $h(x) = -x$ **b.** $g(x) = \sqrt{-x + 1}$ **c.** $f(x) = -x^2 + 5$

Solution

a. The function is _______, so the rate of change, _____, is constant. The average rate of change from $x_1 = -3$ to $x_2 = 0$ is _____.

b. Average rate of change of g from $x_1 = -3$ to $x_2 = 0$:



$$\frac{g(x_2) - g(x_1)}{x_2 - x_1} = \frac{g(0) - g(-3)}{0 - (-3)}$$

$$= \frac{\qquad}{\_\_\_\_\_\_\_}$$

$$= \frac{\qquad}{\_\_\_\_\_}$$

c. Average rate of change of f from $x_1 = -3$ to $x_2 = 0$:

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(0) - f(-3)}{0 - (-3)} = \frac{\qquad}{\_\_\_\_\_\_\_\_\_} = \frac{\quad}{\_\_\_} = \_\_\_$$

The average rate of change of ___ is positive because the function is ____________ on the interval. The average rates of change of ___ and ___ are negative because the functions are _____________ on the intervals. The graph of $h(x) = -x$ is steeper, so the absolute value of its average rate of change is greater.

✔ ***Checkpoint*** **Complete the following exercise.**

3. Compare the average rates of change from Checkpoints 1 and 2.

Your Notes

Example 3 ***Solve an equation using a graph***

Solve $x^3 = 4x$.

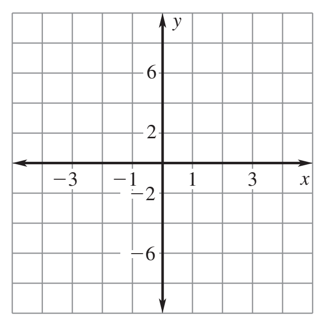
Solution

Method 1 Solve the equation by factoring.

$x^3 = 4x$		**Write original equation.**
________ = ___		**Subtract ____ from each side.**
___(________) = __		**Factor out ___.**
___(_______)(_______) = ___		**Difference of two squares pattern**
___ = 0	*or* _______ = 0	**Zero-product property**
	or _______ = 0	
x = ___	*or* x = _____	**Solve equations.**
	or x = ___	

The solutions are ___, _____, and ___.

Method 2 Solve the equation by finding the intersection of two graphs. The solution will be the __________ of the intersection points.



Graph both sides of the equation and find the points of intersection.

The solutions are ___, _____, and ___.

Checkpoint **Solve the equation by graphing. If necessary, use a graphing calculator and round your answer to the nearest hundredth.**

4. $2x = -x^2$	**5.** $\sqrt{x-2} = x^2 - 8x + 16$

Homework

Name ______________________ Date ______________

LESSON 3.12

Practice

Find the average rate of change of the function from x_1 to x_2.

1. $f(x) = -5x + 2, x_1 = 2, x_2 = 4$

2. $f(x) = \frac{1}{2}x - 3, x_1 = -3, x_2 = -1$

3. $f(x) = 2\sqrt{x} + 1, x_1 = 0, x_2 = 9$

4. $f(x) = \sqrt{x - 5} + 1, x_1 = 5, x_2 = 6$

5. **Multiple Choice** Which values give an average rate of change of the function $f(x) = x^2$ from x_1 to x_2 that is positive?

A. $x_1 = -3, x_2 = 3$ **B.** $x_1 = 0, x_2 = 3$ **C.** $x_1 = -3, x_2 = 0$ **D.** $x_1 = -3, x_2 = 2$

Match the given characteristic of a function with a possible average rate of change of the function from $x_1 = 0$ to $x_2 = 2$.

6. The graph of function f is decreasing on its entire domain.

7. The line through the two points $(0, g(0))$ and $(2, g(2))$ is a horizontal line.

8. The graph of function h is steeper than the graph of a linear function with a slope of 3.

A. 7

B. 0

C. -2

Name ______________________ Date ______________

LESSON 3.12

Practice *continued*

9. *Compare* the average rates of change of $f(x) = 3.5x$ and $g(x) = 2x^3 - 1$ from $x_1 = 2$ to $x_2 = 4$.

Solve the equation by graphing. If necessary, use a graphing calculator and round your answer to the nearest hundredth.

10. $x^2 = 4x$

11. $x^3 - 3 = \frac{1}{4}x^2$

12. $\sqrt{x + 2} - 4 = x + 8$

13. $-\sqrt{x - 1} + 1 = x^3$

14. **Roofing** The height of a shingle tossed from the top of a building can be modeled by the function $h(t) = -16t^2 - 5t + 74$, where t is the number of seconds since the shingle was tossed.

a. Find the average rate of change of the function from $t_1 = 0$ to $t_2 = 1$.

b. Find the average rate of change of the function from $t_1 = 1$ to $t_2 = 2$.

c. Compare the average rates of change in parts (a) and (b). *Explain* what this tells you about the distance that the shingle fell during each time interval.

3.13 Use Sequences

Georgia Performance Standard(s)

MM1A1f

Your Notes

Goal • Write terms of sequences and use terms to write rules.

VOCABULARY
Sequence
Terms

Example 1 ***Write terms of a sequence***

Write the first six terms of the sequence. Identify the domain and range.

a. $a_n = 3n - 1$ **b.** $a_n = 32\left(-\frac{1}{2}\right)^{n-1}$

Solution

a. $a_1 = 3(\_\_\_) - 1 = \_\_\_$ $a_2 = 3(\_\_\_) - 1 = \_\_\_$

$a_3 = 3(\_\_\_) - 1 = \_\_\_$ $a_4 = 3(\_\_\_) - 1 = \_\_\_$

$a_5 = 3(\_\_\_) - 1 = \_\_\_$ $a_6 = 3(\_\_\_) - 1 = \_\_\_$

Domain: __________

Range: __________

b. $a_1 = 32\left(-\frac{1}{2}\right)^{\_\_\_ - 1} = \_\_\_$

$a_2 = 32\left(-\frac{1}{2}\right)^{\_\_\_ - 1} = \_\_\_$

$a_3 = 32\left(-\frac{1}{2}\right)^{\_\_\_ - 1} = \_\_\_$

$a_4 = 32\left(-\frac{1}{2}\right)^{\_\_\_ - 1} = \_\_\_$

$a_5 = 32\left(-\frac{1}{2}\right)^{\_\_\_ - 1} = \_\_\_$

$a_6 = 32\left(-\frac{1}{2}\right)^{\_\_\_ - 1} = \_\_\_$

Domain: __________

Range: __________

Your Notes

Example 2 ***Write rules for sequences***

Describe the pattern, write the next term, and write a rule for the *n*th term of the sequence (a) 1, 4, 9, 16, . . . and (b) −7, −14, −21, −28, ...

Solution

a. You can write the terms as ____, ____, ____, ____, ... The next term is a_5 = ____ = ____. A rule for the *n*th term is a_n = ____.

b. You can write the terms as _______, _______, _______, _______, ... The next term is a_5 = _______ = ______. A rule for the *n*th term is a_n = ______.

✔ ***Checkpoint*** **Write the first six terms of the sequence. Identify the domain and range.**

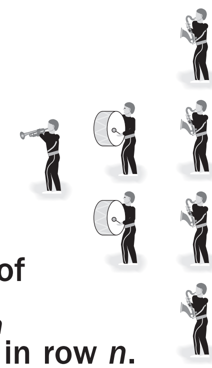
1. $a_n = n + 9$	**2.** $a_n = (-3)^n$

✔ ***Checkpoint*** **Complete the following exercise.**

3. For the sequence 0, −3, −8, −15, . . ., describe the pattern, write the next term, and write a rule for the *n*th term.

Your Notes

Example 3 ***Solve a multi-step problem***



Band A band is arranged in 5 rows. The first 3 rows are shown at the right. Write a rule for the number of musicians in each row. Then graph the sequence.

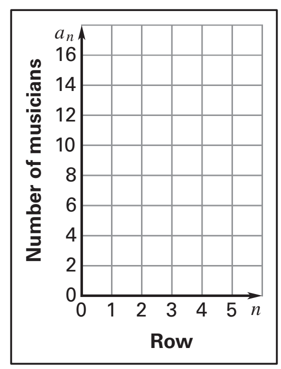
Solution

Step 1 **Make** a table showing the number of musicians in the first 3 rows. Let a_n represent the number of musicians in row n.

Row, n	1	2	3
Number of Musicians, a_n	____ / ____	____ / ____	____ / ____

Step 2 **Write** a rule for the number of musicians in each row. From the table, you can see that a_n = ________.

Step 3 **Plot** the points (______), (______), (______), (______), and (_______). Notice that the graph is a __________.



✔ ***Checkpoint*** **Complete the following exercise.**

4. In Example 3, suppose the band leader wants to add a sixth row. How many musicians are needed for the sixth row?

Homework

Name ______________________ Date ______________

LESSON 3.13 Practice

Write the first six terms of the sequence. Identify the domain and range.

1. $a_n = 6n$

2. $a_n = 2 - n$

3. $a_n = n^2 + 1$

4. $a_n = (n - 1)^2$

5. $a_n = \frac{n}{2}$

6. $a_n = (-1)^n$

7. **Multiple Choice** What is the seventh term of the sequence $a_n = \frac{n + 3}{2n}$?

A. $\frac{5}{7}$ **B.** 5 **C.** 10 **D.** 14

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LESSON 3.13 **Practice** *continued*

For the sequence, describe the pattern, write the next term, and write a rule for the *n*th term.

8. $-2, -5, -8, -11, \ldots$

9. $2, 6, 12, 20, \ldots$

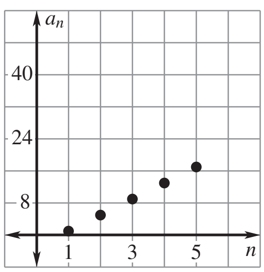
Match the sequence with the graph of its first 5 terms.

10. $a_n = 2n^2 - 1$

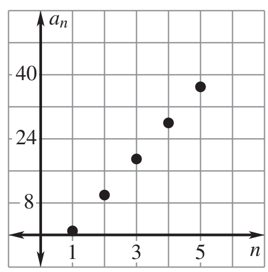
11. $a_n = 4n - 3$

12. $a_n = 9n - 8$

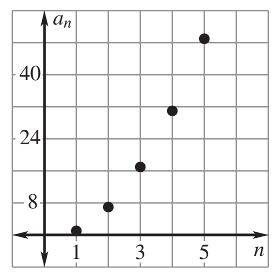
A.



B.



C.



13. Broadcasting A light bulb falls from a broadcasting tower. The height a_n (in feet) is measured each second during its fall. The table shows the first three measurements.

***n*th measurement**	1	2	3
height, a_n	240	192	112

a. Write a rule for the height of each measurement. (*Hint:* The height *h*, in feet, of an object dropped from a height of *s* feet after *t* seconds is given by the function $h(t) = -16t^2 + s$.)

b. What is the height of the light bulb after 4 seconds?

Words to Review

Give an example of the vocabulary word.

Cubic function	Odd function
Even function	End behavior
Radical expression	Radical function
Square root function	Parent square root function
Simplest form of a radical expression	Rationalizing the denominator
Radical conjugates	Radical equation

Extraneous solution	**Rational function**
Asymptote	**Rational expression**
Excluded value	**Simplest form of a rational expression**
Least common denominator of rational expressions	**Rational equation**
Average rate of change	**Sequence**
Terms of sequence	