4.1 Apply the Distance and Midpoint Formulas

Georgia Performance Standard(s)

MM1G1a, MM1G1c

Your Notes

Goal • Use the distance and midpoint formulas.

VOCABULARY

Distance formula

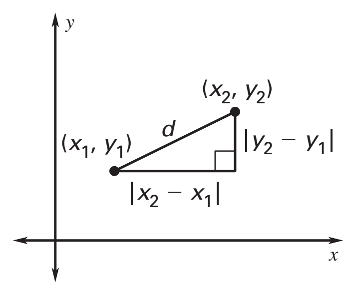
Midpoint

Midpoint formula

THE DISTANCE FORMULA

The distance d between any two points (x_1, y_1) and (x_2, y_2) is

____________________________.



Example 1 ***Find the distance between two points***

Find the distance between (4, −3) and (−7, 2).

Let $(x_1, y_1) = (4, -3)$ and $(x_2, y_2) = (-7, 2)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Distance formula**

$= \sqrt{(\_\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2}$ **Substitute.**

$= \sqrt{(\_\_\_\_\_)^2 + (\_\_\_)^2} = \_\_\_\_\_\_$ **Simplify.**

The distance between the points is ______ units.

Your Notes

Example 2 *Find a missing coordinate*

The distance between (5, a) and (9, 6) is $4\sqrt{2}$ units. Find the possible values of a.

Solution

Use the distance formula with $d = 4\sqrt{2}$. Let $(x_1, y_1) = (5, a)$ and $(x_2, y_2) = (9, 6)$.

d	$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	**Distance formula**
______	$= \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2}$	**Substitute.**
______	$= \sqrt{\_\_\_\_\_\_\_\_\_\_\_\_}$	**Multiply.**
______	$= \sqrt{\_\_\_\_\_\_\_\_\_\_}$	**Simplify.**
_____	= ______________	**Square each side.**
0	= ______________	**Write in standard form.**
0	= ______________	**Factor.**
_________ = 0	*or* ________ = 0	**Zero-product property**
a = _____	*or* a = ___	**Solve for a.**

The value of a is _____ or ___.

✔ *Checkpoint* Complete the following exercises.

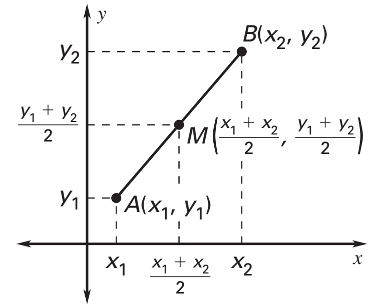
1. Find the distance between (2, −3) and (5, 1).	**2.** The distance between (−1, 2) and (3, b) is $\sqrt{41}$ units. Find the possible values of b.

Your Notes

THE MIDPOINT FORMULA

The midpoint M of the line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$M\left(\_\_\_\_\_\_, \_\_\_\_\_\_\right).$



Example 3 ***Find a midpoint of a line segment***

Find the midpoint of the line segment with endpoints (−3, 7) and (−1, 11).

Solution

Let $(x_1, y_1) = (-3, 7)$ and $(x_2, y_2) = (-1, 11)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{\square + \square}{\square}, \frac{\square + \square}{\square}\right)$$

$$= (\_\_\_\_, \_\_\_)$$

The midpoint of the line segment is (_____, ___).

✔ ***Checkpoint*** **Find the midpoint of the line segment with the given endpoints.**

3. (1, −2), (5, −4)	**4.** (5, 12), (13, 8)

Homework

Name ______________________________ Date ________________

LESSON 4.1 Practice

Match the pair of points with the expression that gives the distance between the points.

1. $(-6, 3), (-4, 2)$

2. $(6, -3), (-4, 2)$

3. $(6, -3), (4, -2)$

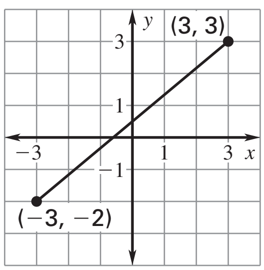
A. $\sqrt{(-4 - 6)^2 + (2 + 3)^2}$

B. $\sqrt{(-2 + 3)^2 + (4 - 6)^2}$

C. $\sqrt{(-4 + 6)^2 + (2 - 3)^2}$

Use the coordinate plane to estimate the distance between the two points. Then use the distance formula to find the distance between the points.

4.



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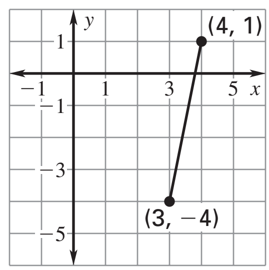

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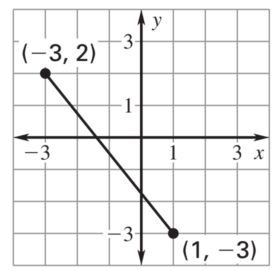


Find the distance between the two points.

7. $(2, 4), (5, 6)$

8. $(7, 3), (1, 5)$

9. $(8, 2), (4, 1)$

Name ______________________________ Date ________________

LESSON 4.1 **Practice** *continued*

The distance *d* between two points is given. Find the possible values of *b*.

10. $(0, b), (5, 12); d = 13$

11. $(1, b), (4, 5); d = 5$

12. $(2, 3), (b, 9); d = 10$

13. $(1, 4), (10, b); d = 15$

14. $(5, 2), (-1, b); d = 6$

15. $(b, 6), (3, -2); d = 8$

Find the midpoint of the line segment with the given endpoints.

16. $(5, 3), (7, 11)$

17. $(-3, 10), (9, 2)$

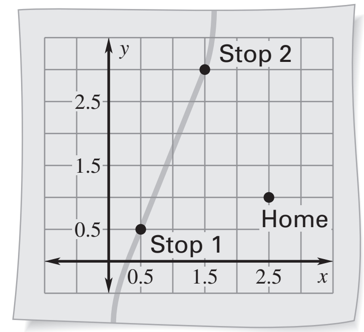
18. $(-2, -4), (8, 4)$

Name ______________________ Date ____________

LESSON 4.1

Practice *continued*

19. Bus Stop A student is taking the bus home. The student can get off at one of two stops, as shown on the map. The distance between consecutive grid lines represents 0.5 mile.



a. Find the distance between stop 1 and home. Round your answer to the nearest hundredth.

b. Find the distance between stop 2 and home. Round your answer to the nearest hundredth.

c. Which distance is shorter? By how much?

20. Sales Use the midpoint formula to estimate the sales of a company in 2000, given the sales in 1995 and 2005. Assume that the sales followed a linear pattern.

Year	1995	2005
Sales (dollars)	740,000	980,000

4.2 Use Inductive Reasoning

Georgia Performance Standard(s)

MM1G2a

Your Notes

Goal • Describe patterns and use inductive reasoning.

VOCABULARY

Conjecture

Inductive Reasoning

Counterexample

Example 1 ***Describe a visual pattern***

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1

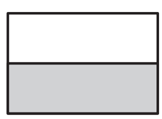


Figure 2

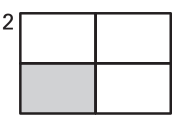
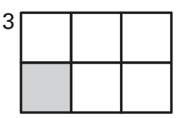


Figure 3



Solution

Each rectangle is divided into ________ as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into __________. Shade the section just ________ the horizontal segment at the _____.

____

Figure 4

✔ ***Checkpoint*** **Complete the following exercise.**

1. Sketch the fifth figure in the pattern in Example 1.

Your Notes

Three dots (. . .) tell you that the pattern continues.

Example 2 ***Describe a number pattern***

Describe the pattern in the numbers −1, −4, −16, −64, . . . and write the next three numbers in the pattern.

Notice that each number in the pattern is ______ times the previous number.

−1, −4, −16, −64, . . .

× ___ × ___ × ___ × ___

The next three numbers are ____________________________.

Checkpoint **Complete the following exercise.**

2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, . . . and write the next three numbers in the pattern.

Example 3 ***Make and test a conjecture***

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Step 1 **Find** a pattern using groups of small numbers.

$1 + 3 + 5 =$ ___ $= 3 \cdot 3$	$3 + 5 + 7 =$ ____ $=$ ___ $\cdot 3$
$5 + 7 + 9 =$ ____ $=$ ___ $\cdot 3$	$7 + 9 + 11 =$ ____ $=$ ___ $\cdot 3$

Conjecture The sum of any three consecutive odd numbers is three times ______________________.

Step 2 **Test** your conjecture using other numbers.

$-1 + 1 + 3 =$ ___ $=$ ___ $\cdot 3$ ✓

$103 + 105 + 107 =$ _____ $=$ _____ $\cdot 3$ ✓

Your Notes

Checkpoint **Complete the following exercise.**

3. Make and test a conjecture about the sign of the product of any four negative numbers.

Example 4 *Find a counterexample*

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

Solution

To find a counterexample, you need to find a difference that is ________ than the ________ number.

$8 - (-4) =$ ____

Because ____ $\nless$ ___, a counterexample exists. The conjecture is false.

Checkpoint **Complete the following exercise.**

4. Find a counterexample to show that the following conjecture is false.

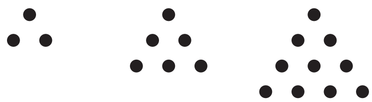
   Conjecture The quotient of two numbers is always smaller than the dividend.

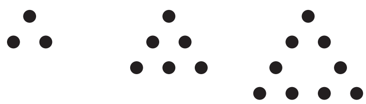
Homework

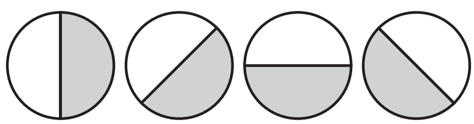
Name ______________________ Date ____________

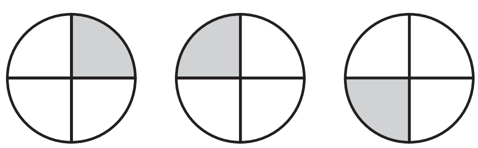
LESSON 4.2 Practice

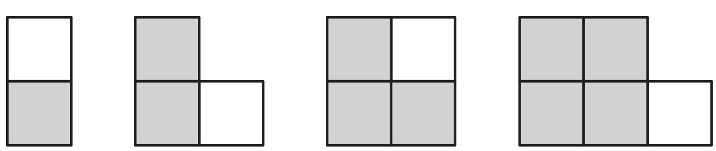
Sketch the next figure in the pattern.

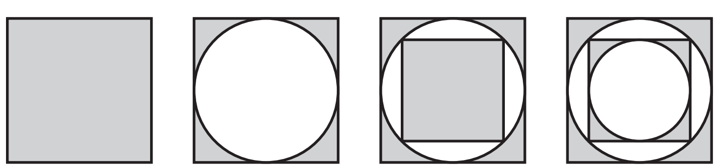
1. 

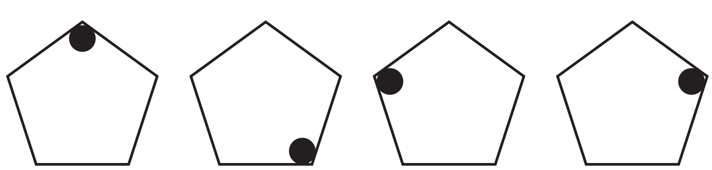
2. 

3. 

4. 

5. 

6. 

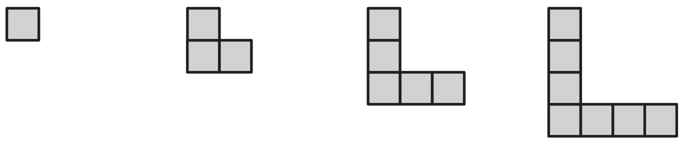
7. 

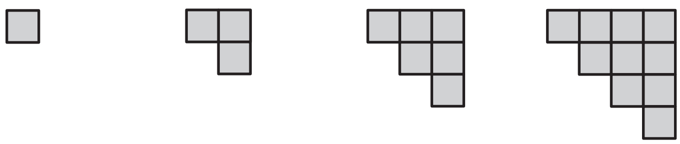
Name ________________________ Date ______________

LESSON 4.2

Practice *continued*

The first four objects in a pattern are shown. How many squares are there in the next object?

8. 

9. 

Describe **a pattern in the numbers. Write the next number in the pattern.**

10. 5, 10, 15, 20, . . .

11. 26, 23, 20, 17, 14, . . .

12. 2, 6, 18, 54, . . .

13. 32, 16, 8, 4, . . .

14. −12, −8, −4, 0, . . .

15. 3, −9, 27, −81, . . .

Complete the conjecture based on the pattern you observe in the specific cases.

16. Use the following products of odd integers to complete the conjecture about the product of any two odd numbers: $1 \times 3 = 3$, $1 \times 5 = 5$, $3 \times 3 = 9$, $3 \times 5 = 15$, $5 \times 1 = 5$, $5 \times 5 = 25$, $5 \times 7 = 35$, $7 \times 1 = 7$, $7 \times 3 = 21$, $7 \times 7 = 49$

Conjecture The product of any two odd integers is __?__.

Name ______________________ Date ______________

LESSON 4.2

Practice *continued*

17. Complete the following table. Then complete the conjecture that follows.

Pair of odd numbers	1, 3	3, 5	5, 7	7, 9	9, 11
Sum of the numbers divided by 2	$\frac{1+3}{2}$	$\frac{3+5}{2}$			
Average of numbers	2				

Conjecture The average of any two consecutive odd whole numbers is __?__.

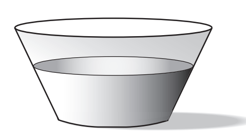
Show the conjecture is false by finding a counterexample.

18. The average of any two consecutive even numbers is an even number.

19. Any four-sided polygon is a square.

20. The square of any integer is a positive integer.

21. **Evaporation** You are performing an experiment to explore the effects of surface area on evaporation. Each day you record the depth (in millimeters) of the water in the bowl pictured. The table below shows your results.



Day	0	1	2	3	4	5
Water level (mm)	180	169	158	147	136	125

a. Predict the height of the water surface in the bowl on day 6.

b. Based on these results, make a conjecture about how the surface area of a body of water affects the rate of change of its depth by evaporation.

4.3 Analyze Conditional Statements

Georgia Performance Standard(s)

MM1G2b

Goal • Write definitions as conditional statements.

Your Notes

VOCABULARY
Conditional statement
If-then form
Hypothesis
Conclusion
Negation
Converse
Inverse
Contrapositive
Equivalent statements
Perpendicular lines
Biconditional statement

Your Notes

Example 1 ***Rewrite a statement in if-then form***

Rewrite the conditional statement in if-then form.

All vertebrates have a backbone.

Solution

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

If ________________________, then __________________.

Checkpoint **Rewrite the conditional statement in if-then form.**

1. All triangles have 3 sides.	**2.** When $x = 2$, $x^2 = 4$.

Example 2 ***Write four related conditional statements***

Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Olympians are athletes." Decide whether each statement is *true* or *false*.

Solution

If-then form ______________________________________

____

Converse ______________________________________

____

Inverse ______________________________________

____

____

Contrapositive ______________________________________

____

____

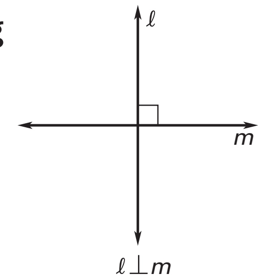
Your Notes

PERPENDICULAR LINES

Definition If two lines intersect to form a ______ angle, then they are perpendicular lines.

The definition can also be written using the converse: If any two lines are perpendicular lines, then they intersect to form a ______ angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.

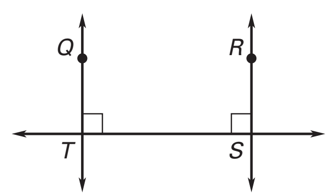


Example 3 *Use definitions*

Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.

a. $\overleftrightarrow{QT} \perp \overleftrightarrow{TS}$

b. $\angle QTS$ and $\angle RST$ are supplementary.



Solution

a. This statement is ______ . The right angle symbol in the diagram indicates that the lines intersect to form a ____________. So the lines are ______________.

b. This statement is ______. Both angles are right angles, so the sum of their measures is ______.

Example 4 *Write a biconditional*

Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel.

Solution

Converse: ____

____

Biconditional: ____________________________________

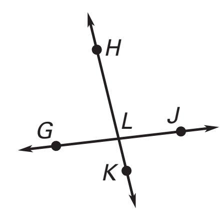
____

Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

3. Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Squares are rectangles." Decide whether each statement is *true* or *false*.

4. Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.



a. $m\angle GLJ = 180°$

b. $\overleftrightarrow{GJ} \perp \overleftrightarrow{HK}$

5. Write the statement below as a biconditional.

Statement: If a student is a boy, he will be in group A. If a student is in group A, the student must be a boy.

Homework

Name ______________________ Date ____________

LESSON 4.3

Practice

Rewrite the conditional statement in if-then form.

1. You have a fever if your body temperature is 103°F.

2. A deer is albino if it has white fur and pink eyes.

3. I'll buy that CD for you if you want it.

4. A pickup truck is a vehicle with a high utility value.

Write the converse, inverse, and contrapositive of each statement.

5. If water is frozen, then its temperature is below 0°C.

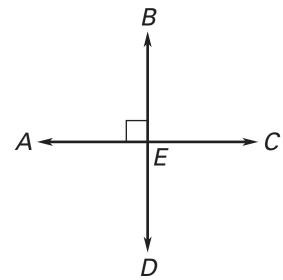
6. If $x + 3 = 5$, then $x = 2$.

Name ______________________ Date ______________

LESSON 4.3

Practice *continued*

Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.



7. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

8. $\angle BEC$ is a right angle.

9. $\angle AEB$ and $\angle BEC$ are supplementary angles.

10. $m\angle AEC = 180°$

11. $\angle AEB$ is an obtuse angle.

Rewrite the definition as an if-then statement. Then write the converse of the if-then statement. Finally, write the definition as a biconditional statement.

12. The midpoint of a segment is a point that divides the segment into two congruent segments.

13. Two angles are complementary angles if the sum of their measures is 90°.

14. In an equilateral polygon, all sides are congruent.

Name ______________________________ Date ________________

LESSON 4.3

Practice *continued*

Decide whether the statement is a valid definition.

15. If a polygon is both equilateral and equiangular, then the polygon is a regular polygon.

16. If a polygon is a square, then the polygon has four congruent sides.

17. If a figure is a line, then the figure has one dimension.

18. **Scuba Diving** The word scuba originated as an acronym for "Self Contained Underwater Breathing Apparatus." Here is a definition of scuba diving.

If a person is scuba diving, then the person is using independent breathing equipment to stay underwater for long periods of time.

Decide whether the converse, the inverse, and the contrapositive of this definition are *true* or *false*. If *false*, explain why.

19. **Skydiving** The statement below describes one of the reasons that overconfidence in a novice skydiver can add danger to a jump.

If a skydiver attempts high speed maneuvers close to the ground, then the jump will have a high risk factor.

Decide whether the converse, the inverse, and the contrapositive of this statement are *true* or *false*. If *false*, explain why.

4.4 Apply Deductive Reasoning

Georgia Performance Standard(s)

MM1G2a

Goal • Use deductive reasoning to form a logical argument.

Your Notes

VOCABULARY

Deductive Reasoning

The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

LAWS OF LOGIC

Law of Detachment If the hypothesis of a true conditional statement is true, then the __________ is also true.

Law of Syllogism

If hypothesis p, then conclusion q.
If hypothesis q, then conclusion r.
If these statements are true,

If hypothesis p, then conclusion r.
then this statement is true.

Example 1 *Use the Law of Detachment*

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two segments have the same length, then they are congruent. You know that $AB = QR$.

b. Jesse goes to the gym every weekday. Today is Monday.

Solution

a. Because $AB = QR$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, __________.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "__________ __________," and the conclusion is "__________ __________."

"Today is Monday" satisfies the hypothesis of a true conditional statement, so you can conclude that ______________________________.

Your Notes

The order in which the statements are given does not affect whether you can use the Law of Syllogism.

Example 2 *Use the Law of Syllogism*

If possible, use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

b. If $x^2 > 36$, then $x^2 > 30$. If $x > 6$, then $x^2 > 36$.

c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following.

If Ron eats lunch today, then ______________________________.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.

If $x > 6$, then __________.

c. Neither statement's conclusion is the same as the other statement's ______________. You cannot use the Law of Syllogism to write a new conditional statement.

✔ *Checkpoint* Complete the following exercises.

1. If $0° < m\angle A < 90°$, then A is acute. The measure of $\angle A$ is 38°. Using the Law of Detachment, what statement can you make?

2. State the law of logic that is illustrated below.

If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.

If you do your homework, then you can watch your favorite show.

Your Notes

Example 3 *Use inductive and deductive reasoning*

What conclusion can you make about the sum of two odd integers?

Solution

Step 1 **Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$-3 + 5 =$ ___, $-1 + 5 =$ ___, $3 + 5 =$ ___

$-3 + (-5) =$ _____, $1 + (-5) =$ _____, $3 + (-5) =$ _____

Conjecture: Odd integer + Odd integer = _______ integer

Step 2 **Let** n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ and $2m$ are _______ integers because any integer multiplied by 2 is _______.

$2n -$ ___ and $2m +$ ___ are ______ integers because $2n$ and $2m$ are _______ integers.

$(2n -$ ___$) + (2m +$ ___$)$ represents the sum of an ______ integer $2n -$ ___ and an ______ integer $2m +$ ___.

$(2n -$ ___$) + (2m +$ ___$) =$ ___$(n + m)$

The result is the product of ___ and an integer $n + m$. So, ___$(n + m)$ is an _______ integer.

The sum of two odd integers is an _______ integer.

Checkpoint **Complete the following exercise.**

3. Use inductive reasoning to make a conjecture about the sum of a negative integer and itself. Then use deductive reasoning to show the conjecture is true.

Homework

Name ______________________ Date ______________

LESSON 4.4

Practice

Use the Law of Detachment to make a valid conclusion in the true situation.

1. If you get a hit, then your baseball team will win. You hit a home run.

2. If Rylee gets promoted, then Callie will also be promoted. Rylee is promoted.

3. Any time Kendra runs in a cross country race, if she runs a strong race, then she wins. In the cross country race last Saturday, Kendra ran her best race.

4. If two integers are added together, then the result is an integer. You add an integer x to another integer y.

5. If you double a negative number, then the result is a smaller number. You calculate $2x$, where $x < 0$.

6. If an integer is divided by one of its factors, then the result is another one of the integer's factors. You divide an even integer x by 2.

Name ______________________________ Date ______________

LESSON 4.4

Practice *continued*

Use the Law of Syllogism to write the conditional statement that follows from the pair of true statements that are given.

7. If Moose is hungry when he goes to the pizza shop, then he'll finish a whole pizza. If Moose eats a whole pizza, then he goes through a pitcher of soda.

8. If you mail the payment by noon, then it will arrive by tomorrow. If your payment arrives by tomorrow, then you won't be charged a late fee.

9. If Estelle takes her broker's advice, she'll invest in stock X. If Estelle invests in stock X, she'll earn 50% on her investment by next year.

10. If a triangle has two angles of 60°, then the triangle is equiangular. If a triangle is equiangular, then it is also equilateral.

Decide whether the conclusion reached from the two statements demonstrates the *Law of Detachment,* the *Law of Syllogism,* or *neither.*

11. If Cedric plays in a big game, then he gets nervous. If Cedric gets nervous, then he performs well.

Conclusion: If Cedric plays in a big game, then he performs well.

12. If Leanne spends more than $30 on her car, then she'll have to wait until next week to buy Michael's birthday gift. Leanne spent $40 on her car.

Conclusion: Leanne will have to wait until next week to buy Michael's birthday gift.

13. If Lavonne gets money, she gives half of it to Sid. If Sid gets money, he gives half of it to Lavonne.

Conclusion: Lavonne and Sid share their money equally.

Name ______________________ Date ______________

LESSON 4.4

Practice *continued*

Decide whether *inductive* or *deductive* reasoning is used to reach the conclusion. *Explain* your reasoning.

14. While shopping for a product, you notice that brand A is more expensive than brand B. You conclude that brand A is of higher quality than brand B.

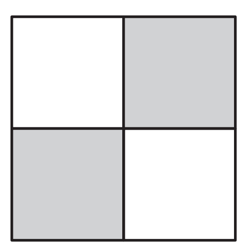
15. Because the brand A product costs $1.50 and the brand B product costs $1.00, you conclude that the brand A product is 50% more expensive.

16. It normally takes you 20 minutes to walk home from school. By walking faster one day, you make it in 15 minutes. The following day, you make it in 12 minutes. You conclude that you could make the trip in as little as 10 minutes.

17. On the first meet of the year, JD, Bob, and Raul finish their race in a tie. In the final meet of the year, Raul finishes well ahead of Bob and JD. Having seen both races, you conclude that Raul trained the hardest.

In Exercises 18 and 19, use the figure at the right.

18. Based on what you see in the figure, use inductive reasoning to make a conjecture about how the area of one square compares to the area of another square with sides that are twice as long.



19. Use deductive reasoning to prove your conjecture by using side lengths of $s = x$ and $s = 2x$ in the formula for the area of a square and comparing the result.

Name ______________________ Date ____________

LESSON 4.4

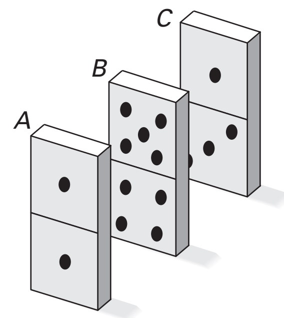
Practice *continued*

Use the figure showing three standing dominos, A, B, and C.

20. Is the *Law of Detachment* or the *Law of Syllogism* used to reach the conclusion below?

Statements: If A is pushed into B, then B will be knocked into C. A is pushed into B.

Conclusion: B is knocked into C.



21. Write a set of statements and a conclusion that demonstrate the Law of Syllogism.

22. Suppose domino D is placed behind domino C. Write a set of statements and a conclusion that demonstrate the Law of Syllogism being used to connect more than two conditional statements.

4.5 Prove Statements about Segments and Angles

Georgia Performance Standard(s)

MM1G2a

Goal • Write proofs using geometric theorems.

Your Notes

VOCABULARY

Proof

Two-column proof

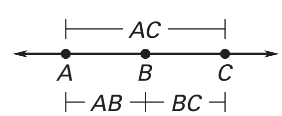
Postulate

Theorem

SEGMENT ADDITION POSTULATE

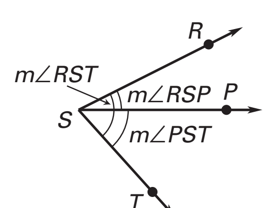
If B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



ANGLE ADDITION POSTULATE

Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle$______ and $\angle$______.



Symbols If P is in the interior of $\angle RST$, then $m\angle RST = m\angle$______ $+ m\angle$______.

Your Notes

THEOREM 4.1 CONGRUENCE OF SEGMENTS

Segment congruence is reflexive, symmetric, and transitive.

Reflexive For any segment AB, ________ .

Symmetric If $\overline{AB} \cong \overline{CD}$, then ________ .

Transitive If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then ________ .

THEOREM 4.2 CONGRUENCE OF ANGLES

Angle congruence is reflexive, symmetric, and transitive.

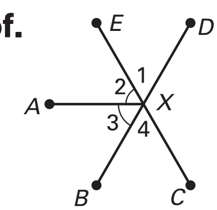
Reflexive For any angle A, ________.

Symmetric If $\angle A \cong \angle B$, then ________.

Transitive If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then ________.

Example 1 ***Write a two-column proof***

Use the diagram to write a two-colum proof.



Given $m\angle 2 = m\angle 3$, $m\angle AXD = m\angle AXC$

Prove $m\angle 1 = m\angle 4$

Writing a two-column proof is a formal way of organizing your reasons to show a statement is true.

Statements	Reasons
1. $m\angle AXC = m\angle AXD$	1. ______
2. $m\angle AXD = m\angle$ ___ $+ m\angle$ ___	2. Angle Addition Postulate
3. $m\angle AXC = m\angle$ ___ $+ m\angle$ ___	3. Angle Addition Postulate
4. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	4. ________________
5. $m\angle 2 = m\angle 3$	5. ______
6. $m\angle 1 + m\angle$ ___ $= m\angle 3 + m\angle 4$	6. Substitution Property of Equality
7. $m\angle 1 = m\angle 4$	7. ________________

Your Notes

Example 2 *Symmetric Property of Congruence*

Prove the Symmetric Property of Segment Congruence.

Given $\overline{AB} \cong \overline{CD}$

Prove $\overline{CD} \cong \overline{AB}$

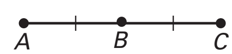
Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. ______
2. ________	2. Definition of congruent segments
3. ________	3. Symmetric Property of Equality
4. $\overline{CD} \cong \overline{AB}$	4. Definition of congruent segments

✔ ***Checkpoint*** **Complete the following exercises.**

1. Three steps of a proof are shown. Give the reasons for the last two steps.

Given $BC = AB$

Prove $AC = AB + AB$



Statements	Reasons
1. $BC = AB$	1. Given
2. $AC = AB + BC$	2. ________________ ________
3. $AC = AB + AB$	3. ________________ ________

2. Prove the Reflexive Property of Segment Congruence.

Given $\overline{AB}$ is a line segment.

Prove $\overline{AB} \cong \overline{AB}$

Statements	Reasons
1. $\overline{AB}$ is a line segment.	1. ______
2. AB is the length of ____.	2. Definition of the length of a segment
3. $AB = AB$	3. ________________ ________
4. $\overline{AB} \cong \overline{AB}$	4. ________ ________________

Homework

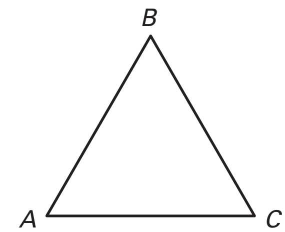
Name ______________________ Date ______________

LESSON 4.5 Practice

In Exercises 1 and 2, complete the proof.

1. **GIVEN:** $m\angle A = m\angle B, m\angle B = m\angle C$

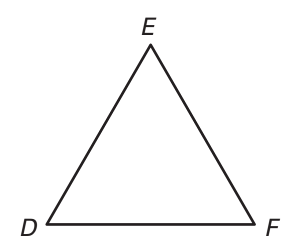
PROVE: $\angle A \cong \angle C$



Statements	Reasons
1. $m\angle A = m\angle B, m\angle B = m\angle C$	**1.** Given
2. $m\angle A = m\angle C$	**2.**
3.	**3.** Definition of congruent angles

2. **GIVEN:** $DE = EF, EF = DF$

PROVE: $\overline{DF} \cong \overline{DE}$



Statements	Reasons
1. $DE = EF, EF = DF$	**1.**
2.	**2.** Transitive Property of Equality
3. $DF = DE$	**3.**
4.	**4.** Definition of congruent segments

Name ______________________ Date ______________

LESSON 4.5

Practice *continued*

Use the property to complete the statement.

3. Reflexive Property of Congruence: __?__ $\cong \angle 4$

4. Symmetric Property of Congruence: If __?__ $\cong$ __?__, then $\overline{CD} \cong \overline{DX}$.

In Exercises 5–8, name the property illustrated by the statement.

5. If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 4$, then $\angle 1 \cong \angle 4$.

6. $\overline{XY} \cong \overline{XY}$

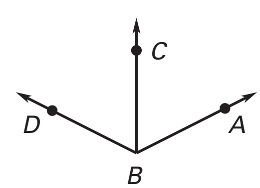
7. If $\angle CDE \cong \angle RST$, then $\angle RST \cong \angle CDE$.

8. If $\overline{AB} \cong \overline{BC}$, then $\overline{BC} \cong \overline{AB}$.

9. Use the given information and the diagram to prove the statement.

GIVEN: $2m\angle ABC = m\angle ABD$

PROVE: $\angle ABC \cong \angle CBD$



Statements	Reasons

Name ______________________ Date ____________

LESSON 4.5

Practice *continued*

10. **Bicycle Tour** You take part in a three day bicycle tour. On the first day, you ride 95 miles. On the third (final) day, you also ride 95 miles. Use the following steps to prove that the distance you ride in the first two days is equal to the distance that you ride in the last two days.

a. Draw a diagram for the situation by using a line segment to represent the total distance of the three days and dividing the line segment into three parts that represent the daily distances.

b. State what is given and what is to be proved.

c. Write a two-column proof.

4.6 Prove Angle Pair Relationships

Georgia Performance Standard(s)

MM1G2a

Goal • Use properties of special pairs of angles.

Your Notes

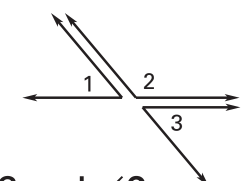
VOCABULARY
Adjacent angles
Linear pair

THEOREM 4.3 RIGHT ANGLES CONGRUENCE THEOREM

All right angles are ___________.

THEOREM 4.4 CONGRUENT SUPPLEMENTS THEOREM

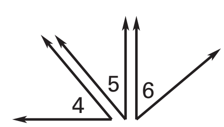
If two angles are supplementary to the same angle (or to congruent angles), then they are ___________.



If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then __________.

THEOREM 4.5 CONGRUENT COMPLEMENTS THEOREM

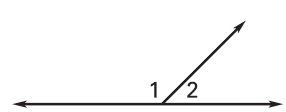
If two angles are complementary to the same angle (or to congruent angles), then they are ___________.



If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then __________.

LINEAR PAIR POSTULATE

If two angles form a linear pair, then they are ________________.

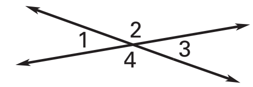


$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 =$ ______.

Your Notes

THEOREM 4.6 VERTICAL ANGLES CONGRUENCE THEOREM

Vertical angles are ___________.

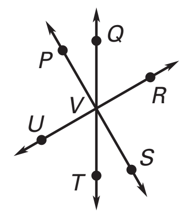


Example 1 ***Find angle measures***

Complete the statement given that $m\angle RVS = 90°$.

a. $m\angle PVU = \underline{\ ?\ }$

b. If $m\angle QVU = 120°$, then $m\angle SVT = \underline{\ ?\ }$.



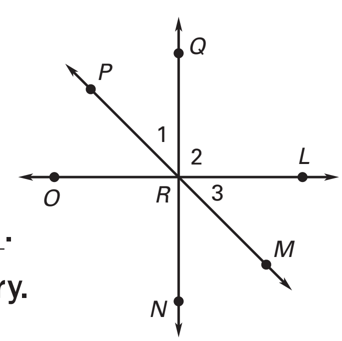
Solution

a. Because $\angle RVS$ and $\angle PVU$ are ______________, $\angle RVS \cong \angle PVU$. By the definition of congruent angles, $m\angle RVS = m\angle PVU$. So, $m\angle PVU =$ _____.

b. By the Angle Addition Postulate, $m\angle QVU =$ ____________________. Substitute to get $120° = 90° + m\angle PVQ$. By the Subtraction Property of Equality, $m\angle PVQ =$ _____. Because $\angle SVT$ and $\angle PVQ$ are _________ angles, $\angle SVT \cong \angle PVQ$. By the definition of __________________, $m\angle SVT = m\angle PVQ$. So, $m\angle SVT =$ _____.

Example 2 ***Find angle measures***

If $m\angle QRO = 90°$ and $m\angle PRO = 45°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.



Solution

$\angle QRP$ and $\angle PRO$ are complementary.

So, $m\angle 1 =$ _____ $-$ _____ $=$ _____.

$\angle QRL$ and $\angle QRO$ are supplementary.

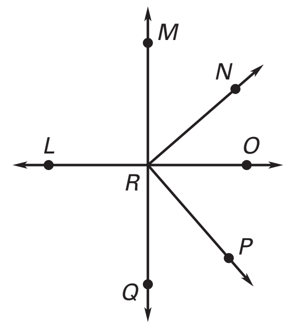
So, $m\angle 2 =$ ______ $-$ _____ $=$ _____.

$\angle LRM$ and $\angle PRO$ are vertical angles.

So, $m\angle 3 =$ _____.

Your Notes

✔ ***Checkpoint*** **In Exercises 1–4, use the diagram where $m\angle LRM = m\angle NRP = 90°$.**



1. Find $m\angle QRO$.

---

2. Find $m\angle LRQ$.

---

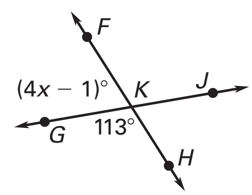
3. If $m\angle NRO = 41°$, find $m\angle MRN$.

---

4. If $m\angle PRO = 49°$, find $m\angle NRO$.

Example 3 *Use algebra*

Solve for x in the diagram.



Solution

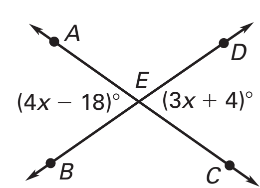
Because $\angle FKG$ and $\angle GKH$ form a linear pair, the sum of their measures is ______.

$(4x - 1)° + 113° =$ ______	**Definition of supplementary angles**
$4x +$ ______ $=$ ______	**Combine like terms.**
$4x =$ ______	**Subtract ______ from each side.**
$x =$ ______	**Divide each side by 4.**

Homework

✔ ***Checkpoint*** **Complete the following exercise.**

5. Solve for x in the diagram.



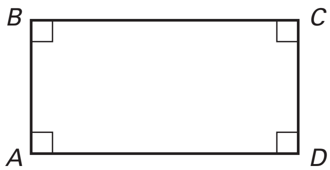
Name ______________________ Date ______________

LESSON 4.6

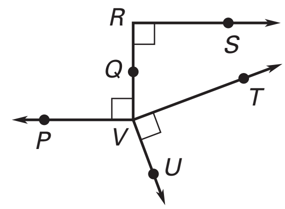
Practice

Identify the congruent angles in the figure. *Explain* how you know they are congruent.

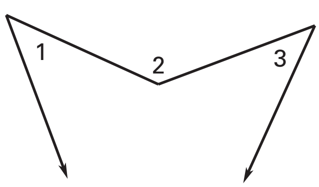
1.



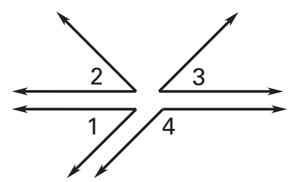
2.



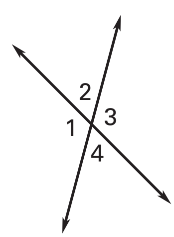
3. $\angle 1$ and $\angle 3$ are complementary.
$\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 2$ are supplementary.



4. $\angle 1$ and $\angle 2$ are complementary.
$\angle 2$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 4$ are supplementary.



Use the diagram at the right.



5. If $m\angle 1 = 115°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

6. If $m\angle 2 = 64°$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.

7. If $m\angle 3 = 112°$, find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

8. If $m\angle 4 = 67°$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.

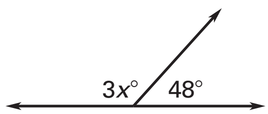
Name ______________________ Date ______________

LESSON 4.6

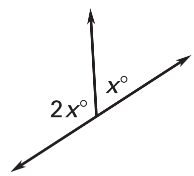
Practice *continued*

Find the value of *x*.

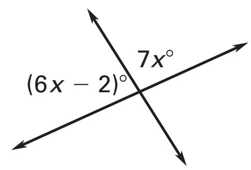
9.



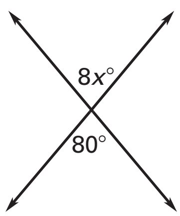
10.



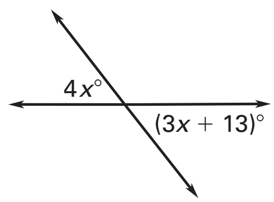
11.



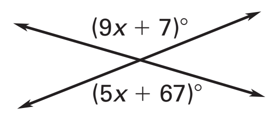
12.



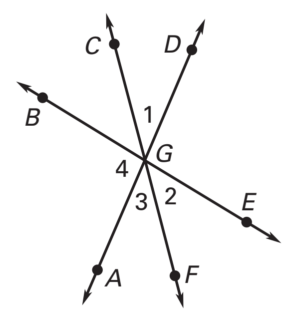
13.



14.



In the diagram at the right, $m\angle 1 = 38°$ and $m\angle 4 = 98°$. Find the indicated angle measure.



15. Find $m\angle 3$.

16. Find $m\angle DGE$.

17. Find $m\angle CGE$.

18. Find $m\angle 2$.

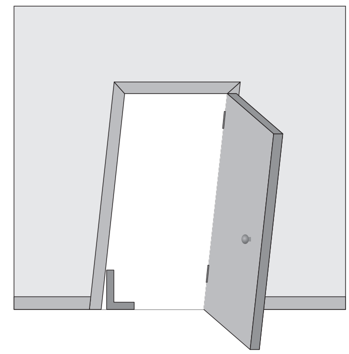
19. Find $m\angle AGC$.

Name ______________________ Date ______________

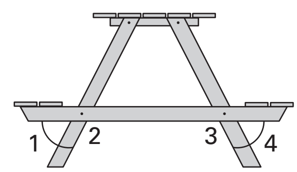
LESSON 4.6

Practice *continued*

20. Door Frame You are using a carpenter's square to check whether a corner of a door frame forms a right angle. The square is basically a ruler in the form of a right angle. When you try to fit the square into the corner, there is a gap as shown in the figure. *Explain* whether there is a right angle in this corner by using a theorem from this lesson.



21. Picnic table The figure shows the side view of a picnic table. Given that ∠1 ≅ ∠4, complete the proof showing that ∠2 ≅ ∠3.



GIVEN: ∠1 ≅ ∠4

PROVE: ∠2 ≅ ∠3

Statements	**Reasons**
1. ∠1 ≅ ∠4	**1.**
2. ∠1 and _____ are a linear pair. ∠3 and _____ are a linear pair.	**2.**
3.	**3.** Linear Pair Postulate
4. ∠2 ≅ ∠3	**4.**

4.7 Prove Theorems About Perpendicular Lines

Georgia Performance Standard(s)
MM1G1b

Your Notes

Goal • Find the distance between a point and a line.

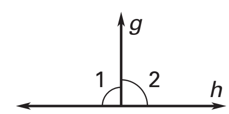
VOCABULARY

Distance from a point to a line

Transversal

THEOREM 4.7

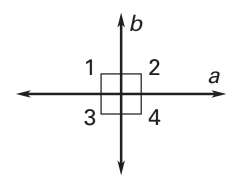
If two lines intersect to form a linear pair of congruent angles, then the lines are ________________.



If $\angle 1 \cong \angle 2$, then g ____ h.

THEOREM 4.8

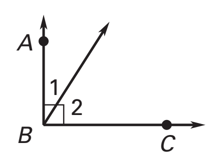
If two lines are perpendicular, then they intersect to form four ______________.



If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are ______________.

THEOREM 4.9

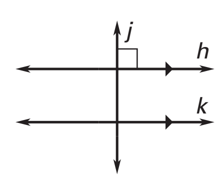
If two sides of two adjacent acute angles are perpendicular, then the angles are __________________.



If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are __________________.

THEOREM 4.10 PERPENDICULAR TRANSVERSAL THEOREM

If a transversal is perpendicular to one of two parallel lines, then it is ________________ to the other.

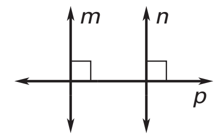


If $h \parallel k$ and $j \perp h$, then j ____ k.

Your Notes

THEOREM 4.11 LINES PERPENDICULAR TO A TRANSVERSAL THEOREM

In a plane, if two lines are perpendicular to the same line, then they are _________ to each other.

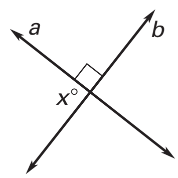


If $m \perp p$ and $n \perp p$, then m __ n.

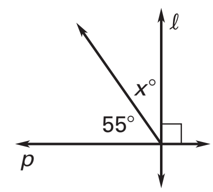
Example 1 *Applications of the theorems*

Find the value of x.

a.



b.



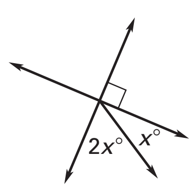
Solution

a. Because a and b are _______________, all four angles formed are right angles by _______________. By definition of a right angle, $x =$ _____.

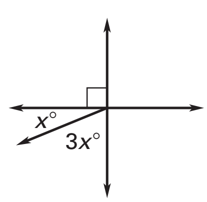
b. Because ℓ and p are perpendicular, all four angles formed are right angles by _______________. By _______________, the 55° angle and the $x°$ angle are _________________. Thus $x + 55 = 90$, so $x =$ _____.

Checkpoint Find the value of x.

1.



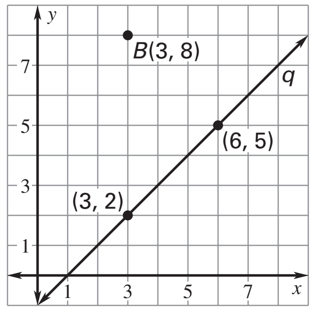
2.



Your Notes

Example 2 ***Find the distance between a point and a line***

What is the distance from point *B* to line *q*?



Solution

You need to find the slope of line *q*. Using the points (3, 2) and (6, 5), the slope of line *q* is

$$m = \frac{\square - 2}{6 - \square} = \_\_\_.$$

The distance from point *B* to line *q* is the length of the perpendicular segment from point *B* to line *q*. The slope of a perpendicular segment from point *B* to line *q* is the negative reciprocal of ___, or $\frac{\_\_\_}{\_\_\_}$ = _____. The segment from (6, 5) to (3, 8) has a slope of _____. So, the segment is perpendicular to line *q*.

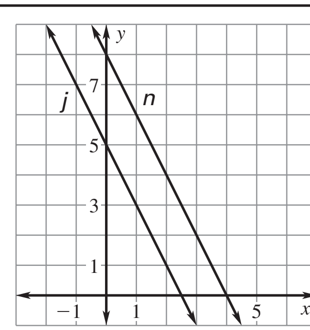
Find the distance between (6, 5) and (3, 8).

$$d = \sqrt{(\_\_\_ - \_\_\_)^2 + (\_\_\_ - \_\_\_)^2} \approx \_\_\_\_\_$$

The distance from point *B* to line *q* is about _____ units.

✔ ***Checkpoint*** **Complete the following exercise.**

3. What is the distance from line *n* to line *j*?



Homework

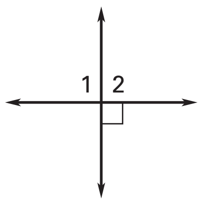
Name ______________________ Date ______________

LESSON 4.7

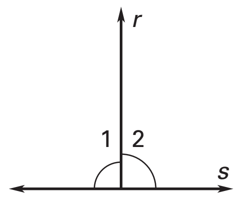
Practice

Write the theorem that justifies the statement.

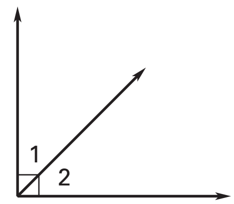
1. $\angle 1$ and $\angle 2$ are right angles.



2. $r \perp s$

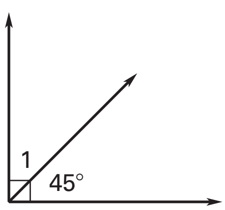


3. $\angle 1$ and $\angle 2$ are complementary.

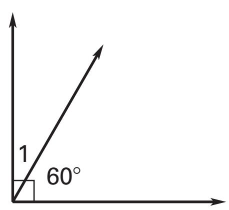


Find $m\angle 1$.

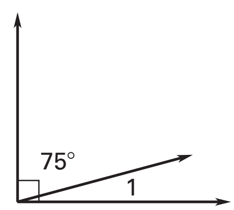
4.



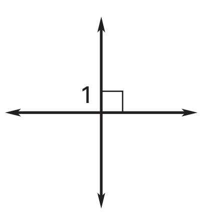
5.



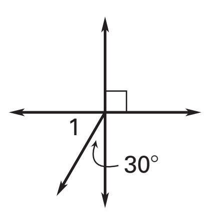
6.



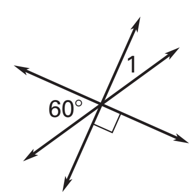
7.



8.



9.

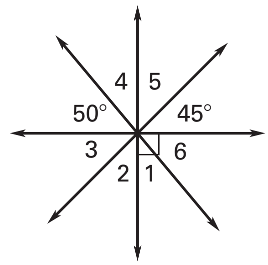


Name ______________________ Date ______________

LESSON 4.7

Practice *continued*

Find the measure of the indicated angle.



10. $\angle 1$

11. $\angle 2$

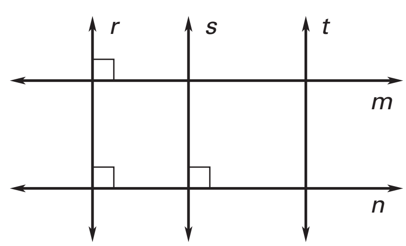
12. $\angle 3$

13. $\angle 4$

14. $\angle 5$

15. $\angle 6$

In Exercises 16–18, use the diagram.



16. Is $r \parallel s$?

17. Is $m \parallel n$?

18. Is $r \parallel t$?

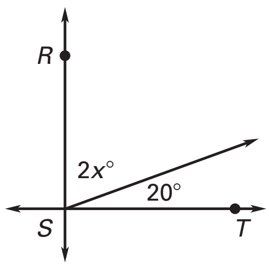
Name ______________________ Date ____________

LESSON 4.7

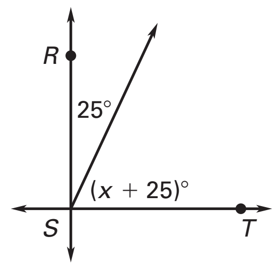
Practice *continued*

In the diagram, $\overleftrightarrow{RS} \perp \overleftrightarrow{ST}$. Find the value of *x*.

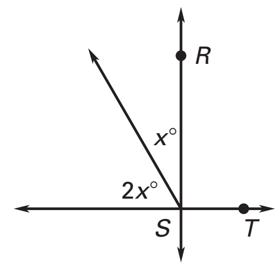
19.



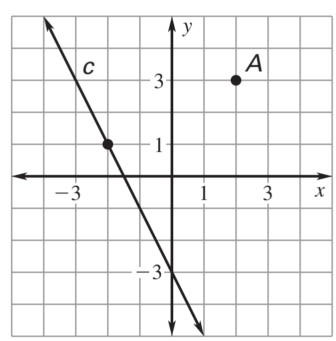
20.



21.



22. Find the distance from point *A* to line *c*. Round your answer to the nearest tenth.

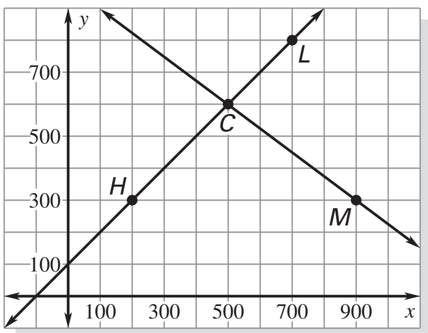


Name ____________________ Date ____________

LESSON 4.7

Practice *continued*

23. Maps A partial map of a town is drawn on a graph where units are measured in feet. Line $\overleftrightarrow{HL}$ represents Main Street and line $\overleftrightarrow{CM}$ represents 4th Avenue. Point L represents the library, point C represents the center of town, point H represents the high school, and point M represents the medical center.



a. Find the distance between the medical center and the high school.

b. How far away is the medical center from the center of town along 4th Avenue?

c. What distance do you walk if you go from the medical center to the library along 4th Avenue and Main Street? Round your answer to the nearest foot.

d. Is 4th Avenue perpendicular to Main Street?

4.8 Prove Triangles Congruent by SSS

Georgia Performance Standard(s)

MM1G1e, MM1G3c

Your Notes

Goal • Use side lengths to prove triangles are congruent.

VOCABULARY
Congruent figures
Corresponding parts
Coordinate proof

SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE

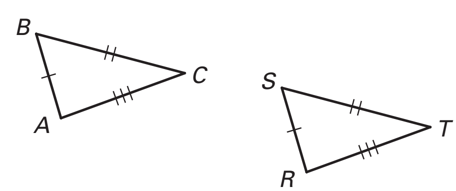
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{AB} \cong$ _____,

Side $\overline{BC} \cong$ _____, and

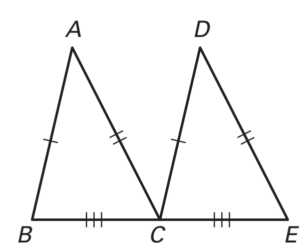
Side $\overline{CA} \cong$ _____,

then $\triangle ABC \cong$ ________.



Example 1 ***Use the SSS Congruence Postulate***

Show that $\triangle ABC \cong \triangle DCE$.



Solution

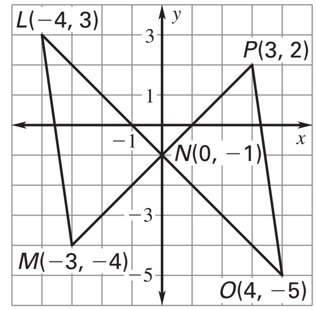
It is given that $\overline{AB} \cong \overline{DC}$, $\overline{BC} \cong \overline{CE}$, and ___________. So, by the ______________________________,

$\triangle ABC \cong$ ________.

Your Notes

Example 2 ***Congruent triangles in a coordinate plane***

Use the SSS Congruence Postulate to show that $\triangle LMN \cong \triangle OPN$.



Solution

Use the Distance Formula to show that corresponding sides are the same length.

$LM = \sqrt{(-3 - (-4))^2 + (-4 - 3)^2}$

$= \sqrt{\_\_\_\_\_^2 + \_\_\_\_\_^2}$

$= \_\_\_\_\_$

$OP = \sqrt{(3 - 4)^2 + (2 - (-5))^2}$

$= \sqrt{\_\_\_\_\_^2 + \_\_\_\_\_^2}$

$= \_\_\_\_\_$

So, $LM = OP$, and hence _____ $\cong$ _____.

$MN = \sqrt{(0 - (-3))^2 + ((-1) - (-4))^2}$

$= \sqrt{\_\_\_\_\_^2 + \_\_\_\_\_^2}$

$= \_\_\_\_\_$

$PN = \sqrt{(0 - 3)^2 + (-1 - 2)^2}$

$= \sqrt{\_\_\_\_\_^2 + \_\_\_\_\_^2}$

$= \_\_\_\_\_$

So, $MN = PN$, and hence _____ $\cong$ _____.

$NL = \sqrt{(-4 - 0)^2 + (3 - (-1))^2}$

$= \sqrt{\_\_\_\_\_^2 + \_\_\_^2}$

$= \_\_\_\_\_$

$NO = \sqrt{(4 - 0)^2 + (-5 - (-1))^2}$

$= \sqrt{\_\_\_\_\_^2 + \_\_\_\_\_^2}$

$= \_\_\_\_\_$

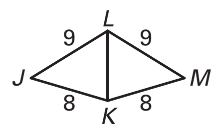
So, $NL = NO$, and hence _____ $\cong$ _____.

So, by the ______________________________, you know that $\triangle LMN \cong$ _________.

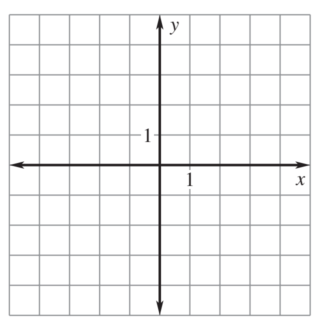
Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

1. Decide whether $\triangle JKL \cong \triangle MKL$ is true. *Explain* your reasoning.



2. $\triangle DFG$ has vertices $D(-2, 4)$, $F(4, 4)$, and $G(-2, 2)$. $\triangle LMN$ has vertices $L(-3, -3)$, $M(-3, 3)$, and $N(-1, -3)$. Graph the triangles in the same coordinate plane and show that they are congruent.



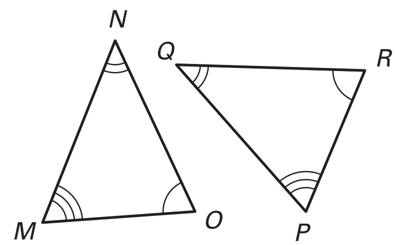
Homework

Name ______________________ Date ____________

LESSON 4.8 Practice

Tell whether the angles or sides are *corresponding angles, corresponding sides,* or *neither*.

1. $\angle N$ and $\angle P$



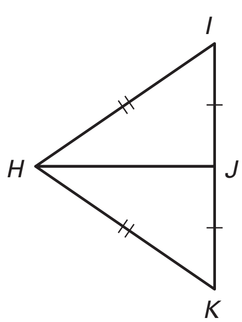
2. $\angle M$ and $\angle P$

3. $\overline{OM}$ and $\overline{RP}$

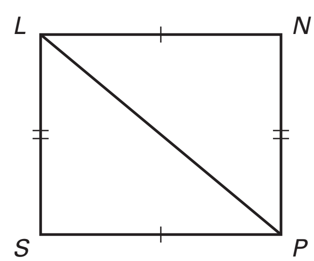
4. $\overline{NO}$ and $\overline{QP}$

Decide whether the congruence statement is true. *Explain* your reasoning.

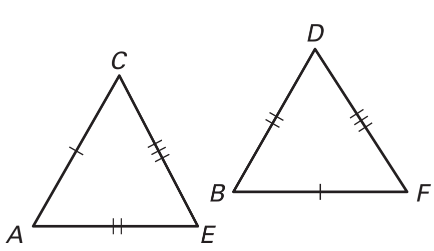
5. $\triangle IHJ \cong \triangle JHK$



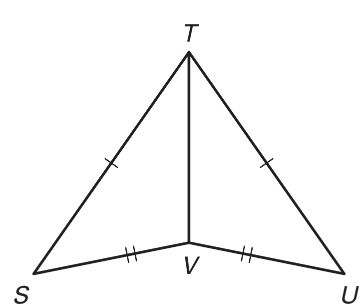
6. $\triangle LPS \cong \triangle PLN$



7. $\triangle ACE \cong \triangle BDF$



8. $\triangle STV \cong \triangle UTV$



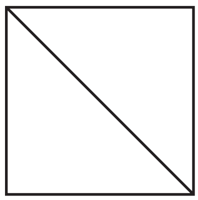
Name ______________________ Date ______________

LESSON 4.8

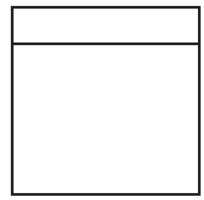
Practice *continued*

Use the SSS Congruence Postulate to decide whether the figure is stable. *Explain* your reasoning.

9.



10.



Use the given coordinates to determine if $\triangle ABC \cong \triangle DEF$.

11. $A(1, 1), B(2, 0), C(1, -1), D(3, 1), E(4, 0), F(3, -1)$

12. $A(1, 2), B(4, 1), C(3, 4), D(5, 2), E(8, 1), F(6, 4)$

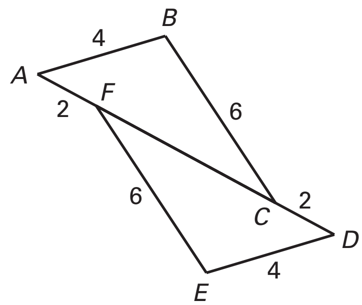
Name ______________________ Date ______________

LESSON 4.8

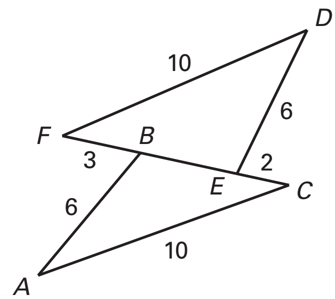
Practice *continued*

Determine whether $\triangle ABC \cong \triangle DEF$. *Explain* your reasoning.

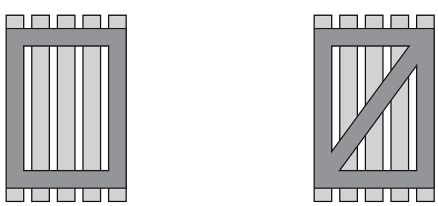
13.



14.



15. **Gate** Two different gate doors are shown below. Which door frame is stable? *Explain* your reasoning.



4.9 Prove Triangles Congruent by SAS and HL

Georgia Performance Standard(s)

MM1G3c

Your Notes

Goal • Use sides and angles to prove congruence.

VOCABULARY

Leg of a right triangle

Hypotenuse

SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE

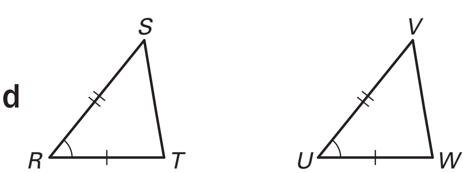
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{RS} \cong$ _____,

Angle $\angle R \cong$ _____, and

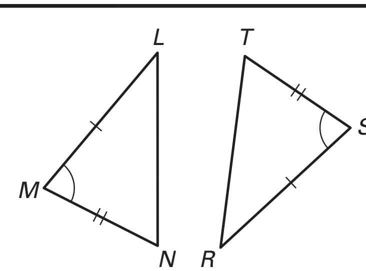
Side $\overline{RT} \cong$ _____,

then $\triangle RST \cong$ ________.



Example 1 ***Use the SAS Congruence Postulate***

Show that $\triangle LMN \cong \triangle RST$.



Solution

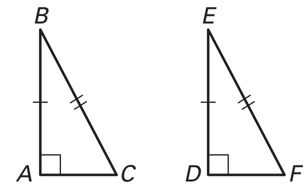
It is given that $\overline{LM} \cong \overline{RS}$, $\overline{MN} \cong \overline{ST}$, and _____ $\cong$ _____.

So, by the ________________ __________, $\triangle LMN \cong$ ________.

Your Notes

THEOREM 4.12: HYPOTENUSE-LEG CONGRUENCE THEOREM

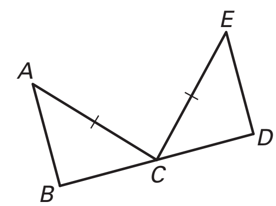
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are ____________.



Example 2 ***Use the Hypotenuse-Leg Congruence Theorem***

Write a proof.

Given $\overline{AC} \cong \overline{EC}$,
$\overline{AB} \perp \overline{BD}$,
$\overline{ED} \perp \overline{BD}$,
$\overline{AC}$ is a bisector of $\overline{BD}$.



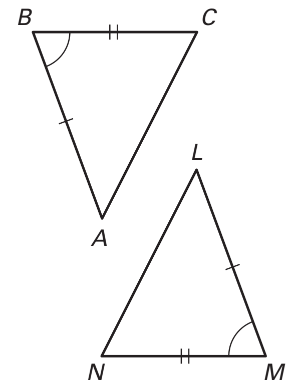
Prove $\triangle ABC \cong \triangle EDC$

Statements	Reasons
1. $\overline{AC} \cong \overline{EC}$	**1.** ________
2. $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$	**2.** ________
3. $\angle B$ and $\angle D$ are ______________.	**3.** Definition of $\perp$ lines
4. $\triangle ABC$ and $\triangle EDC$ are ________________.	**4.** Definition of a ______ ________
5. $\overline{AC}$ is a bisector of $\overline{BD}$.	**5.** ________
6. $\overline{BC} \cong \overline{DC}$	**6.** Definition of segment bisector
7. $\triangle ABC \cong \triangle EDC$	**7.** ________________ __________

Your Notes

✔ *Checkpoint* Complete the following exercises.

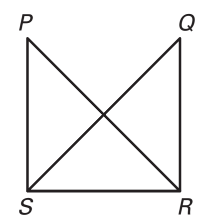
1. Decide whether enough information is given to prove that $\triangle ABC \cong \triangle LMN$ using the SAS Congruence Postulate.



2. Write a proof.

Given $\overline{PR} \cong \overline{QS}$, $\overline{PS} \perp \overline{SR}$, $\overline{QR} \perp \overline{SR}$

Prove $\triangle PRS \cong \triangle QSR$



Statements	Reasons
1. $\overline{PR} \cong \overline{QS}$	1. ________
2. $\overline{PS} \perp \overline{SR}$, $\overline{QR} \perp \overline{SR}$	2. ________
3. $\angle S$ and $\angle R$ are right angles.	3. ________________________
4. $\triangle PRS$ and $\triangle QSR$ are right triangles.	4. ________________________
5. $\overline{SR} \cong \overline{RS}$	5. ________________________
6. $\triangle PRS \cong \triangle QSR$	6. ________________________

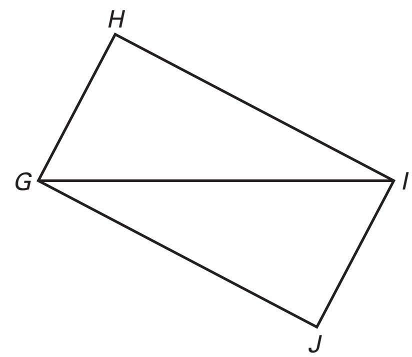
Homework

Name ______________________ Date ____________

LESSON 4.9

Practice

Use the diagram to name the included angle between the given pair of sides.



1. $\overline{GH}$ and $\overline{HI}$

2. $\overline{HI}$ and $\overline{IG}$

3. $\overline{IG}$ and $\overline{HG}$

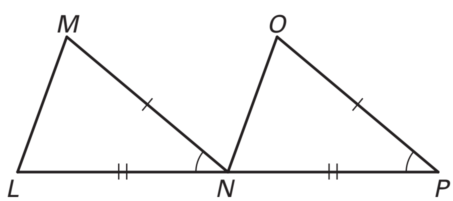
4. $\overline{GI}$ and $\overline{IJ}$

5. $\overline{JG}$ and $\overline{IG}$

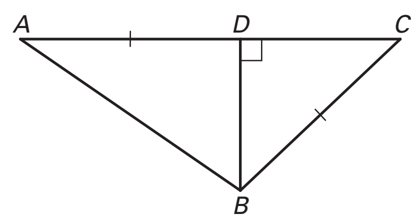
6. $\overline{IJ}$ and $\overline{GJ}$

Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.

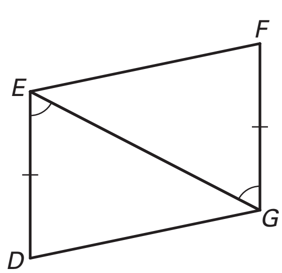
7. $\triangle LMN$, $\triangle NOP$



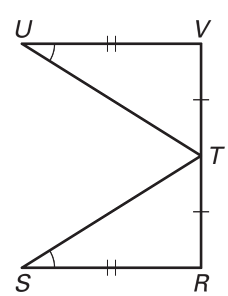
8. $\triangle ABD$, $\triangle CBD$



9. $\triangle DEG$, $\triangle FGE$



10. $\triangle RST$, $\triangle VUT$



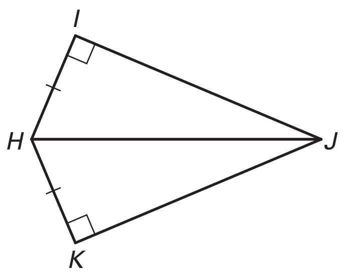
Name ______________________ Date ______________

LESSON 4.9

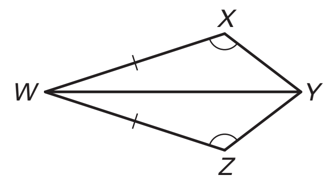
Practice *continued*

Decide whether enough information is given to prove that the triangles are congruent using the HL Congruence Theorem.

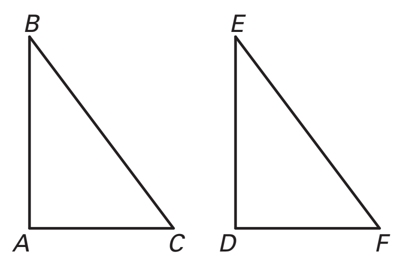
11. $\triangle HIJ, \triangle HKJ$



12. $\triangle WXY, \triangle WZY$



State the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$ using the indicated postulate or theorem.



13. **GIVEN:** $\angle B \cong \angle E, \overline{BC} \cong \overline{EF}$, __?__ $\cong$ __?__
Use the SAS Congruence Postulate.

14. **GIVEN:** $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$, __?__ $\cong$ __?__
Use the SSS Congruence Postulate.

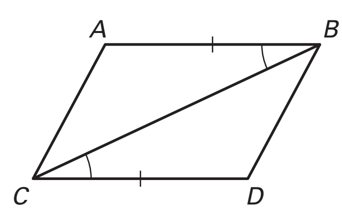
15. **GIVEN:** $\overline{AC} \cong \overline{DF}$, $\angle A$ is a right angle and $\angle A \cong \angle D$, __?__ $\cong$ __?__

Use the HL Congruence Theorem.

16. **Proof** Complete the proof.

GIVEN: $\overline{AB} \cong \overline{DC}, \angle ABC \cong \angle DCB$

PROVE: $\triangle ABC \cong \triangle DCB$



Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1.
2. $\angle ABC \cong \angle DCB$	2.
3. $\overline{CB} \cong \overline{CB}$	3.
4. $\triangle ABC \cong \triangle DCB$	4.

4.10 Prove Triangles Congruent by ASA and AAS

Georgia Performance Standard(s)

MM1G3c

Goal • Use two more methods to prove congruences.

Your Notes

VOCABULARY

Flow proof

ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE

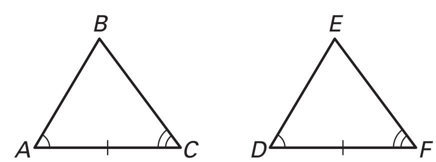
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If **A**ngle $\angle A \cong$ _____,

Side $\overline{AC} \cong$ _____, and

Angle $\angle C \cong$ _____,

then $\triangle ABC \cong$ ________.



THEOREM 4.13: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM

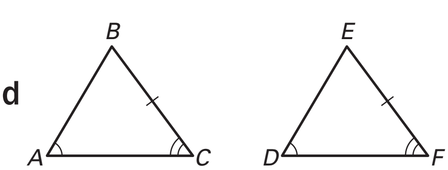
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If **A**ngle $\angle A \cong$ _____,

Angle $\angle C \cong$ _____, and

Side $\overline{BC} \cong$ _____,

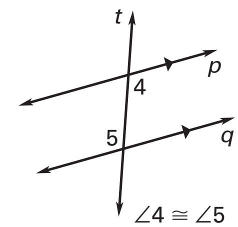
then $\triangle ABC \cong$ ________.



Your Notes

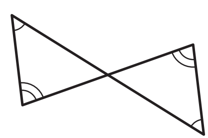
THEOREM 4.14: ALTERNATE INTERIOR ANGLES THEOREM

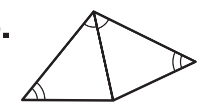
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are ___________.

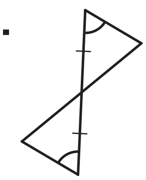


Example 1 ***Identify congruent triangles***

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a. 

b. 

c. 

Solution

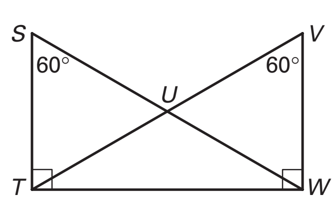
a. There is not enough information to prove the triangles are congruent, because no _______ are known to be congruent.

b. Two pairs of angles and a _______________ pair of sides are congruent. The triangles are congruent by the _____________________________.

c. The vertical angles are congruent, so two pairs of angles and their ________________ are congruent. The triangles are congruent by the __________________ __________.

✔ ***Checkpoint*** **Complete the following exercise.**

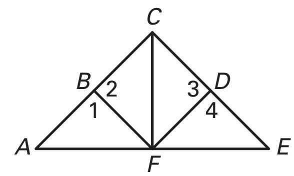
1. Can $\triangle STW$ and $\triangle VWT$ be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.



Your Notes

Example 2 ***Write a flow proof***

In the diagram, $\angle 1 \cong \angle 4$ and $\overline{CF}$ bisects $\angle ACE$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.



Solution

Given $\angle 1 \cong \angle 4$, $\overline{CF}$ bisects $\angle ACE$.

Prove $\triangle CBF \cong \triangle CDF$

$\angle 1 \cong \angle 4$

$\angle 1$ and $\angle 2$ are ______________.
$\angle 3$ and $\angle 4$ are ______________.

$\overline{CF}$ bisects. $\angle ACE$.

________ Def. of ______________ angles ________

$\angle 2 \cong$ _____

$\overline{CF} \cong \overline{CF}$

$\angle ACF \cong$ ________

Congruent Supps. Thm. ______________ Def. of $\angle$ bisector

$\triangle CBF \cong \triangle CDF$

____

✔ ***Checkpoint*** **Complete the following exercise.**

2. In Example 2, suppose it is given that $\overline{CF}$ bisects $\angle ACE$ and $\angle BFD$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.

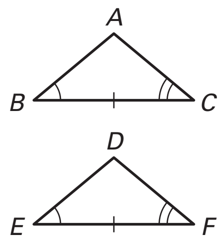
Homework

Name ______________________________ Date ______________

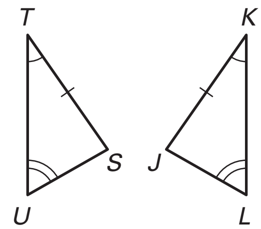
LESSON 4.10 Practice

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.

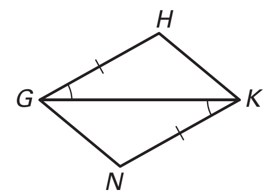
1.



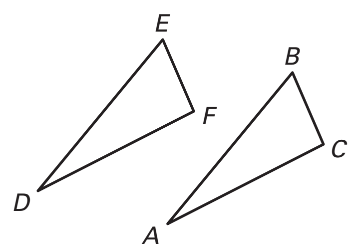
2.



3.



State the third congruence that is needed to prove that $\triangle DEF \cong \triangle ABC$ using the given postulate or theorem.

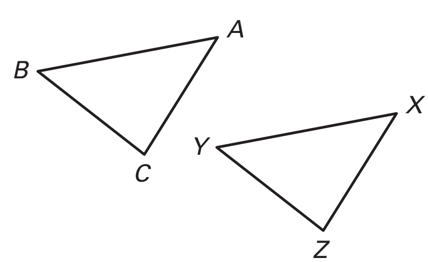


4. **GIVEN:** $\overline{DE} \cong \overline{AB}$, $\angle D \cong \angle A$, __?__ $\cong$ __?__
Use the AAS Congruence Theorem.

5. **GIVEN:** $\overline{FE} \cong \overline{CB}$, $\angle F \cong \angle C$, __?__ $\cong$ __?__
Use the ASA Congruence Postulate.

6. **GIVEN:** $\overline{DF} \cong \overline{AC}$, $\angle F \cong \angle C$, __?__ $\cong$ __?__
Use the SAS Congruence Theorem.

State the third congruence that is needed to prove that $\triangle ABC \cong \triangle XYZ$ using the given postulate or theorem.



7. **GIVEN:** $\angle C \cong \angle Z$, $\overline{AC} \cong \overline{XZ}$, __?__ $\cong$ __?__
Use the AAS Congruence Theorem.

8. **GIVEN:** $\angle B \cong \angle Y$, $\overline{AB} \cong \overline{XY}$, __?__ $\cong$ __?__
Use the ASA Congruence Postulate.

9. **GIVEN:** $\overline{BC} \cong \overline{YZ}$, $\angle B \cong \angle Y$, __?__ $\cong$ __?__
Use the SAS Congruence Theorem.

Name ______________________ Date ______________

LESSON 4.10

Practice *continued*

Tell whether you can use the given information to determine whether $\triangle JKL \cong \triangle RST$.

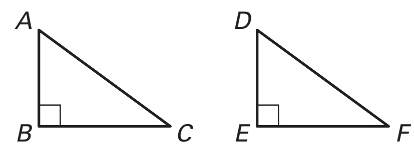
10. $\angle J \cong \angle R, \angle K \cong \angle S, \angle L \cong \angle T$

11. $\overline{JK} \cong \overline{RS}, \angle J \cong \angle R, \angle L \cong \angle T$

12. $\angle K \cong \angle S, \angle L \cong \angle T, \overline{KL} \cong \overline{ST}$

13. $\angle J \cong \angle R, \overline{KL} \cong \overline{ST}$

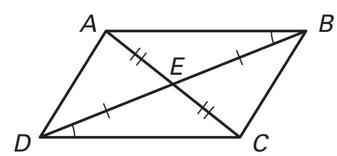
14. **Multiple Choice** Which postulate or theorem can you use to prove that $\triangle ABC \cong \triangle DEF$?



A. AAS **B.** ASA
C. SAS **D.** Not enough information

Explain **how you can prove that the indicated triangles are congruent using the given postulate or theorem.**

15. $\triangle ABE \cong \triangle CDE$ by SAS



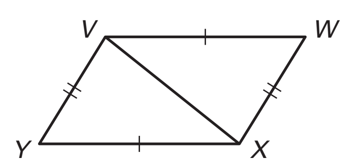
16. $\triangle ABE \cong \triangle CDE$ by ASA

17. $\triangle ABE \cong \triangle CDE$ by AAS

18. **Proof** Complete the proof.

GIVEN: $\overline{VW} \cong \overline{XY}, \overline{WX} \cong \overline{YV}$

PROVE: $\triangle VWX \cong \triangle XYV$



Statements	Reasons
1. $\overline{VW} \cong \overline{XY}$	1.
2. $\overline{WX} \cong \overline{YV}$	2.
3. $\overline{VX} \cong \overline{VX}$	3.
4. $\triangle VWX \cong \triangle XYV$	4.

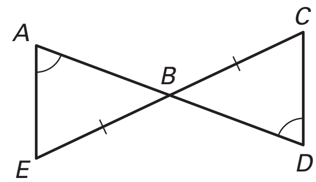
Name ______________________ Date ____________

LESSON 4.10 **Practice** *continued*

19. Proof Write a proof.

GIVEN: $\overline{BE} \cong \overline{BC}$, $\angle A \cong \angle D$

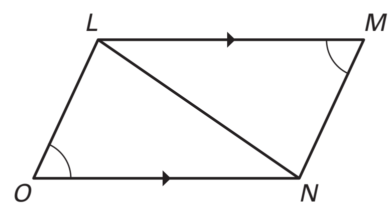
PROVE: $\triangle ABE \cong \triangle DBC$



20. Proof Complete the proof.

GIVEN: $\overline{LM} \parallel \overline{NO}$, $\angle LMN \cong \angle NOL$

PROVE: $\triangle LMN \cong \triangle NOL$



Statements	Reasons
1. $\overline{LM} \parallel \overline{NO}$	**1.**
2. $\angle LMN \cong \angle NOL$	**2.**
3. $\overline{NL} \cong \overline{NL}$	**3.**
4. $\angle MLN \cong \angle ONL$	**4.**
5. $\triangle LMN \cong \triangle NOL$	**5.**

Words to Review

Give an example of the vocabulary word.

Distance formula	Midpoint
Midpoint formula	Conjecture
Inductive reasoning	Counterexample
Conditional statement	If-then form
Hypothesis	Conclusion

Negation	**Converse**
Inverse	**Contrapositive**
Equivalent statements	**Perpendicular lines**
Biconditional statement	**Deductive reasoning**

Proof	
Two-column proof	
Postulate	**Theorem**
Adjacent angles	**Linear pair**

Distance from a point to a line.	Transversal
Congruent figures	Corresponding parts
Coordinate proof	Flow proof
Leg of a right triangle, hypotenuse	