

5.1

Midsegment Theorem and Coordinate Proof



Georgia Performance Standard(s)

MM1G1c,
MM1G1e

Your Notes

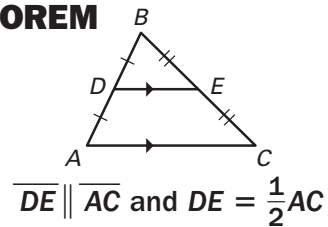
- Goal** • Use properties of midsegments and write coordinate proofs.

VOCABULARY

Midsegment of a triangle

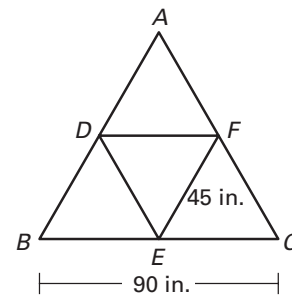
THEOREM 5.1: MIDSEGMENT THEOREM

The segment connecting the midpoints of two sides of a triangle is _____ to the third side and is _____ as long as that side.



Example 1 Use the Midsegment Theorem to find lengths

Windows A large triangular window is segmented as shown. In the diagram, \overline{DF} and \overline{EF} are midsegments of $\triangle ABC$. Find DF and AB .



In the diagram for Example 1, midsegment \overline{DF} can be called “the midsegment opposite BC .”

Solution

$$DF = \frac{1}{2} \cdot BC = \frac{1}{2} (90) = 45$$

$$AB = 2 \cdot EF = 2(45) = 90$$

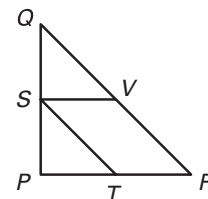
- ✓ **Checkpoint** Complete the following exercise.

- In Example 1, consider $\triangle ADF$. What is the length of the midsegment opposite DF ?

Your Notes

Example 2 Use the Midsegment Theorem

In the diagram at the right, $QS = SP$ and $PT = TR$. Show that $\overline{QR} \parallel \overline{ST}$.



Solution

Because $QS = SP$ and $PT = TR$, S is the _____ of \overline{QP} and T is the _____ of \overline{PR} by definition. Then \overline{ST} is a _____ of $\triangle PQR$ by definition and $\overline{QR} \parallel \overline{ST}$ by the _____.

Example 3 Place a figure in a coordinate plane

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

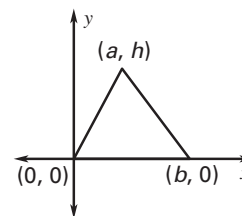
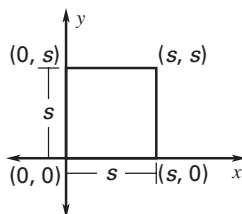
- a. A square
- b. An acute triangle

Solution

It is easy to find lengths of horizontal and vertical segments and distances from _____, so place one vertex at the _____ and one or more sides on an _____.

- a. Let s represent the _____.
- b. You need to use _____ different variables.

The square represents a general square because the coordinates are based only on the definition of a square. If you use this square to prove a result, the result will be true for all squares.



Your Notes

✔ **Checkpoint** Complete the following exercises.

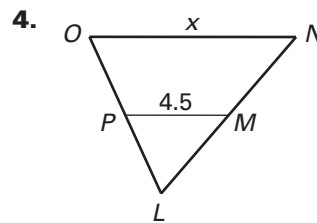
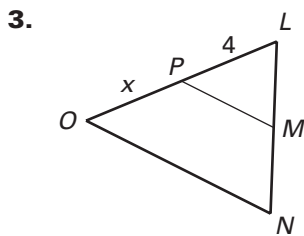
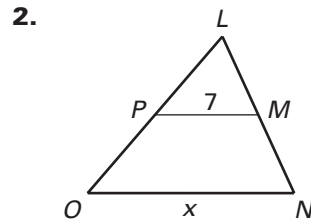
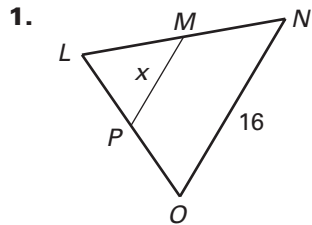
2. In Example 2, if V is the midpoint of \overline{QR} , what do you know about \overline{SV} ?

3. Place an obtuse scalene triangle in a coordinate plane that is convenient for finding side lengths. Assign coordinates to each vertex.

Homework

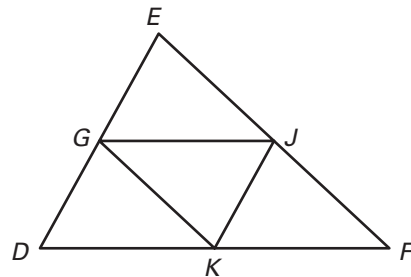
LESSON 5.1 Practice

\overline{MP} is a midsegment of $\triangle LNO$. Find the value of x .



In $\triangle DEF$, $\overline{EJ} \cong \overline{JF}$, $\overline{FK} \cong \overline{KD}$, and $\overline{DG} \cong \overline{GE}$. Complete the statement.

5. $\overline{GJ} \parallel$ _____
6. $\overline{EJ} \cong$ _____ \cong _____
7. $\overline{DE} \parallel$ _____
8. $\overline{GJ} \cong$ _____ \cong _____

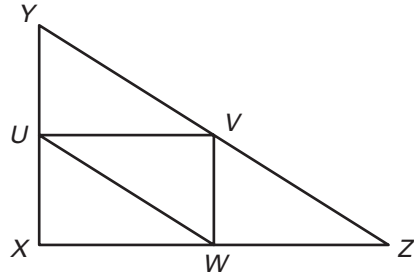


LESSON
5.1

Practice *continued*

Use the diagram of $\triangle XYZ$ where U , V , and W are the midpoints of the sides.

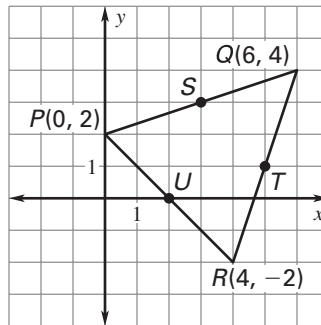
9. If $UW = 4x - 1$ and $YZ = 5x + 4$, what is UW ?



10. Find UV .

Use the graph shown.

11. Find the coordinate of the endpoints of each midsegment of $\triangle PQR$.

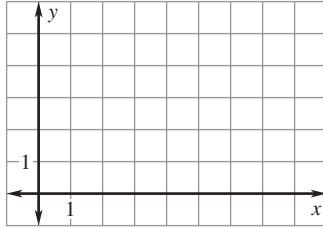


12. Use the slope and the Distance Formula to verify that the Midsegment Theorem is true for \overline{ST} .

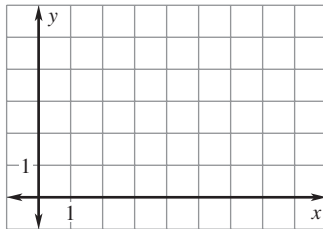
LESSON
5.1
Practice *continued*

Place the figure in the coordinate plane. Assign coordinates to each vertex.

- 13.** A 4 unit by 7 unit rectangle with one vertex at $(0, 0)$.

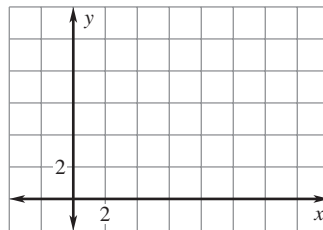


- 14.** A square with side length 4 and one vertex at $(4, 0)$.

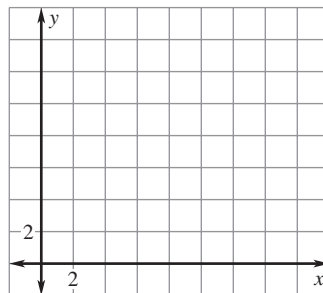


Place the figure in the coordinate plane. Assign coordinates to each vertex. Explain the advantage of your placement.

- 15.** Right triangle: leg lengths are 5 units and 9 units



- 16.** Isosceles right triangle: leg length is 14 units



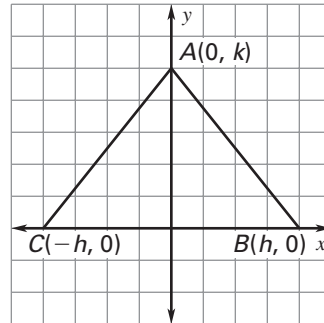
LESSON
5.1

Practice *continued*

In Exercises 17 and 18, describe a plan for the proof.

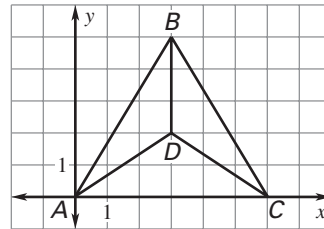
17. GIVEN: Coordinates of vertices of $\triangle ABC$

PROVE: $\triangle ABC$ is isosceles.



18. GIVEN: \overline{BD} bisects $\angle ABC$.

PROVE: $\triangle BDA \cong \triangle BDC$



5.2

Use Perpendicular Bisectors



Georgia
Performance
Standard(s)

MM1G3e

Your Notes

Goal • Use perpendicular bisectors to solve problems.

VOCABULARY

Perpendicular bisector

Equidistant

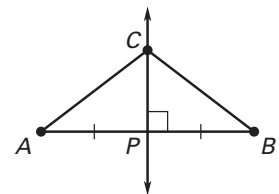
Concurrent

Point of concurrency

Circumcenter

THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM

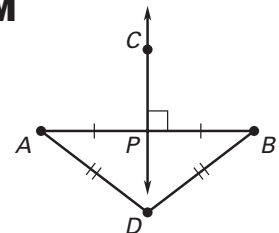
In a plane, if a point is on the perpendicular bisector of a segment, then it is _____ from the endpoints of the segment.



If \overleftrightarrow{CP} is the \perp bisector of \overline{AB} , then $CA = \underline{\hspace{1cm}}$.

THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the _____ of the segment.

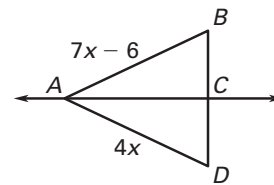


If $DA = DB$, then D lies on the _____ of \overline{AB} .

Your Notes

Example 1 Use the Perpendicular Bisector Theorem

\overleftrightarrow{AC} is the perpendicular bisector of \overline{BD} . Find AD .



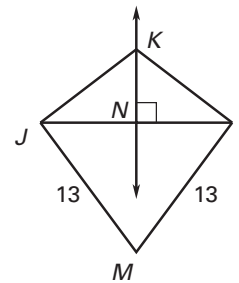
Solution

$AD =$ _____ Perpendicular Bisector Theorem
 _____ = _____ Substitute.
 $x =$ _____ Solve for x .
 $AD =$ _____ = _____ = _____.

Example 2 Use perpendicular bisectors

In the diagram, \overleftrightarrow{KN} is the perpendicular bisector of \overline{JL} .

- What segment lengths in the diagram are equal?
- Is M on \overleftrightarrow{KN} ?



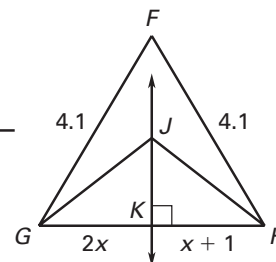
Solution

- \overleftrightarrow{KN} bisects \overline{JL} , so _____ = _____. Because K is on the perpendicular bisector of \overline{JL} , _____ = _____ by Theorem 5.2. The diagram shows that _____ = _____ = 13.
- Because $MJ = ML$, M is _____ from J and L . So, by the _____, M is on the perpendicular bisector of \overline{JL} , which is \overleftrightarrow{KN} .

✓ **Checkpoint** In the diagram, \overleftrightarrow{JK} is the perpendicular bisector of \overline{GH} .

1. What segment lengths are equal?

2. Find GH .

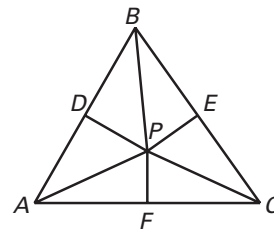


Your Notes

THEOREM 5.4: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE

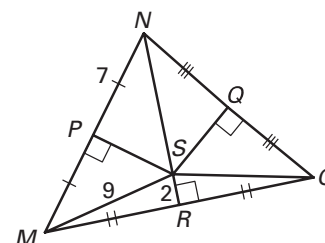
The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If \overline{PD} , \overline{PE} , and \overline{PF} are perpendicular bisectors, then $PA = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.



Example 3 Use the concurrency of perpendicular bisectors

The perpendicular bisectors of $\triangle MNO$ meet at point S. Find SN.



Solution

Using _____, you know that point S is _____ from the vertices of the triangle.

So, _____ = _____ = _____.

_____ = _____

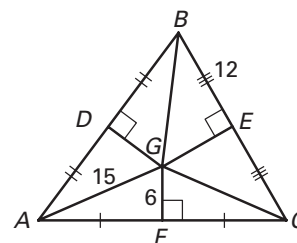
Theorem 5.4

_____ = _____

Substitute.

Checkpoint Complete the following exercise.

3. The perpendicular bisectors of $\triangle ABC$ meet at point G. Find GC.

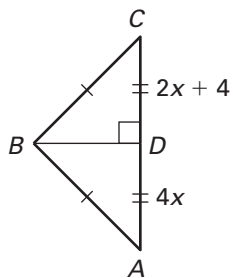


Homework

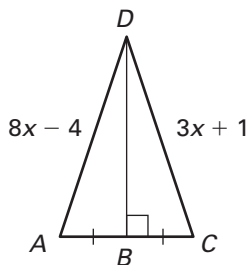
LESSON 5.2 Practice

Find the length of \overline{CD} .

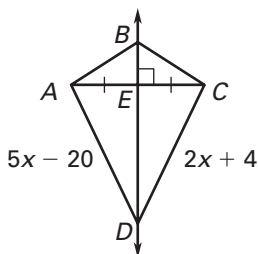
1.



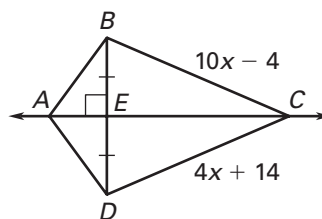
2.



3.



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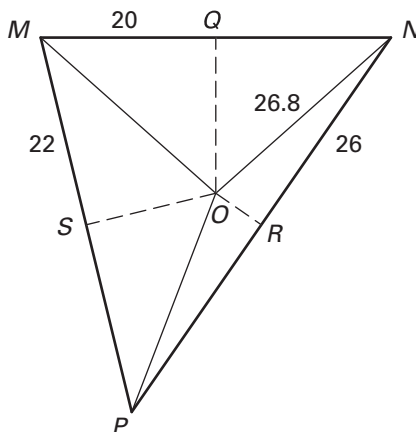


In the diagram, the perpendicular bisectors of $\triangle MNP$ meet at point O and are shown dashed. Find the indicated measure.

5. Find MO .

6. Find PR .

7. Find MN .

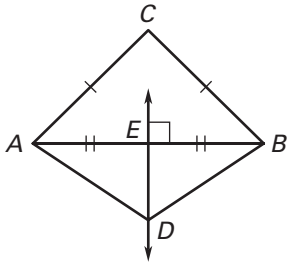


LESSON
5.2

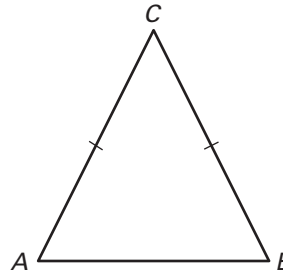
Practice *continued*

Tell whether the information in the diagram allows you to conclude that C is on the perpendicular bisector of \overline{AB} . Explain.

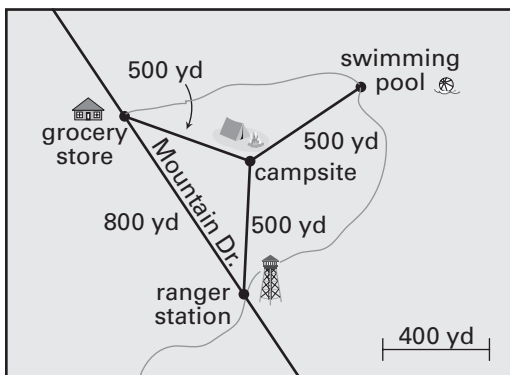
8.



9.



- 10. Camping** Your campsite is located 500 yards from the ranger station, the grocery store, and the swimming pool, as shown on the map. The ranger station and the grocery store are located 800 yards apart along Mountain Drive. How far is your campsite from Mountain Drive?



5.3

Use Angle Bisectors of Triangles



Georgia Performance Standard(s)

MM1G3e

Your Notes

Goal • Use angle bisectors to find distance relationships.

VOCABULARY

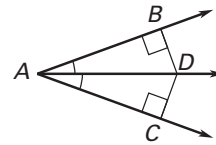
Angle bisector

Incenter

THEOREM 5.5: ANGLE BISECTOR THEOREM

If a point is on the bisector of an angle, then it is equidistant from the two _____ of the angle.

If \overrightarrow{AD} bisects $\angle BAC$ and $\overline{DB} \perp \overline{AB}$ and $\overline{DC} \perp \overline{AC}$, then $DB = \underline{\hspace{1cm}}$.

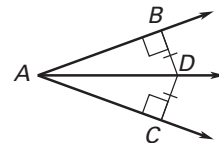


In Geometry, *distance* means the *shortest* length between two objects.

THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the _____ of the angle.

If $\overline{DB} \perp \overline{AB}$ and $\overline{DC} \perp \overline{AC}$ and $DB = DC$, then \overrightarrow{AD} _____ $\angle BAC$.

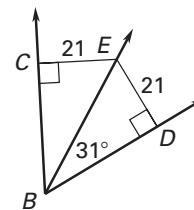


Example 1 Use the Angle Bisector Theorems

Find the measure of $\angle CBE$.

Solution

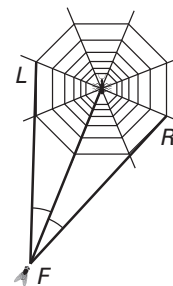
Because $\overline{EC} \perp \underline{\hspace{1cm}}$, $\overline{ED} \perp \underline{\hspace{1cm}}$, and $EC = ED = 21$, \overline{BE} bisects $\angle CBD$ by the _____ . So, $m\angle CBE = m\angle \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.



Your Notes

Example 2 Solve a real-world problem

Web A spider's position on its web relative to an approaching fly and the opposite sides of the web form congruent angles, as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?



Solution

The congruent angles tell you that the spider is on the _____ of $\angle LFR$. By the _____, the spider is equidistant from \overrightarrow{FL} and \overrightarrow{FR} .

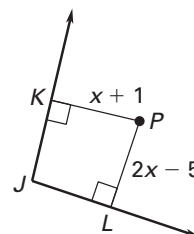
So, the spider must move the _____ to reach each edge.

Example 3 Use algebra to solve a problem

For what value of x does P lie on the bisector of $\angle J$?

Solution

From the Converse of the Angle Bisector Theorem, you know that P lies on the bisector of $\angle J$ if P is equidistant from the sides of $\angle J$, so when _____ = _____.



_____ = _____ Set segment lengths equal.

_____ = _____ Substitute expressions for segment lengths.

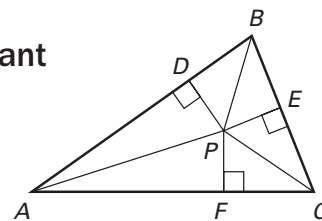
_____ = x Solve for x .

Point P lies on the bisector of $\angle J$ when $x =$ _____.

THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If \overline{AP} , \overline{BP} , and \overline{CP} are angle bisectors of $\triangle ABC$, then $PD =$ _____ = _____.

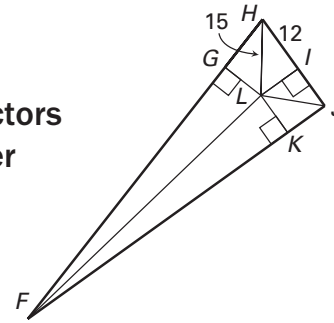


Your Notes

Example 4 Use the concurrency of angle bisectors

In the diagram, L is the incenter of $\triangle FHJ$. Find LK .

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter L is _____ from the sides of $\triangle FHJ$. So, to find LK , you can find ____ in $\triangle LHI$. Use the Pythagorean Theorem.



_____ = _____

Pythagorean Theorem

_____ = _____

Substitute known values.

_____ = _____

Simplify.

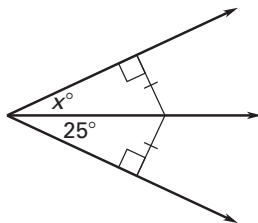
____ = ____

Take the positive square root of each side.

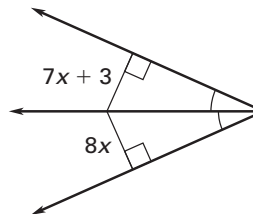
Because _____ = LK , $LK =$ _____.

✔ **Checkpoint** In Exercises 1 and 2, find the value of x .

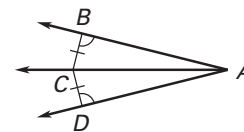
1.



2.



3. Do you have enough information to conclude that \overrightarrow{AC} bisects $\angle DAB$? Explain.



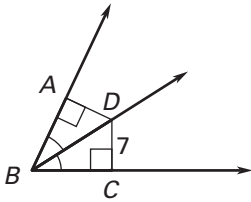
4. In Example 4, suppose you are not given HL or HI , but you are given that $JL = 25$ and $JL = 20$. Find LK .

Homework

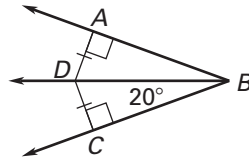
LESSON 5.3 Practice

Use the information in the diagram to find the measure.

1. Find AD .

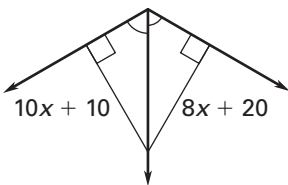


2. Find $m\angle DBA$.

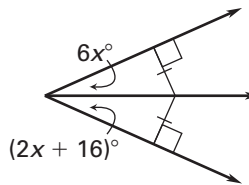


Find the value of x .

- 3.

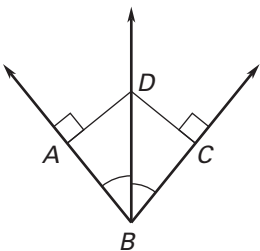


- 4.

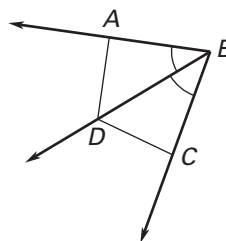


Is $DA = DC$? Explain.

- 5.



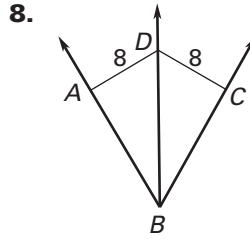
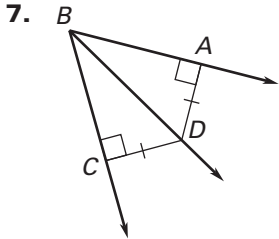
- 6.



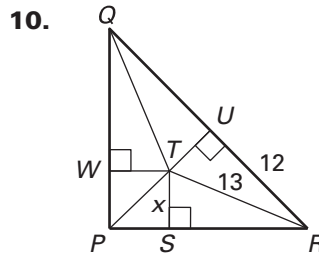
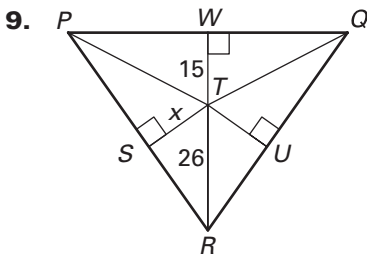
LESSON
5.3

Practice *continued*

Can you conclude that \overline{BD} bisects $\angle ABC$? *Explain.*

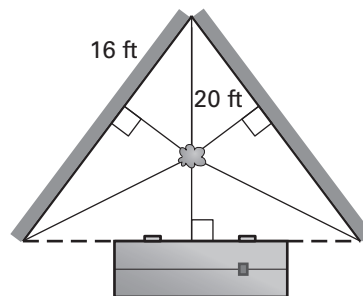


In Exercises 9 and 10, point T is the incenter of $\triangle PQR$. Find the value of x .



11. **Bird Bath** Your neighbor is moving a new bird bath to his triangular back yard. He wants the bird bath to be the same distance from each edge of the yard. Where should your neighbor place the bird bath? *Explain.*

12. **Landscaping** You are planting a tree at the incenter of your triangular front yard. Use the diagram to determine how far the tree is from the house.



5.4

Use Medians and Altitudes



Georgia
Performance
Standard(s)

MM1G1c,
MM1G1e,
MM1G3e

Your Notes

Goal • Use medians and altitudes of triangles.

VOCABULARY

Median of a triangle

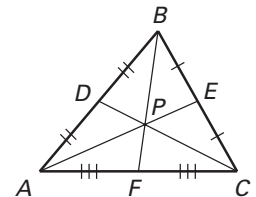
Centroid

Altitude of a triangle

Orthocenter

THEOREM 5.8: CONCURRENCY OF MEDIANS OF A TRIANGLE

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.



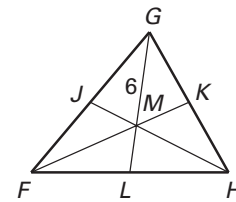
The medians of $\triangle ABC$ meet at P and

$$AP = \frac{2}{3}\text{---}, BP = \frac{2}{3}\text{---}, \text{ and } CP = \frac{2}{3}\text{---}.$$

Your Notes

Example 1 Use the centroid of a triangle

In $\triangle FGH$, M is the centroid and $GM = 6$. Find ML and GL .



_____ = _____ GL Concurrency of Medians of a Triangle Theorem

_____ = _____ GL Substitute _____ for GM .

_____ = GL Multiply each side by the reciprocal, _____.

Then $ML = GL - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.

So, $ML = \underline{\hspace{1cm}}$ and $GL = \underline{\hspace{1cm}}$.

✓ **Checkpoint** Complete the following exercise.

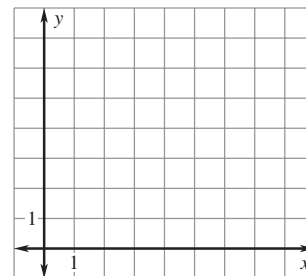
1. In Example 1, suppose $FM = 10$. Find MK and FK .

Example 2 Find the centroid of a triangle

The vertices of $\triangle JKL$ are $J(1, 2)$, $K(4, 6)$, and $L(7, 4)$. Find the coordinates of the centroid P of $\triangle JKL$.

Sketch $\triangle JKL$. Then use the Midpoint Formula to find the midpoint M of \overline{JL} and sketch median \overline{KM} .

$$M\left(\frac{\boxed{\hspace{1cm}}}{2}, \frac{\boxed{\hspace{1cm}}}{2}\right) = \underline{\hspace{2cm}}$$



The centroid is _____ of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex K to point M is $6 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$ units. So, the centroid is

$\underline{\hspace{1cm}}$ ($\underline{\hspace{1cm}}$) = $\underline{\hspace{1cm}}$ units down from K on \overline{KM} .

The coordinates of the centroid P are $(4, 6 - \underline{\hspace{1cm}})$, or $(\underline{\hspace{1cm}})$.

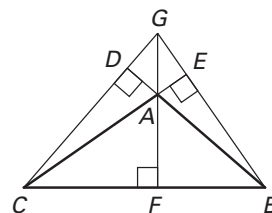
Median \overline{KM} is used in Example 2 because it is easy to find distances on a vertical segment. You can check by finding the centroid using a different median.

Your Notes

THEOREM 5.9: CONCURRENCY OF ALTITUDES OF A TRIANGLE

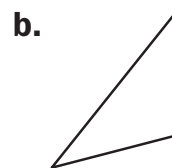
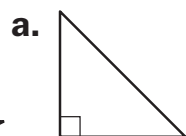
The lines containing the altitudes of a triangle are _____.

The lines containing \overline{AF} , \overline{BE} , and \overline{CD} meet at G .



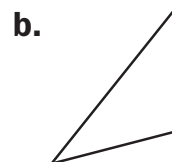
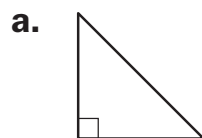
Example 3 Find the orthocenter

Find the orthocenter P of the triangle.



Notice that in a right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

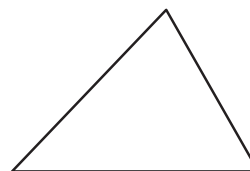
Solution



Checkpoint Complete the following exercises.

2. In Example 2, where do you need to move point K so that the centroid is $P(4, 5)$?

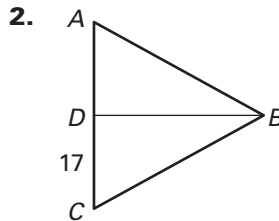
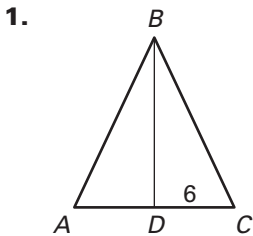
3. Find the orthocenter P of the triangle.



Homework

LESSON 5.4 Practice

\overline{BD} is a median of $\triangle ABC$. Find the length of \overline{AD} .

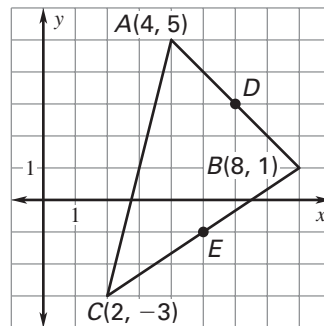


Use the graph shown.

3. Find the coordinates of D , the midpoint of \overline{AB} .

4. Find the length of the median \overline{CD} .

5. Find the coordinates of E , the midpoint of \overline{BC} .



6. Find the length of the median \overline{AE} .

Complete the statement for $\triangle MNP$ with medians \overline{MT} , \overline{NR} , and \overline{PS} , and centroid Q .

7. $QR = \underline{\hspace{1cm}} NR$

8. $MQ = \underline{\hspace{1cm}} MT$

S is the centroid of $\triangle RTW$, $RS = 4$, $VW = 6$, and $TV = 9$. Find the length of the segment.

9. \overline{RV}

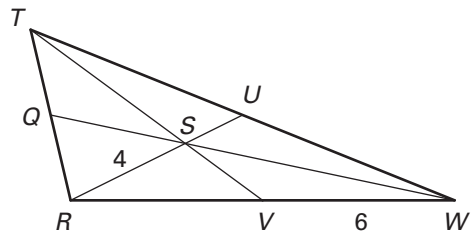
10. \overline{SU}

11. \overline{RU}

12. \overline{RW}

13. \overline{TS}

14. \overline{SV}

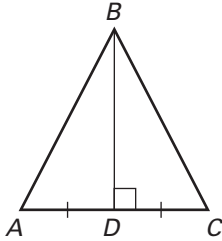


LESSON
5.4

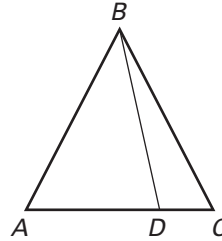
Practice *continued*

Is \overline{BD} a median of $\triangle ABC$? Is \overline{BD} an altitude? Is \overline{BD} a perpendicular bisector?

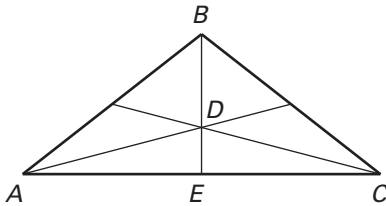
15.



16.



17. **Error Analysis** D is the centroid of $\triangle ABC$. Your friend wants to find DE . The median \overline{BE} has length 24. Describe and correct the error. Explain your reasoning.



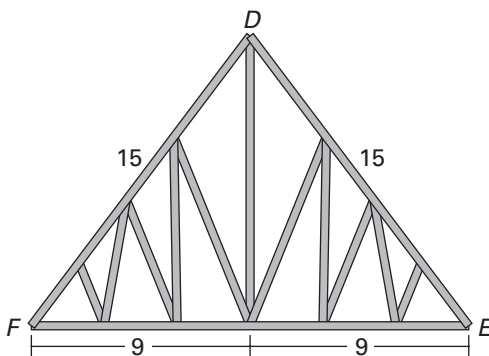
$$DE = \frac{2}{3}BE$$

$$DE = \frac{2}{3}(24)$$

$$DE = 16$$

In Exercises 18 and 19, use the following information.

Roof Trusses Some roofs are built using several triangular wooden trusses.



18. Find the altitude (height) of the truss.

19. How far down from D is the centroid of $\triangle DEF$?

5.5

Use Inequalities in a Triangle



Georgia
Performance
Standard(s)

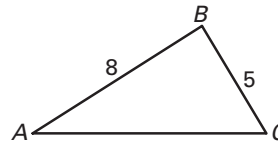
MM1G3b

Your Notes

Goal • Find possible side lengths of a triangle.

THEOREM 5.10

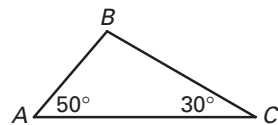
If one side of a triangle is longer than another side, then the angle opposite the longer side is _____ than the angle opposite the shorter side.



$AB > BC$, so
 $m\angle \underline{\quad} > m\angle \underline{\quad}$.

THEOREM 5.11

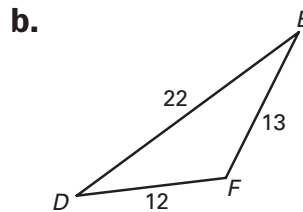
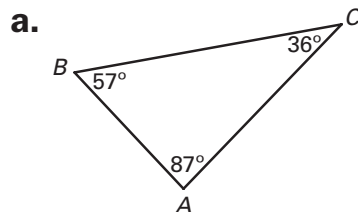
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is _____ than the side opposite the smaller angle.



$m\angle A > m\angle C$,
so $\underline{\quad} > \underline{\quad}$.

Example 1 Write measurements in order from least to greatest

Write the measurements of the triangle in order from least to greatest.



Solution

a. _____ $< m\angle B <$ _____

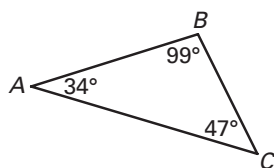
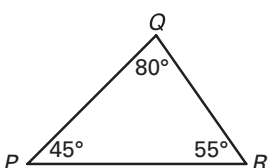
_____ $< AC <$ _____

b. _____ $< m\angle D <$ _____

_____ $< EF <$ _____

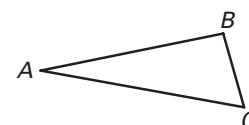
Your Notes

✓ Checkpoint Write the measurements of the triangle in order from least to greatest.

<p>1.</p> 	<p>2.</p> 
--	--

THEOREM 5.12: TRIANGLE INEQUALITY THEOREM

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} > AC$$

$$AC + \underline{\hspace{2cm}} > \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} + AC > \underline{\hspace{2cm}}$$

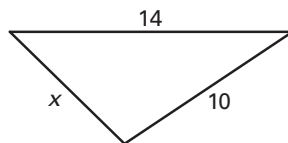
Example 2 Find possible side lengths

A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x . Then use the Triangle Inequality Theorem to write and solve inequalities.

Small values of x



$$x + \underline{\hspace{2cm}} > \underline{\hspace{2cm}}$$

$$x > \underline{\hspace{2cm}}$$

Large values of x

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} > x$$

$$\underline{\hspace{2cm}} > x, \text{ or } x < \underline{\hspace{2cm}}$$

The length of the third side must be _____.

Your Notes

✔ **Checkpoint** Complete the following exercise.

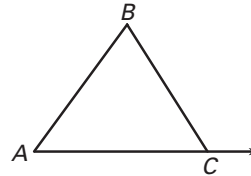
3. A triangle has one side of 23 meters and another of 17 meters. Describe the possible lengths of the third side.

THEOREM 5.13: EXTERIOR ANGLE INEQUALITY THEOREM

The measure of an exterior angle of a triangle is greater than the measure of either of the nonadjacent interior angles.

_____ $> m\angle 2$

_____ $> m\angle 3$

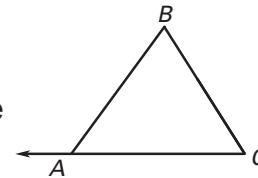


Example 3 Relate exterior and interior angles

Write inequalities that relate $m\angle 1$ to $m\angle B$ and $m\angle C$.

Solution

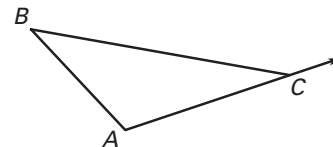
$\angle B$ and $\angle C$ are nonadjacent interior angles to $\angle 1$. So, by the Exterior Angle Inequality Theorem, _____ $> 70^\circ$ and _____ $> \underline{\hspace{1cm}}$.



✔ **Checkpoint** Complete the following exercise.

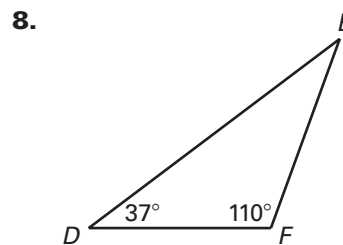
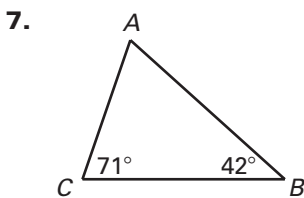
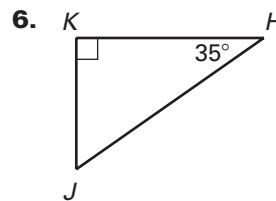
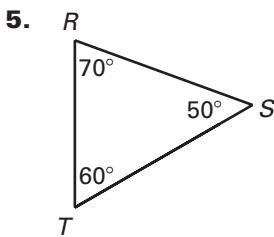
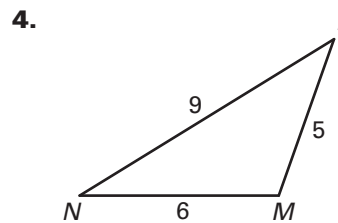
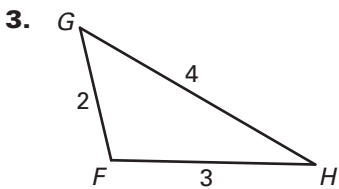
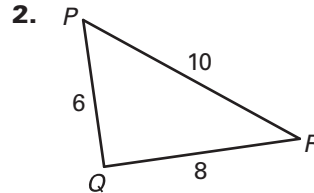
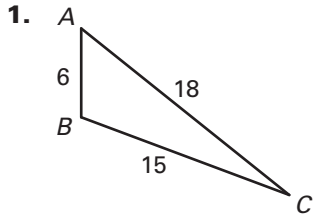
Homework

4. Write inequalities that relate $m\angle 1$ to $m\angle B$ and $m\angle C$.



LESSON
5.5**Practice**

List the sides in order from shortest to longest and the angles in order from smallest to largest.



Use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice?

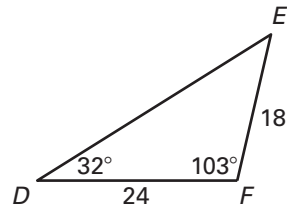
9. Obtuse scalene

10. Right scalene

LESSON
5.5**Practice** *continued*

For Exercises 11 and 12, use the following diagram.

11. Name the smallest and largest angles of $\triangle DEF$.



12. Name the shortest and longest sides of $\triangle DEF$.

**Is it possible to construct a triangle with the given side lengths?
If not, explain why not.**

13. 6, 10, 15

14. 11, 16, 32

Describe the possible lengths of the third side of the triangle given the lengths of the other two sides.

15. 12 in., 6 in.

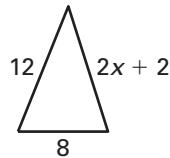
16. 3 ft, 8 ft

17. 12 cm, 17 cm

18. 7 yd, 13 yd

LESSON
5.5**Practice** *continued*

19. Describe the possible values of x .



In Exercises 20–22, you are given a 12-inch piece of wire. You want to bend the wire to form a triangle so that the length of each side is a whole number.

20. Sketch two possible isosceles triangles and label each side length.
21. Sketch a possible scalene triangle.
22. List two combinations of segment lengths that will not produce triangles.
23. **Distance** Union Falls is 60 miles NE of Harnedville. Titus City is 40 miles SE of Harnedville. Is it possible that Union Falls and Titus City are less than 100 miles apart? *Justify* your answer.

5.6

Inequalities in Two Triangles and Indirect Proof



Georgia Performance Standard(s)

MM1G2a

Your Notes

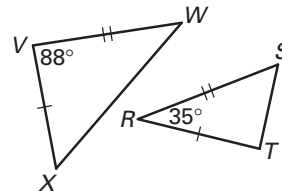
- Goal** • Use inequalities to make comparisons in two triangles.

VOCABULARY

Indirect Proof

THEOREM 5.14: HINGE THEOREM

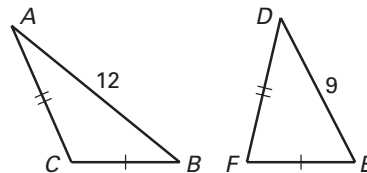
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is _____ than the third side of the second.



$$WX > \underline{\hspace{1cm}}$$

THEOREM 5.15: CONVERSE OF THE HINGE THEOREM

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is _____ than the included angle of the second.



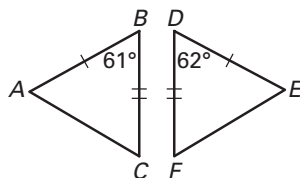
$$m\angle C > m\angle \underline{\hspace{1cm}}$$

Your Notes

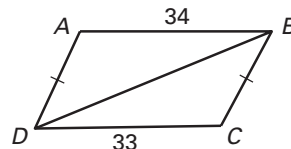
Example 1 Use the Hinge Theorem and its converse

Complete the statement with $<$, $>$, or $=$. Explain.

a. AC $\underline{\quad?}$ EF



b. $m\angle ADB$ $\underline{\quad?}$ $m\angle CBD$



Solution

a. You are given that $\overline{AB} \cong$ _____ and $\overline{BC} \cong$ _____. Because $61^\circ <$ _____, by the Hinge Theorem, $AC <$ _____.

b. You are given that $AD \cong$ _____ and you know that $\overline{BD} \cong$ _____ by the Reflexive Property. Because $34 > 33$, _____ $>$ _____. So, by the Converse of the Hinge Theorem, _____ $>$ _____.

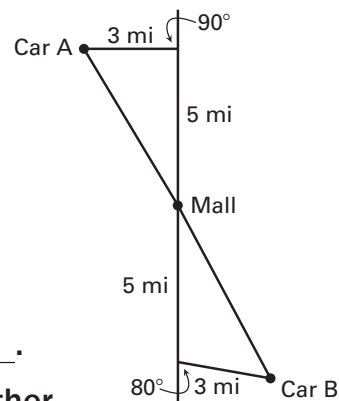
Example 2 Solve a multi-step problem

Travel Car A leaves a mall, heads due north for 5 mi and then turns due west for 3 mi. Car B leaves the same mall, heads due south for 5 mi and then turns 80° toward east for 3 mi. Which car is farther from the mall?

Draw a diagram. The distance driven and the distance back to the mall form two triangles, with _____ 5 mile sides and _____ 3 mile sides. Add the third side to the diagram.

Use linear pairs to find the included angles of _____ and _____.

Because $100^\circ > 90^\circ$, Car _____ is farther from the mall than Car A by the _____.



Your Notes

Example 3 Write an indirect proof

Write an indirect proof to show that an odd number is not divisible by 6.

Given x is an odd number.

Prove x is not divisible by 6.

Solution

Step 1 Assume temporarily that _____.

This means that $\underline{\hspace{2cm}} = n$ for some whole number n .

So, multiplying both sides by 6 gives $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

Step 2 If x is odd, then, by definition, x cannot be divided evenly by $\underline{\hspace{2cm}}$. However, $\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ so

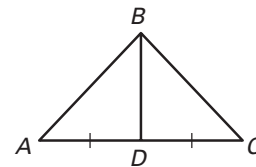
$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$. We know that $\underline{\hspace{2cm}}$ is a whole number because n is a whole number, so x can be divided evenly by $\underline{\hspace{2cm}}$. This contradicts the given statement that _____.

Step 3 Therefore, the assumption that x is divisible by 6 is _____, which proves that _____.

You have reached a contradiction when you have two statements that cannot both be true at the same time.

✓ Checkpoint Complete the following exercises.

1. If $m\angle ADB > m\angle CDB$ which is longer, \overline{AB} or \overline{CB} ?



2. In Example 2, car C leaves the mall and goes 5 miles due west, then turns 85° toward south for 3 miles. Write the cars in order from the car closest to the mall to the car farthest from the mall.

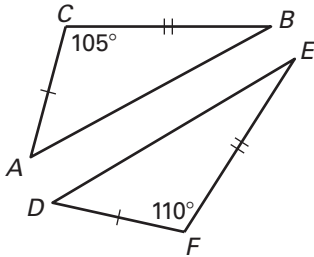
3. Suppose you want to prove the statement "If $x + y \neq 5$ and $y = 2$, then $x \neq 3$." What temporary assumption could you make to prove the conclusion indirectly?

Homework

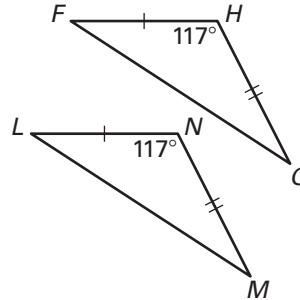
LESSON 5.6 Practice

Complete with $<$, $>$, or $=$.

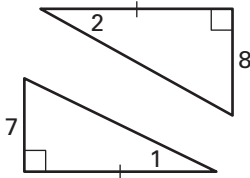
1. AB ____ DE



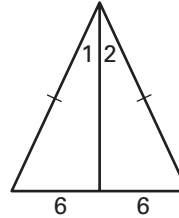
2. FG ____ LM



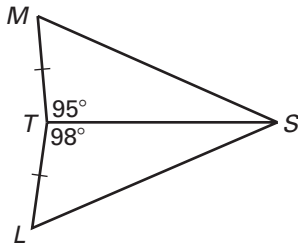
3. $m\angle 1$ ____ $m\angle 2$



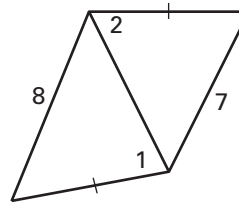
4. $m\angle 1$ ____ $m\angle 2$



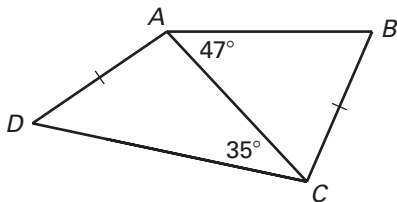
5. MS ____ LS



6. $m\angle 1$ ____ $m\angle 2$



7. **Error Analysis** Explain why the student's reasoning is not correct.



By the Hinge Theorem, $AB > DC$.

LESSON
5.6

Practice *continued*

Match the conclusion on the right with the given information.

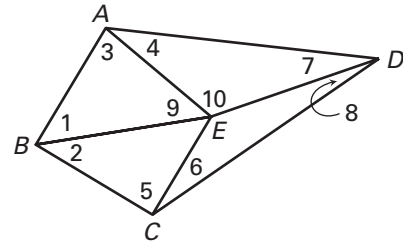
Explain your reasoning.

8. $AB = BC, m\angle 1 > m\angle 2$ **A.** $m\angle 7 > m\angle 8$

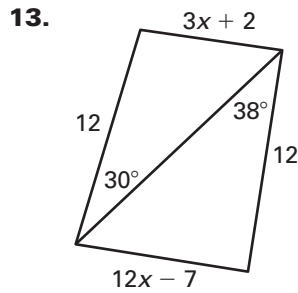
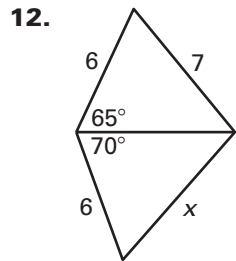
9. $AE > EC, AD = CD$ **B.** $AD > AB$

10. $m\angle 9 < m\angle 10, BE = ED$ **C.** $m\angle 3 + m\angle 4 = m\angle 5 + m\angle 6$

11. $AB = BC, AD = CD$ **D.** $AE > EC$

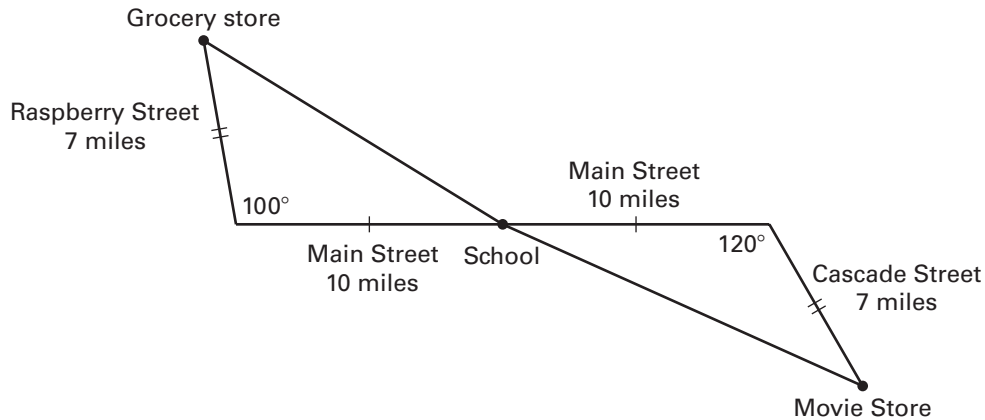


Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of x .



LESSON
5.6
Practice *continued*

- 14. Shopping** You and a friend are going shopping. You leave school and drive 10 miles due west on Main Street. You then drive 7 miles NW on Raspberry Street to the grocery store. Your friend leaves school and drives 10 miles due east on Main Street. He then drives 7 miles SE on Cascade Street to the movie store. Each of you has driven 17 miles. Which of you is farther from your school?



- 15.** Write the first statement for an indirect proof of the situation. In $\triangle MNO$, if \overline{MP} is perpendicular to \overline{NO} , then \overline{MP} is an altitude.

5.7

Find Angle Measures in Polygons



Georgia Performance Standard(s)

MM1G3a

Your Notes

Goal • Find angle measures in polygons.

VOCABULARY

Diagonal

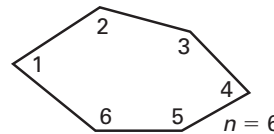
Interior angles of a polygon

Exterior angles of a polygon

THEOREM 5.16: POLYGON INTERIOR ANGLES THEOREM

The sum of the measures of the interior angles of a convex n -gon is $(n - \underline{\quad}) \cdot \underline{\quad}$.

$$m\angle 1 + m\angle 2 + \dots + m\angle n = (n - \underline{\quad}) \cdot \underline{\quad}$$



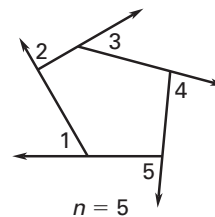
COROLLARY TO THEOREM 5.16: INTERIOR ANGLES OF A QUADRILATERAL

The sum of the measures of the interior angles of a quadrilateral is $\underline{\quad}$.

THEOREM 5.17: POLYGON EXTERIOR ANGLES THEOREM

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is $\underline{\quad}$.

$$m\angle 1 + m\angle 2 + \dots + m\angle n = \underline{\quad}$$



Your Notes

Example 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex octagon.



Solution

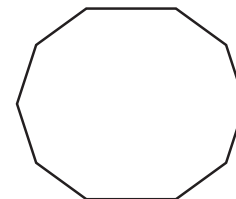
A octagon has ___ sides. Use the Polygon Interior Angles Theorem.

$$\begin{aligned}
 (n - \underline{\quad}) \cdot \underline{\quad} &= (\underline{\quad} - \underline{\quad}) \cdot \underline{\quad} && \text{Substitute} \\
 & && \text{for } n. \\
 &= \underline{\quad} \cdot \underline{\quad} && \text{Subtract.} \\
 &= \underline{\quad} && \text{Multiply.}
 \end{aligned}$$

The sum of the measures of the interior angles of a hexagon is _____.

✓ Checkpoint Complete the following exercise.

- Find the sum of the measures of the interior angles of the convex decagon.



Example 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 1260° . Classify the polygon by the number of sides.

Solution

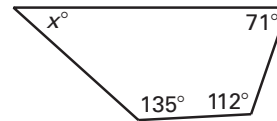
Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n . Then solve the equation to find the number of sides.

$$\begin{aligned}
 (n - \underline{\quad}) \cdot \underline{\quad} &= \underline{\quad} && \text{Polygon Interior Angles} \\
 & && \text{Theorem} \\
 n - \underline{\quad} &= \underline{\quad} && \text{Divide each side by } \underline{\quad}. \\
 n &= \underline{\quad} && \text{Add } \underline{\quad} \text{ to each side.}
 \end{aligned}$$

The polygon has ___ sides. It is a _____.

Example 3 Find an unknown interior angle measure

Find the value of x in the diagram shown.



Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x . Then solve the equation.

$$x^\circ + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Corollary to Theorem 5.16

$$x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Combine like terms.

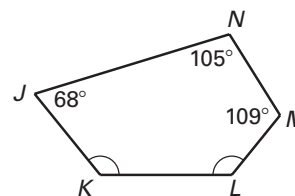
$$x = \underline{\hspace{1cm}}$$

Subtract $\underline{\hspace{1cm}}$ from each side.

Checkpoint Complete the following exercises.

2. The sum of the measures of the interior angles of a convex polygon is 1620° . Classify the polygon by the number of sides.

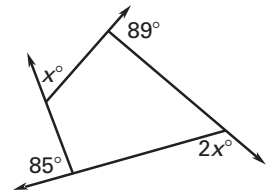
3. Use the diagram at the right. Find $m\angle K$ and $m\angle L$.



Your Notes

Example 4 Find unknown exterior angle measures

Find the value of x in the diagram shown.



Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$$\begin{aligned}x^\circ + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} &= \underline{\hspace{1cm}} && \text{Polygon Exterior Angles Theorem} \\ \underline{\hspace{1cm}}x + \underline{\hspace{1cm}} &= \underline{\hspace{1cm}} && \text{Combine like terms.} \\ x &= \underline{\hspace{1cm}} && \text{Solve for } x.\end{aligned}$$

✔ **Checkpoint** Complete the following exercises.

4. A convex pentagon has exterior angles with measures 66° , 77° , 82° , and 62° . What is the measure of an exterior angle at the fifth vertex?

5. Find the measure of (a) each interior angle and (b) each exterior angle of a regular nonagon.

Homework

LESSON
5.7

Practice

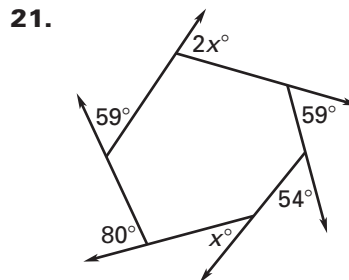
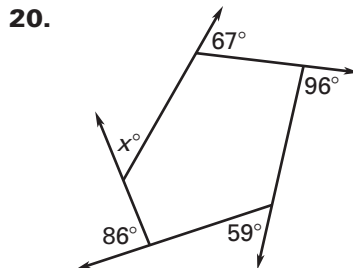
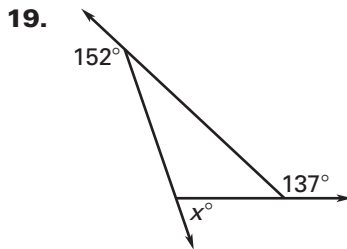
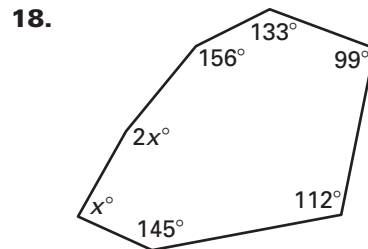
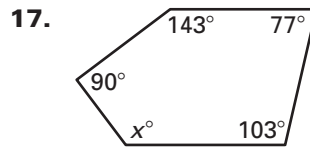
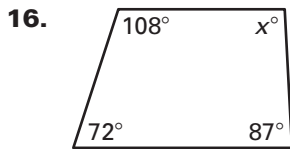
Find the sum of the measures of the interior angles of the indicated convex polygon.

- | | | |
|-------------|-----------|-----------|
| 1. Heptagon | 2. 13-gon | 3. 17-gon |
| 4. 18-gon | 5. 22-gon | 6. 25-gon |
| 7. 30-gon | 8. 34-gon | 9. 39-gon |

The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.

- | | | |
|------------------|------------------|------------------|
| 10. 1260° | 11. 2160° | 12. 3240° |
| 13. 4680° | 14. 5400° | 15. 7560° |

Find the value of x .



22. The measures of the interior angles of a convex quadrilateral are x° , $2x^\circ$, $4x^\circ$, and $5x^\circ$. What is the measure of the largest interior angle?
23. The measures of the exterior angles of a convex pentagon are $2x^\circ$, $4x^\circ$, $6x^\circ$, $8x^\circ$, and $10x^\circ$. What is the measure of the smallest exterior angle?

LESSON
5.7**Practice** *continued*

Find the measures of an interior angle and an exterior angle of the indicated regular polygon.

24. Regular hexagon 25. Regular decagon 26. Regular 15-gon
27. Regular 20-gon 28. Regular 30-gon 29. Regular 36-gon

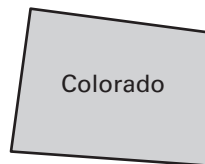
In Exercises 30–37, find the value of n for each regular n -gon described.

30. Each interior angle of the regular n -gon has a measure of 90° .
31. Each interior angle of the regular n -gon has a measure of 108° .
32. Each interior angle of the regular n -gon has a measure of 135° .
33. Each interior angle of the regular n -gon has a measure of 144° .
34. Each exterior angle of the regular n -gon has a measure of 90° .
35. Each exterior angle of the regular n -gon has a measure of 60° .
36. Each exterior angle of the regular n -gon has a measure of 40° .
37. Each exterior angle of the regular n -gon has a measure of 30° .

LESSON
5.7**Practice** *continued*

38. Geography The shape of Colorado can be approximated by a polygon, as shown.

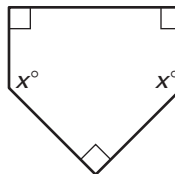
a. How many sides does the polygon have? Classify the polygon.



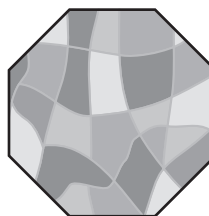
b. What is the sum of the measures of the interior angles of the polygon?

c. What is the sum of the measures of the exterior angles of the polygon?

39. Softball A home plate marker for a softball field is a pentagon, as shown. Three of the interior angles of the pentagon are right angles and the remaining two interior angles are congruent. What is the value of x ?



40. Stained Glass Window Part of a stained-glass window is a regular octagon, as shown. Find the measure of an interior angle of the regular octagon. Then find the measure of an exterior angle.



5.8

Use Properties of Parallelograms



Georgia Performance Standard(s)

MM1G1e,
MM1G3d

Your Notes

Goal • Find angle and side measures in parallelograms.

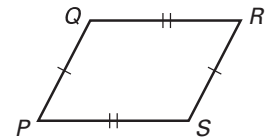
VOCABULARY

Parallelogram

THEOREM 5.18

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

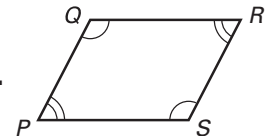
If $PQRS$ is a parallelogram, then _____ $\cong \overline{RS}$ and $\overline{QR} \cong$ _____.



THEOREM 5.19

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong$ _____ and _____ $\cong \angle S$.

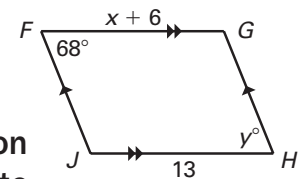


Example 1 Use properties of parallelograms

Find the values of x and y .

Solution

$FGHJ$ is a parallelogram by the definition of a parallelogram. Use Theorem 5.18 to find the value of x .



$FG =$ _____ Opposite sides of a \square are \cong .

$x + 6 =$ _____ Substitute $x + 6$ for FG and _____ for _____.

$x =$ _____ Subtract 6 from each side.

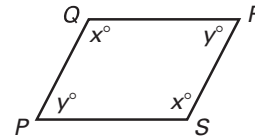
By Theorem 5.19, $\angle F \cong$ _____, or $m\angle F =$ _____. So, $y^\circ =$ _____.

In $\square FGHJ$, $x =$ _____ and $y =$ _____.

Your Notes

THEOREM 5.20

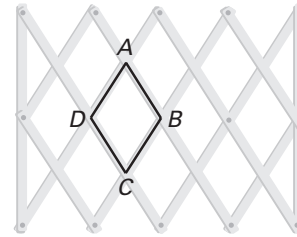
If a quadrilateral is a parallelogram, then its consecutive angles are _____.



If $PQRS$ is a parallelogram, then $x^\circ + y^\circ = \underline{\hspace{2cm}}$.

Example 2 Use properties of a parallelogram

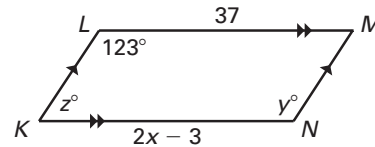
Gates As shown, a gate contains several parallelograms. Find $m\angle ADC$ when $m\angle DAB = 65^\circ$.



Solution

By Theorem 5.20, the consecutive angle pairs in $\square ABCD$ are _____. So, $m\angle ADC + m\angle DAB = \underline{\hspace{2cm}}$. Because $m\angle DAB = 65^\circ$, $m\angle ADC = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.

✓ **Checkpoint** Find the indicated measure in $\square KLMN$ shown at the right.



1. x

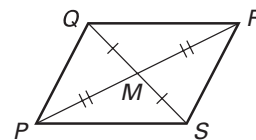
2. y

3. z

Your Notes

THEOREM 5.21

If a quadrilateral is a parallelogram, then its diagonals _____ each other.

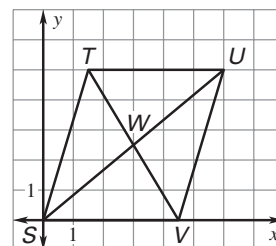


$$\overline{QM} \cong \underline{\hspace{2cm}} \text{ and}$$

$$\overline{PM} \cong \underline{\hspace{2cm}}$$

Example 3 Find the intersection of diagonals

The diagonals of $\square STUV$ intersect at point W . Find the coordinates of W .



Solution

By Theorem 5.21, the diagonals of a parallelogram _____ each other.

So, W is the _____ of the diagonals \overline{TV} and \overline{SU} .

Use the _____.

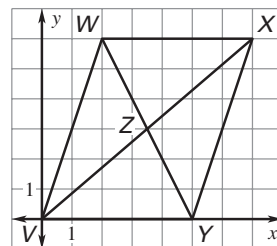
Coordinates of midpoint W of

$$\overline{SU} = \left(\underline{\hspace{2cm}} \right) = \left(\underline{\hspace{2cm}} \right)$$

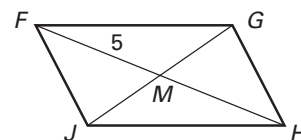
In Example 3, you can use either diagonal to find the coordinates of W . Using \overline{SU} simplifies calculations because one endpoint is $(0, 0)$.

✓ Checkpoint Complete the following exercises.

4. The diagonals of $\square VWXY$ intersect at point Z . Find the coordinates of Z .



5. Given that $\square FGHI$ is a parallelogram, find MH and FH .



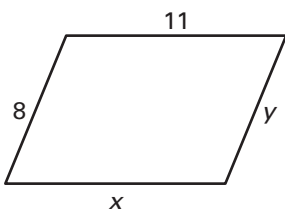
Homework

LESSON
5.8

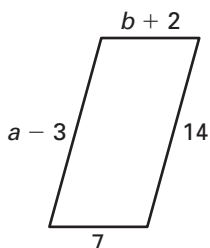
Practice

Find the value of each variable in the parallelogram.

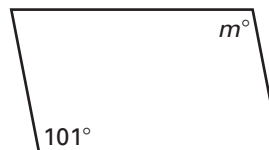
1.



2.



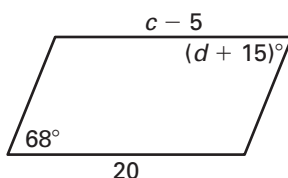
3.



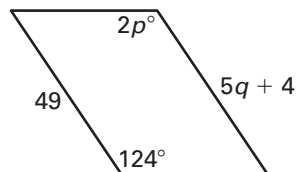
4.



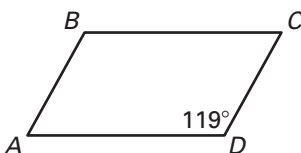
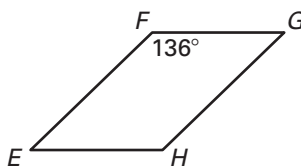
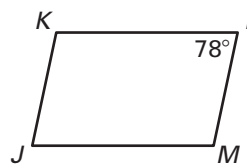
5.



6.



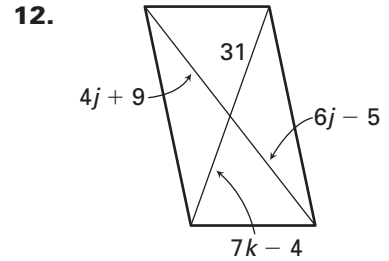
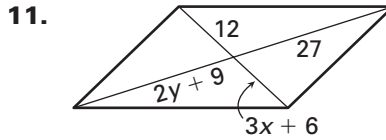
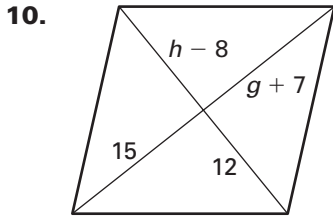
Find the measure of the indicated angle in the parallelogram.

7. Find $m\angle C$.8. Find $m\angle E$.9. Find $m\angle K$.

LESSON
5.8

Practice *continued*

Find the value of each variable in the parallelogram.



Use the diagram of parallelogram *MNOP* at the right to complete the statement. *Explain.*

13. $\overline{MN} \cong$ _____

14. $\overline{MN} \parallel$ _____

15. $\overline{ON} \cong$ _____

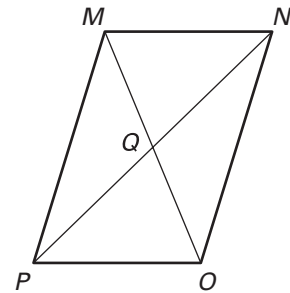
16. $\angle MPO \cong$ _____

17. $\overline{PQ} \cong$ _____

18. $\overline{QM} \cong$ _____

19. $\angle MQN \cong$ _____

20. $\angle NPO \cong$ _____



Find the indicated measure in $\square HIJK$. *Explain.*

21. HI

22. KH

23. GH

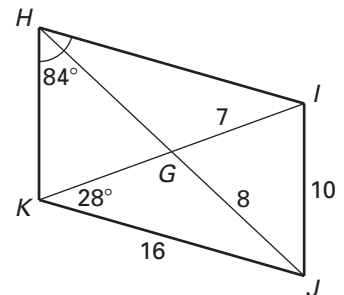
24. HJ

25. $m\angle KIH$

26. $m\angle JIH$

27. $m\angle KJI$

28. $m\angle HKI$



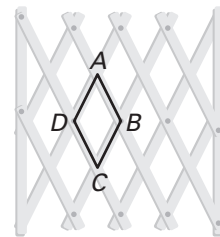
LESSON
5.8

Practice *continued*

- 29. The measure of one interior angle of a parallelogram is twice the measure of another angle. Find the measure of each angle.

- 30. The measure of one interior angle of a parallelogram is 30 degrees more than the measure of another angle. Find the measure of each angle.

The crossing slats of a gate form parallelograms that move together to make the gate wider. In Exercises 31–34, use the figure at the right.



- 31. What is $m\angle A$ when $m\angle B = 110^\circ$?

- 32. What is $m\angle D$ when $m\angle B = 130^\circ$?

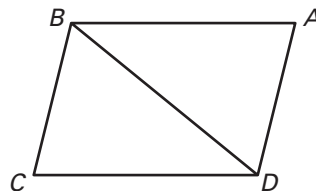
- 33. What happens to $m\angle A$ when $m\angle B$ decreases?

- 34. What happens to AC when $m\angle B$ increases?

35. Complete the proof.

GIVEN: $ABCD$ is a \square .

PROVE: $\triangle ABD \cong \triangle CDB$



Statements	Reasons
1. $ABCD$ is a \square .	1.
2.	2. Opposite sides of \square are \cong .
3.	3. Opposite sides of \square are \cong .
4. $\angle A \cong \angle C$	4.
5. $\triangle ABD \cong \triangle CBD$	5.

5.9

Show that a Quadrilateral is a Parallelogram



Georgia Performance Standard(s)

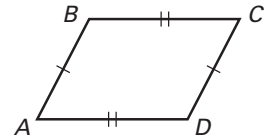
MM1G1a,
MM1G1e,
MM1G3d

Your Notes

Goal • Use properties to identify parallelograms.

THEOREM 5.22

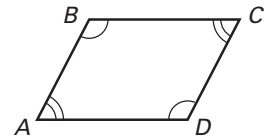
If both pairs of opposite _____ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

THEOREM 5.23

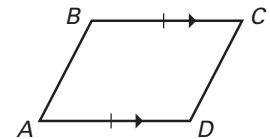
If both pairs of opposite _____ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

THEOREM 5.24

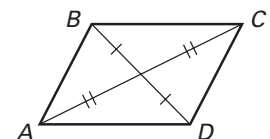
If one pair of opposite sides of a quadrilateral are _____ and _____, then the quadrilateral is a parallelogram.



If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

THEOREM 5.25

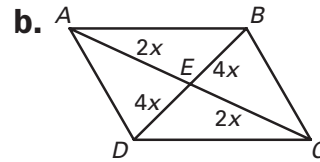
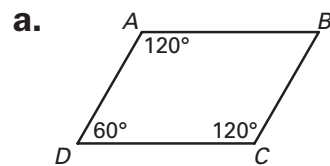
If the diagonals of a quadrilateral _____ each other, then the quadrilateral is a parallelogram.



If \overline{BD} and \overline{AC} _____ each other, then $ABCD$ is a parallelogram.

Example 1 Identify parallelograms

Explain how you know that quadrilateral $ABCD$ is a parallelogram.



- a. By the _____ you know that $m\angle A + m\angle B + m\angle C + m\angle D = \underline{\hspace{2cm}}$, so $m\angle B = \underline{\hspace{2cm}}$. Because both pairs of opposite angles are _____, then $ABCD$ is a parallelogram by _____.
- b. In the diagram, $AE = \underline{\hspace{2cm}}$ and $BE = \underline{\hspace{2cm}}$. So, the diagonals bisect each other, and $ABCD$ is a parallelogram by _____.

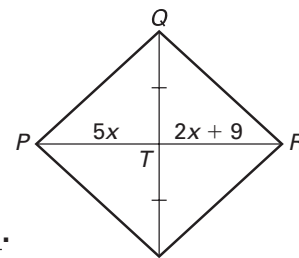
✓ **Checkpoint** Complete the following exercise.

1. In quadrilateral $GHJK$, $m\angle G = 55^\circ$, $m\angle H = 125^\circ$, and $m\angle J = 55^\circ$. Find $m\angle K$. What theorem can you use to show that $GHJK$ is a parallelogram?

Example 2 Use algebra with parallelograms

For what value of x is quadrilateral $PQRS$ a parallelogram?

By Theorem 5.25, if the diagonals of $PQRS$ _____ each other, then it is a parallelogram. You are given that $QT \cong \underline{\hspace{2cm}}$. Find x so that $PT \cong \underline{\hspace{2cm}}$.



- $PT = \underline{\hspace{2cm}}$ Set the segment lengths equal. ^S
- $5x = \underline{\hspace{2cm}}$ Substitute for PT and for _____.
- $\underline{\hspace{2cm}}x = \underline{\hspace{2cm}}$ Subtract _____ from each side.
- $x = \underline{\hspace{2cm}}$ Divide each side by _____.

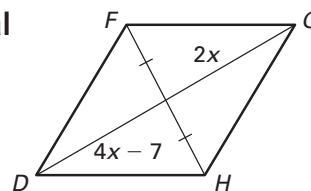
When $x = \underline{\hspace{2cm}}$, $PT = 5(\underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$ and $RT = 2(\underline{\hspace{2cm}}) + 9 = \underline{\hspace{2cm}}$.

Quadrilateral $PQRS$ is a parallelogram when $x = \underline{\hspace{2cm}}$.

Your Notes

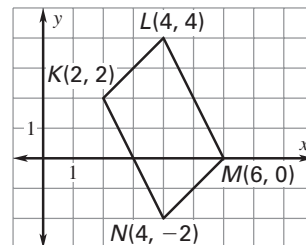
✔ **Checkpoint** Complete the following exercise.

2. For what value of x is quadrilateral $DFGH$ a parallelogram?



Example 3 Use coordinate geometry

Show that quadrilateral $KLMN$ is a parallelogram.



One way is to show that a pair of sides are congruent and parallel. Then apply _____.

First use the Distance Formula to show that \overline{KL} and \overline{MN} are _____.

$$KL = \sqrt{\quad} = \sqrt{\quad}$$

$$MN = \sqrt{\quad} = \sqrt{\quad}$$

Because $KL = MN = \sqrt{\quad}$, $\overline{KL} \cong \overline{MN}$.

Then use the slope formula to show that $\overline{KL} \parallel \overline{MN}$.

$$\text{Slope of } \overline{KL} = \frac{\boxed{\quad}}{\boxed{\quad}} = \underline{\quad}$$

$$\text{Slope of } \overline{MN} = \frac{\boxed{\quad}}{\boxed{\quad}} = \underline{\quad}$$

\overline{KL} and \overline{MN} have the same slope, so they are _____.

\overline{KL} and \overline{MN} are congruent and parallel. So, $KLMN$ is a parallelogram by _____.

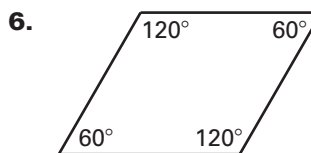
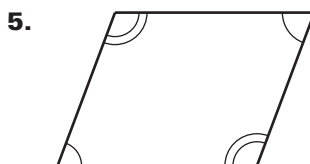
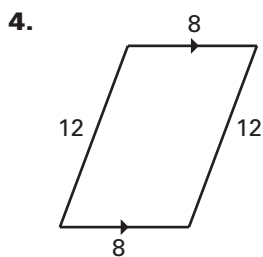
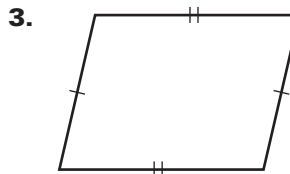
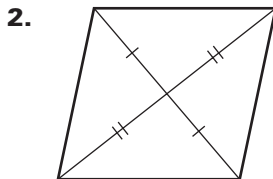
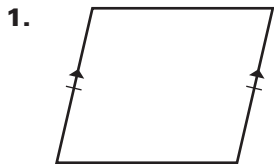
✔ **Checkpoint** Complete the following exercise.

Homework

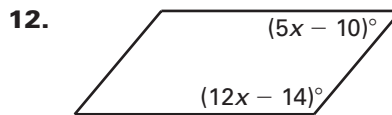
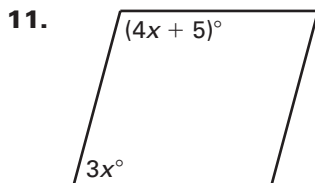
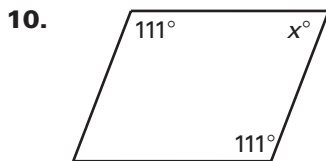
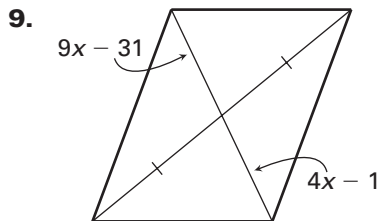
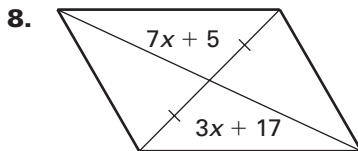
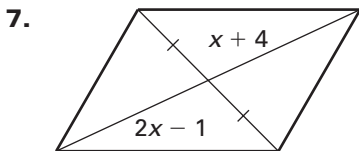
3. Explain another method that can be used to show that quadrilateral $KLMN$ in Example 3 is a parallelogram.

LESSON 5.9 Practice

What theorem can you use to show that the quadrilateral is a parallelogram?



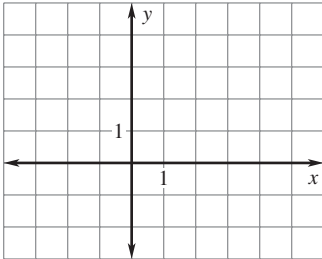
For what value of x is the quadrilateral a parallelogram?



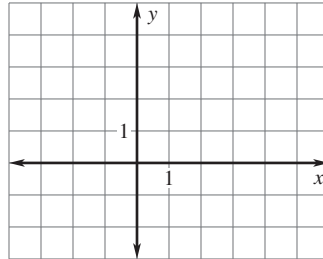
LESSON
5.9
Practice *continued*

The vertices of quadrilateral $ABCD$ are given. Draw $ABCD$ in a coordinate plane and show that it is a parallelogram.

13. $A(-1, 3), B(4, 3), C(2, -1), D(-3, -1)$



14. $A(-2, 3), B(3, 2), C(3, -1), D(-2, 0)$



What additional information is needed in order to prove that quadrilateral $ABCD$ is a parallelogram?

15. $\overline{AB} \parallel \overline{DC}$

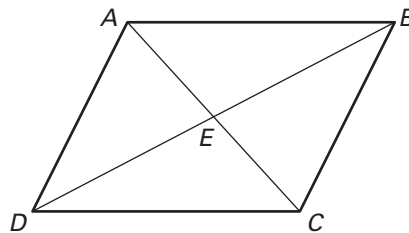
16. $\overline{AB} \cong \overline{DC}$

17. $\angle DCB \cong \angle DAB$

18. $\overline{DE} \cong \overline{EB}$

19. $m\angle CDA + m\angle DAB = 180^\circ$

20. $\angle DCA \cong \angle BAC$

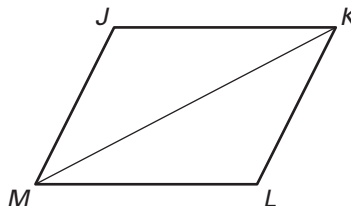


LESSON
5.9**Practice** *continued*

In Exercises 21 and 22, use the diagram below to complete the proof using two different methods.

GIVEN: $\triangle MJK \cong \triangle KLM$

PROVE: $MJKL$ is a parallelogram.



21. Statements	Reasons
1.	1. Given
2. $\overline{JK} \cong \overline{LM}$ $\overline{JM} \cong \overline{LK}$	2.
3. $MJKL$ is a \square .	3.

22. Statements	Reasons
1.	1. Given
2. $\overline{JK} \cong \overline{LM}$ $\angle JKM \cong \angle KML$	2.
3.	3. Alternate Interior \angle 's Converse
4. $MJKL$ is a \square .	4.

5.10

Properties of Rhombuses, Rectangles, and Squares



Georgia
Performance
Standard(s)

MM1G3d

Your Notes

- Goal** • Use properties of rhombuses, rectangles, and squares.

VOCABULARY

Rhombus

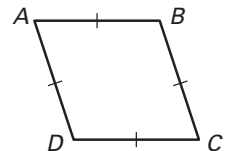
Rectangle

Square

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent _____.

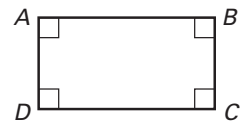
$ABCD$ is a rhombus if and only if
 $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four _____.

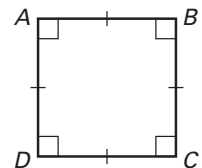
$ABCD$ is a rectangle if and only if
 $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



SQUARE COROLLARY

A quadrilateral is a square if and only if it is a _____ and a _____.

$ABCD$ is a square if and only if
 $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and
 $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



Example 1 Use properties of special quadrilaterals

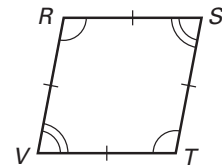
For any rhombus $RSTV$, decide whether the statement is always or sometimes true. Draw a sketch and explain your reasoning.

a. $\angle S \cong \angle V$

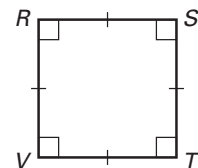
b. $\angle T \cong \angle V$

Solution

a. By definition, a rhombus is a parallelogram with four congruent _____. By Theorem 5.19, opposite angles of a parallelogram are _____. So, $\angle S \cong \angle V$. The statement is _____ true.

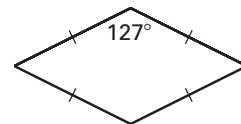


b. If rhombus $RSTV$ is a _____, then all four angles are congruent right angles. So $\angle T \cong \angle V$ if $RSTV$ is a _____. Because not all rhombuses are also _____, the statement is _____ true.



Example 2 Classify special quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



The quadrilateral has four congruent _____. One of the angles is not a _____, so the rhombus is not also a _____. By the Rhombus Corollary, the quadrilateral is a _____.

✔ **Checkpoint** Complete the following exercises.

1. For any square $CDEF$, is it always or sometimes true that $\overline{CD} \cong \overline{DE}$? Explain your reasoning.

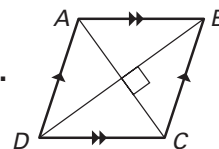
2. A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

Your Notes

THEOREM 5.26

A parallelogram is a rhombus if and only if its diagonals are _____.

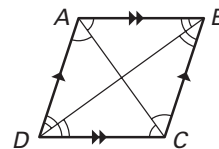
$\square ABCD$ is a rhombus if and only if _____ \perp _____.



THEOREM 5.27

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

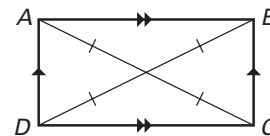
$\square ABCD$ is a rhombus if and only if \overline{AC} bisects \angle _____ and \angle _____ and \overline{BD} bisects \angle _____ and \angle _____.



THEOREM 5.28

A parallelogram is a rectangle if and only if its diagonals are _____.

$\square ABCD$ is a rectangle if and only if _____ \cong _____.



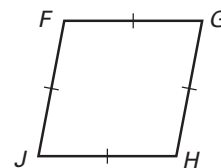
Example 3 List properties of special parallelograms

Sketch rhombus $FGHJ$. List everything you know about it.

Solution

By definition, you need to draw a figure with the following properties:

- The figure is a _____.
- The figure has four congruent _____.



Because $FGHJ$ is a parallelogram, it has these properties:

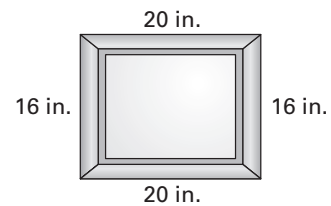
- Opposite sides are _____ and _____.
- Opposite angles are _____. Consecutive angles are _____.
- Diagonals _____ each other.

By Theorem 5.26, the diagonals of $FGHJ$ are _____. By Theorem 5.27, each diagonal bisects a pair of _____.

Your Notes

Example 4 Solve a real-world problem

Framing You are building a frame for a painting. The measurements of the frame are shown at the right.



- The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain.*
- You measure the diagonals of the frame. The diagonals are about 25.6 inches. What can you conclude about the shape of the frame?

Solution

- No, you cannot. The boards on opposite sides are the same length, so they form a _____. But you do not know whether the angles are _____.
- By Theorem 5.28, the diagonals of a rectangle are _____. The diagonals of the frame are _____, so the frame forms a _____.

✔ **Checkpoint** Complete the following exercises.

- Sketch rectangle $WXYZ$. List everything that you know about it.

Homework

- Suppose the diagonals of the frame in Example 4 are not congruent. Could the frame still be a rectangle? *Explain.*

LESSON
5.10

Practice

For any rhombus $ABCD$, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

1. $\angle A \cong \angle C$

2. $\overline{DA} \cong \overline{AB}$

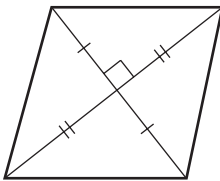
For any rectangle $FGHJ$, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

3. $\angle G \cong \angle H$

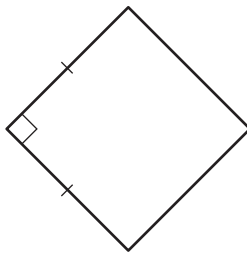
4. $\overline{JF} \cong \overline{FG}$

Classify the parallelogram. *Explain your reasoning.*

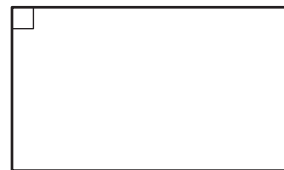
5.



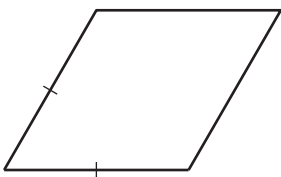
6.



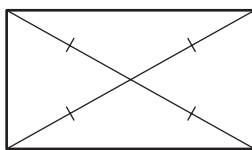
7.



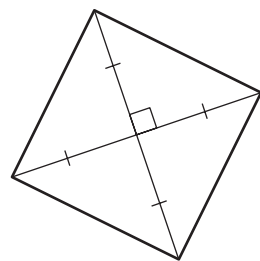
8.



9.



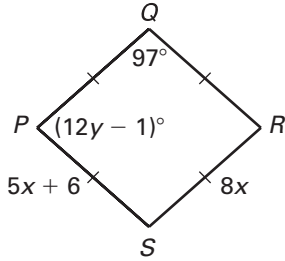
10.



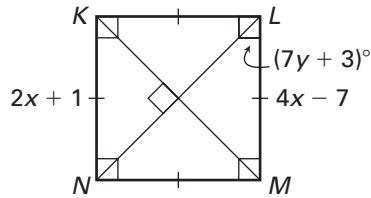
LESSON 5.10 Practice *continued*

Classify the special quadrilateral. Explain your reasoning. Then find the values of x and y .

11.



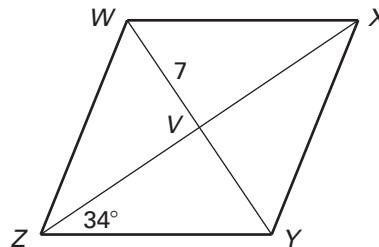
12.



The diagonals of rhombus $WXYZ$ intersect at V . Given that $m\angle XZY = 34^\circ$ and $WV = 7$, find the indicated measure.

13. $m\angle WZV$

14. $m\angle XYZ$



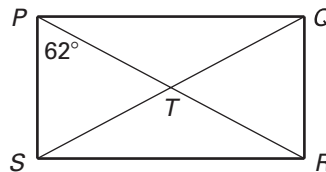
15. VY

16. WY

The diagonals of rectangle $PQRS$ intersect at T . Given that $m\angle RPS = 62^\circ$ and $QS = 18$, find the indicated measure.

17. $m\angle QPR$

18. $m\angle PTQ$



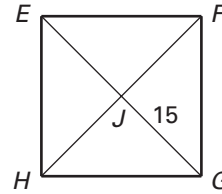
19. ST

20. PR

LESSON
5.10

Practice *continued*

The diagonals of square $EFGH$ intersect at J .
Given that $GJ = 15$, find the indicated measure.



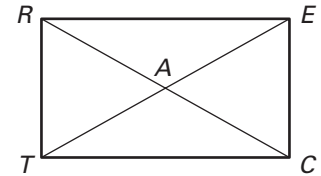
21. $m\angle EJF$ 22. $m\angle JFG$

23. FH 24. EJ

25. Complete the proof.

GIVEN: $RECT$ is a rectangle.

PROVE: $\triangle ART \cong \triangle ACE$



Statements	Reasons
1.	1. Given
2. $\overline{RT} \cong \overline{EC}$ $\overline{RT} \parallel \overline{EC}$	2.
3.	3. Alternate Interior \angle s are \cong .
4.	4. Vertical \angle s are \cong .
5. $\triangle ART \cong \triangle ACE$	5.

Write the corollary as a conditional statement and its converse.
Then explain why each statement is true.

26. Rhombus Corollary

27. Rectangle Corollary

28. Square Corollary

5.11

Use Properties of Trapezoids and Kites



Georgia
Performance
Standard(s)

MM1G1e,
MM1G3d

Your Notes

Goal • Use properties of trapezoids and kites.

VOCABULARY

Trapezoid

Bases of a trapezoid

Base angles of a trapezoid

Legs of a trapezoid

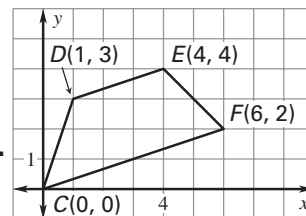
Isosceles trapezoid

Midsegment of a trapezoid

Kite

Example 1 Use a coordinate plane

Show that $CDEF$ is a trapezoid.



Solution

Compare the slopes of opposite sides.

Slope of \overline{DE} = _____ = _____

Slope of \overline{CF} = _____ = _____ = _____

The slopes of \overline{DE} and \overline{CF} are the same, so $\overline{DE} \parallel \overline{CF}$.

Slope of \overline{EF} = _____ = _____ = _____

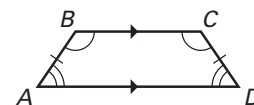
Slope of \overline{CD} = _____ = _____ = _____

The slopes of \overline{EF} and \overline{CD} are not the same, so \overline{EF} is not parallel to \overline{CD} .

Because quadrilateral $CDEF$ has exactly one pair of _____, it is a trapezoid.

THEOREM 5.29

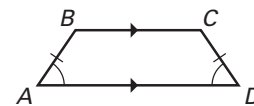
If a trapezoid is isosceles, then each pair of base angles is _____.



If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

THEOREM 5.30

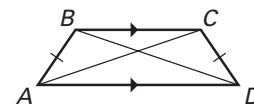
If a trapezoid has a pair of congruent _____, then it is an isosceles trapezoid.



If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

THEOREM 5.31

A trapezoid is isosceles if and only if its diagonals are _____.

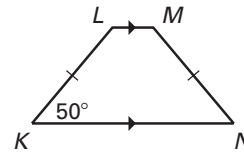


Trapezoid $ABCD$ is isosceles if and only if _____ \cong _____.

Your Notes

Example 2 Use properties of isosceles trapezoids

Kitchen A shelf fitting into a cupboard in the corner of a kitchen is an isosceles trapezoid. Find $m\angle N$, $m\angle L$, and $m\angle M$.



Solution

Step 1 Find $m\angle N$. $KLMN$ is an _____, so $\angle N$ and \angle _____ are congruent base angles, and $m\angle N = m\angle$ _____ = _____.

Step 2 Find $m\angle L$. Because $\angle K$ and $\angle L$ are consecutive interior angles formed by \overleftrightarrow{KL} intersecting two parallel lines, they are _____. So, $m\angle L =$ _____ - _____ = _____.

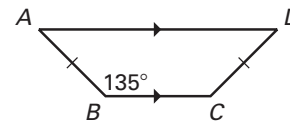
Step 3 Find $m\angle M$. Because $\angle M$ and \angle _____ are a pair of base angles, they are congruent, and $m\angle M = m\angle$ _____ = _____.

So, $m\angle N =$ _____, $m\angle L =$ _____, and $m\angle M =$ _____.

Checkpoint Complete the following exercises.

1. In Example 1, suppose the coordinates of point E are $(7, 5)$. What type of quadrilateral is $CDEF$? Explain.

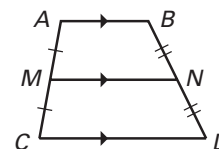
2. Find $m\angle C$, $m\angle A$, and $m\angle D$ in the trapezoid shown.



Your Notes

THEOREM 5.32: MIDSEGMENT THEOREM FOR TRAPEZOIDS

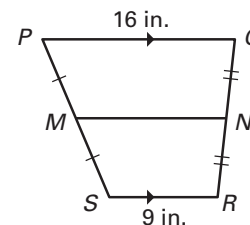
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.



If \overline{MN} is the midsegment of trapezoid $ABDC$, then $\overline{MN} \parallel$ _____, $\overline{MN} \parallel$ _____, and $MN =$ _____ (_____ + _____).

Example 3 Use the midsegment of a trapezoid

In the diagram, \overline{MN} is the midsegment of trapezoid $PQRS$. Find MN .



Solution

Use Theorem 5.32 to find MN .

$MN =$ _____ (_____ + _____)

Apply Theorem 5.32.

$=$ _____ (_____ + _____)

Substitute _____ for PQ and _____ for SR .

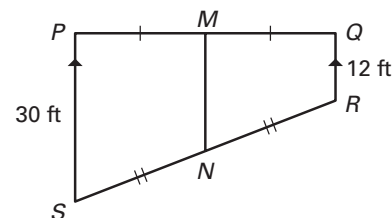
$=$ _____

Simplify.

MN is _____ inches.

Checkpoint Complete the following exercise.

3. Find MN in the trapezoid at the right.

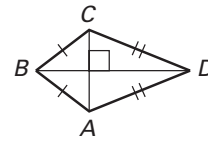


Your Notes

THEOREM 5.33

If a quadrilateral is a kite, then its diagonals are _____.

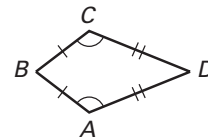
If quadrilateral $ABCD$ is a kite, then _____ \perp _____.



THEOREM 5.34

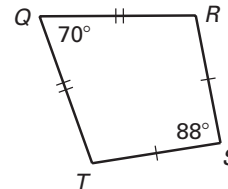
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A$ _____ $\angle C$ and $\angle B$ _____ $\angle D$.



Example 4 Use properties of kites

Find $m\angle T$ in the kite shown at the right.



Solution

By Theorem 5.34, $QRST$ has exactly one pair of _____ opposite angles.

Because $\angle Q \neq \angle S$, \angle _____ and $\angle T$ must be congruent. So, $m\angle$ _____ = $m\angle T$. Write and solve an equation to find $m\angle T$.

$m\angle T + m\angle R + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Corollary to Theorem 5.16

$m\angle T + m\angle T + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Substitute $m\angle T$ for $m\angle R$.

$\underline{\hspace{1cm}}(m\angle T) + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Combine like terms.

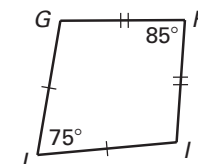
$m\angle T = \underline{\hspace{1cm}}$

Solve for $m\angle T$.

Homework

Checkpoint Complete the following exercise.

4. Find $m\angle G$ in the kite shown at the right.



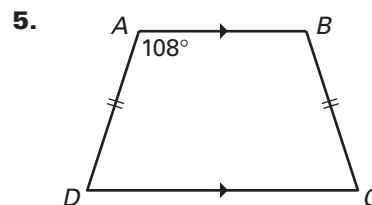
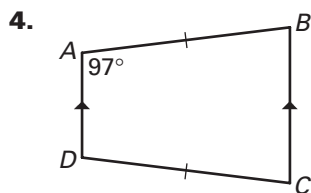
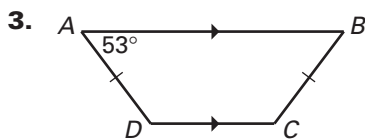
LESSON
5.11**Practice**

Points J , K , L , and M are the vertices of a quadrilateral. Determine whether $JKLM$ is a trapezoid.

1. $J(-1, -1)$, $K(0, 3)$, $L(3, 3)$, $M(4, -1)$

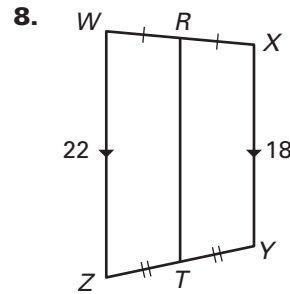
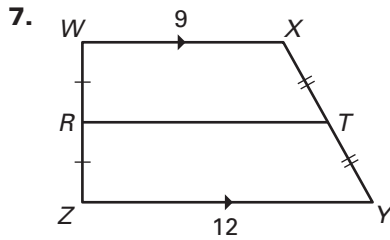
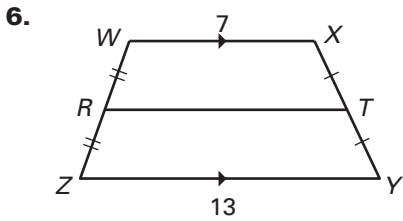
2. $J(-4, -2)$, $K(-4, 3)$, $L(2, 3)$, $M(3, -5)$

Find $m\angle B$, $m\angle C$, and $m\angle D$.



LESSON 5.11 Practice *continued*

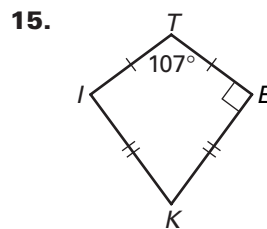
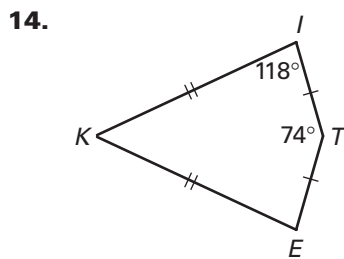
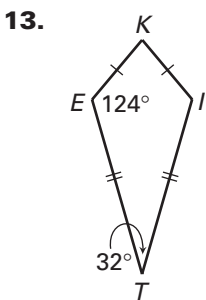
Find the length of the midsegment \overline{RT} .



Tell whether the statement is *always*, *sometimes*, or *never* true.

9. A trapezoid is a parallelogram.
10. The bases of a trapezoid are parallel.
11. The base angles of an isosceles trapezoid are congruent.
12. The legs of a trapezoid are congruent.

KITE is a kite. Find $m\angle K$.

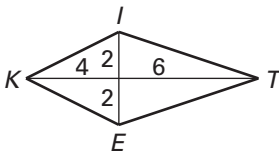


LESSON
5.11

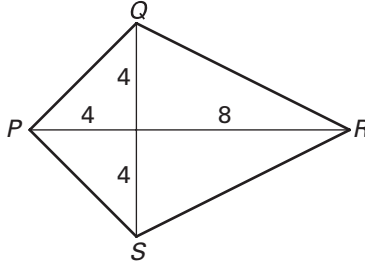
Practice *continued*

Use Theorem 5.33 and the Pythagorean Theorem to find the side lengths of the kite. Write the lengths in simplest radical form.

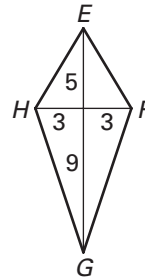
16.



17.

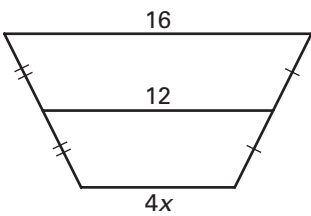


18.

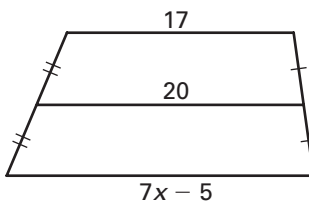


Find the value of x .

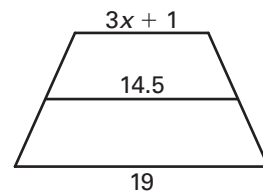
19.



20.



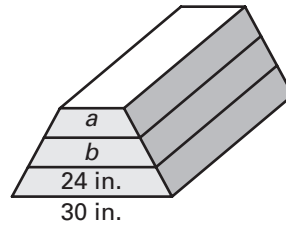
21.



LESSON
5.11

Practice *continued*

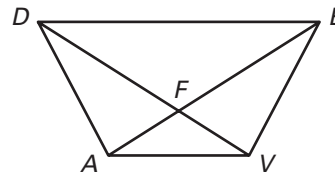
- 22. Vaulting Box** Three vaulting boxes used by a gymnastics team are stacked on top of each other as shown. The sides are in the shape of a trapezoid. Find the lengths of a and b .



- 23.** Complete the proof.

GIVEN: $\overline{DE} \parallel \overline{AV}$,
 $\triangle DAV \cong \triangle EVA$

PROVE: $DAVE$ is an isosceles trapezoid.



Statements	Reasons
1. $\overline{DE} \parallel \overline{AV}$	1.
2. $DAVE$ is a trapezoid.	2.
3.	3. Given
4.	4. Corresponding parts of $\cong \triangle$ are \cong .
5. $DAVE$ is an isosceles trapezoid.	5.

5.12

Identify Special Quadrilaterals

Georgia
Performance
Standard(s)
MM1G3d

Your Notes

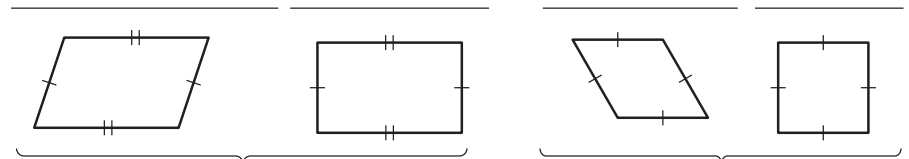
Goal • Identify special quadrilaterals.

Example 1 Identify quadrilaterals

Quadrilateral $ABCD$ has both pairs of opposite sides congruent. What types of quadrilaterals meet this condition?

Solution

There are many possibilities.



Opposite sides are congruent.

All sides are congruent.

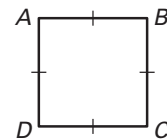
✓ **Checkpoint** Complete the following exercise.

1. Quadrilateral $JKLM$ has both pairs of opposite angles congruent. What types of quadrilaterals meet this condition?

In Example 2, $ABCD$ is shaped like a square. But you must rely only on marked information when you interpret a diagram.

Example 2 Classify a quadrilateral

What is the most specific name for quadrilateral $ABCD$?



Solution

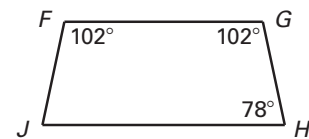
The diagram shows that both pairs of opposite sides are congruent. By Theorem 5.22, $ABCD$ is a _____ . All sides are congruent, so $ABCD$ is a _____ by definition.

_____ are also rhombuses. However, there is no information given about the angle measures of $ABCD$. So, you cannot determine whether it is a _____ .

Your Notes

Example 3 Identify a quadrilateral

Is enough information given in the diagram to show that quadrilateral $FGHJ$ is an isosceles trapezoid? Explain.



Solution

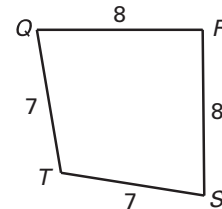
Step 1 Show that $FGHJ$ is a _____. $\angle G$ and $\angle H$ are _____ but $\angle F$ and $\angle G$ are not. So, _____ \parallel _____, but \overline{FJ} is not _____ to \overline{GH} . By definition, $FGHJ$ is a _____.

Step 2 Show that trapezoid $FGHJ$ is _____. $\angle F$ and $\angle G$ are a pair of congruent _____. So, $FGHJ$ is an _____ by Theorem 5.30.

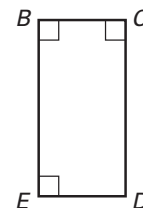
Yes, the diagram is sufficient to show that $FGHJ$ is an isosceles trapezoid.

Checkpoint Complete the following exercises.

2. What is the most specific name for quadrilateral $QRST$? Explain your reasoning.



3. Is enough information given in the diagram to show that quadrilateral $BCDE$ is a rectangle? Explain.



Homework

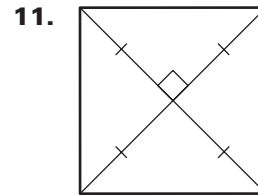
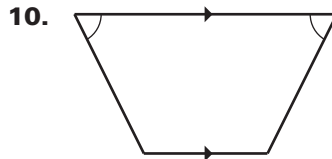
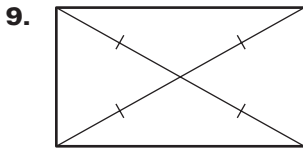
LESSON
5.12

Practice

Match the property on the left with all of the quadrilaterals that have the property.

- | | |
|---|-------------------------------|
| 1. Both pairs of opposite sides are parallel. | A. Parallelogram |
| 2. Both pairs of opposite sides are congruent. | B. Rectangle |
| 3. Both pairs of opposite angles are congruent. | C. Rhombus |
| 4. Exactly one pair of opposite sides are parallel. | D. Square |
| 5. Exactly one pair of opposite sides are congruent. | E. Trapezoid |
| 6. Exactly one pair of opposite angles are congruent. | F. Isosceles Trapezoid |
| 7. Diagonals are congruent. | G. Kite |
| 8. Diagonals are perpendicular. | |

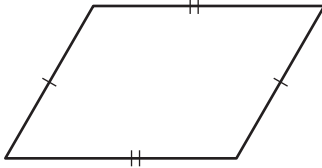
Give the most specific name for the quadrilateral. Explain.



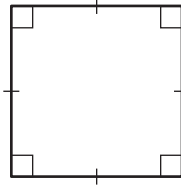
LESSON 5.12 Practice *continued*

Tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. **Explain.**

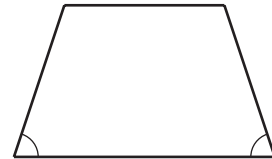
12. Parallelogram



13. Square

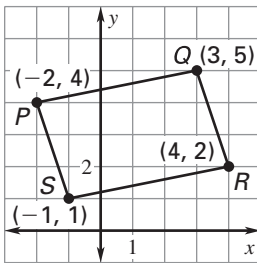


14. Trapezoid

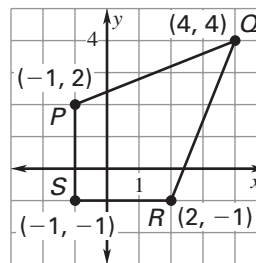


Give the most specific name for quadrilateral **PQRS**. **Justify** your answer.

15.

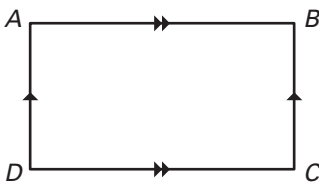


16.

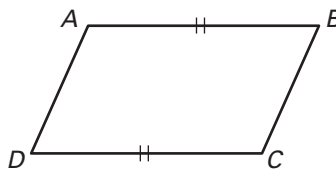


Which pairs of segments or angles must be congruent so that you can prove that **ABCD** is the indicated quadrilateral? **Explain.** There may be more than one right answer.

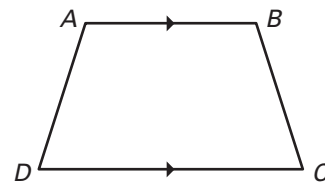
17. Rectangle



18. Parallelogram



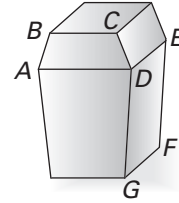
19. Isosceles Trapezoid



LESSON
5.12
Practice *continued*

In Exercises 20 and 21, use the following information.

Gem Cutting There are different ways of cutting gems to enhance the beauty of the jewel. One of the earliest shapes used for diamonds is called the *table cut*, as shown. Each face of a cut gem is called a *facet*.



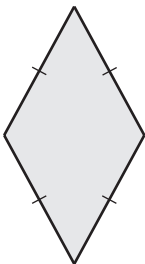
20. $\overline{BC} \parallel \overline{AD}$, \overline{AB} and \overline{DC} are not parallel. What shape is the facet labeled $ABCD$?

21. $\overline{DE} \parallel \overline{GF}$, \overline{DG} and \overline{EF} are congruent, but not parallel. What shape is the facet labeled $DEFG$?

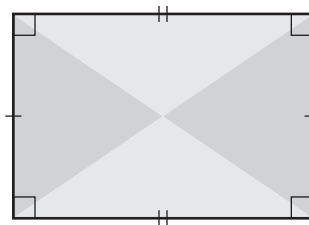
In Exercises 22–24, use the following information.

Wall Hangings Decorative wall hangings are made in a variety of shapes. What type of special quadrilateral is shown?

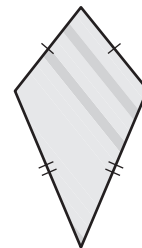
22.



23.



24.



Words to Review

Give an example of the vocabulary word.

Midsegment of a triangle	Perpendicular bisector
Equidistant	Concurrent
Point of concurrency	Circumcenter
Angle bisector	Incenter

Median of a triangle	Centroid
Altitude of a triangle	Orthocenter
Indirect proof	
Diagonal	Interior angles of a polygon

Exterior angles of a polygon	Parallelogram
Rhombus	Rectangle
Square	Trapezoid
Bases, Legs, and Base angles of a trapezoid	Isosceles trapezoid
Midsegment of a trapezoid	Kite