

6.1

Use Counting Principles



Georgia
Performance
Standard(s)

MM1D1a

Your Notes

Goal • Use addition and multiplication counting principles.

VOCABULARY

The Multiplication Counting Principle

The Addition Counting Principle

Example 1 Use the multiplication counting principle

Shoes At a shoe store, shoes are available in 6 different styles. Each style is available in 3 different colors. How many choices does the shoe store offer?

Solution

You can use the multiplication counting principle to find the number of shoe choices. Multiply the number of shoe styles ___ and the number of color variations ___.

The store offers $__ \cdot __ = __$ shoe choices.

✓ Checkpoint Complete the following exercise.

1. A store carries wood stain in 8 different shades. Each shade is available in 4 different finishes. How many choices of wood stain does the store offer?

Example 2 Use the addition counting principle

Identification Cards Suppose that each student at a certain school is assigned an identification card which contains a unique 4 character (letter and digit) barcode. Each barcode contains at most 1 digit. How many unique identification cards are possible?

Solution

0-digits: There are no digits and there are ____ choices for each letter. So, there are ____ • ____ • ____ • ____ = _____ letter-letter-letter-letter possibilities.

1-digit: There are ____ • ____ • ____ • ____ = _____ digit-letter-letter-letter possibilities. The digit can be in any of four positions, so there are ____ • _____ = _____ possibilities.

So, in total there are _____ + _____ = _____ possible identification cards.

Example 3 Find a probability

Computer Games You are looking at action and strategy computer games. The action game is available in 3 versions, easy, medium, and hard. The strategy game is available in 2 versions, easy and medium. If you randomly choose one action game and one strategy game, what is the probability your choice includes an easy action game?

Solution

Because there are 3 action game choices and 2 strategy game choices, the total number of possible choices is ____ • ____ = ____ . If you limit yourself to only easy action games, the number of choices that include an easy action game is ____ • ____ = ____ .

$$P(\text{easy action game}) = \frac{\text{Easy action game choices}}{\text{Total possible choices}}$$

$$= \frac{\square}{\square} = \frac{\square}{\square}$$

Your Notes

Example 4 Solve a multi-step problem

Beads There is 1 red bead, 1 blue bead, and 1 yellow bead in a bag. You randomly select a bead from the bag and then put it back. Your two friends each do the same. What is the probability that you each choose the same color bead?

Solution

Step 1 List the favorable outcomes. There are ____.

R-R-R ____ - ____ - ____ Y-Y-Y

Step 2 Find the total number of outcomes using the multiplication counting principle.

Total outcomes = ____ • ____ • ____ = ____

Step 3 Find the probability.

$$P(\text{all the same}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

✔ **Checkpoint** Complete the following exercises.

2. In Example 2, suppose only even digits can be used in the identification card. How many 4 character identification cards can be made with at most 1 digit?

3. In Example 3, what is the probability your choice includes a medium difficulty strategy game?

4. In Example 4, suppose a green bead is also in the bag. Recalculate the probability you and your two friends choose the same color.

Homework

LESSON
6.1**Practice**

In Exercises 1–4, use the multiplication counting principle to find the number of choices that are available.

1. Choose a raisin, whole grain, or apple muffin with coffee or milk.
2. Choose sneakers, shoes, or sandals in white, black, or gray.
3. Choose small, medium, large, or extra large pants in black, cream, or white.
4. Choose one of 3 group projects and one of 4 individual projects.
5. **Road Trip** Your family is going on a road trip. You can go to Florida, Alabama, or South Carolina. You can stay 5, 6, or 7 days. You are allowed to bring 1 or 2 friends. How many different options do you have?
6. **Lunch** You may choose a sandwich, drink, and dessert for lunch at your school. The menu offers 6 sandwiches, 5 drinks, and 3 desserts. How many different lunches can you choose from?
7. **Yearbook** The yearbook staff has to choose a theme, layout, and color scheme for the yearbook. They can choose a summer, winter, spring, or fall theme. The layout can be wide or narrow. The color scheme can be warm, cool, or neutral. How many different yearbook designs can be made?

LESSON
6.1**Practice** *continued*

- 8. Lockers** The combination for your gym locker consists of 4 symbols (letters and digits). If there are 1 or 0 letters, how many combinations are possible?
- 9. Engraving** you can choose to have a ring engraved with your first name, full name, or initials, and one of 5 symbols. How many different engravings are possible if the symbol can be the first or last part of the engraving?
- 10. Video Games** You and two friends each randomly choose a video game from 6 choices. What is the probability that you all pick the same video game? What is the probability that none of you choose the same video game?
- 11. Ice Cream** You are at the ice cream shop and you can choose a small, medium, or large ice cream cone. You can choose chocolate, vanilla, or strawberry ice cream. If you randomly select an ice cream cone and ice cream flavor, what is the probability your choice includes chocolate ice cream?

6.2

Find Probabilities Using Permutations



Georgia Performance Standard(s)

MM1D1b

Your Notes

Goal • Use the formula for the number of permutations.

VOCABULARY

n Factorial

Permutation

PERMUTATIONS

The number of permutations of n objects is given by ${}_n P_n = n!$. The number of permutations of n objects taken r at a time, where $r \leq n$, is given by

$${}_n P_r = \underline{\hspace{2cm}} .$$

Example 1 *Count permutations*

Consider the number of permutations of the letters in the word WINTER. In how many ways can you arrange 3 of the letters?

Solution

Use the multiplication counting principle when arranging the letters in the word WINTER. Because you are arranging three of the letters, the solution is as follows:

$$\begin{aligned} \text{Number of permutations} &= \text{Choices for 1st letter} \cdot \text{Choices for 2nd letter} \cdot \text{Choices for 3rd letter} \\ &= \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

Your Notes

✓ **Checkpoint** Complete the following exercise.

1. How many ways can you arrange the letters in the word STOVE?

Example 2 Use the permutation formula

Photo Album You have 12 pictures and are selecting 4 of them for a photo album. In how many ways can you select the pictures for the album?

Solution

To find the number of permutations of 4 pictures chosen from 12, find ${}_{12}P_4$.

$${}_{12}P_4 = \frac{\boxed{}!}{(\boxed{} - \boxed{})!} \quad \text{Permutation formula}$$

$$= \frac{\boxed{}!}{\boxed{}!} \quad \text{Subtract.}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot \boxed{}!}{\boxed{}!} \quad \text{Expand and divide.}$$

$$= \underline{\hspace{2cm}} \quad \text{Multiply.}$$

There are ways to select 4 pictures out of 12.

✓ **Checkpoint** Complete the following exercise.

2. In Example 2, suppose that you had 14 pictures and wanted to select 6 of them to put in the photo album. In how many ways can you select the pictures for the album?

Your Notes

Example 3 Find a probability using permutations

Cards A bag contains 5 cards numbered 1–5. You draw one card at a time until you draw all 5 cards. What is the probability of drawing the card numbered 1 first and the card numbered 2 second?

Solution

Step 1 Write the number of possible outcomes as the number of permutations of the 5 cards. This is

$${}_5P_5 = 120.$$

Step 2 Write the number of favorable outcomes as the number of permutations of the other cards, given that the card numbered 1 is drawn first and the card numbered 2 is drawn second. This is

$${}_3P_3 = 6.$$

Step 3 Calculate the probability.

$$P(1 \text{ then } 2) = \frac{\boxed{}}{\boxed{}}$$

Form a ratio of favorable to possible outcomes.

$$= \frac{\boxed{}}{\boxed{}}$$

Expand factorials.
Divide out common factor, ____.

$$= \frac{\boxed{}}{\boxed{}}$$

Simplify.

✔ **Checkpoint** Complete the following exercise.

3. In Example 3, suppose there are 6 cards numbered 1–6. What is the probability of choosing the card numbered 1 first and the card numbered 2 second?

Homework

**LESSON
6.2****Practice**

1. List all the possible permutations for the numbers 3, 5, and 7.

Find the number of ways you can arrange (a) all of the letters in the given word and (b) 2 of the letters in the word.

2. ON

3. TAP

4. STAR

Evaluate the expression.

5. $2!$

6. $4!$

7. $7!$

Write the notation for the given description.

8. The number of permutations of 10 objects taken 3 at a time

9. The number of permutations of 14 objects taken 4 at a time

Write the meaning of the notation in words.

10. ${}_{13}P_2$

11. ${}_{17}P_8$

LESSON
6.2**Practice** *continued***Match the notation with the correct expression.**

12. ${}_{10}P_4$

13. ${}_{10}P_6$

14. ${}_{10}P_2$

A. $\frac{10!}{8!}$

B. $\frac{10!}{6!}$

C. $\frac{10!}{4!}$

Evaluate the expression.

15. ${}_4P_3$

16. ${}_6P_2$

17. ${}_7P_4$

18. **Concert** Seven friends go to a concert. In how many different ways can they sit together in a row of 7 empty seats?
19. **Appliance Delivery** An appliance delivery person has 5 deliveries to make. The destinations are all so close, it doesn't matter the order in which the appliances are delivered. In how many orders can the deliveries be made?
20. **Side Work** You and three of your friends work together at a restaurant. You choose your side jobs by each of you drawing one of four straws, each with a different length.
- List all of the possible ways the straws can be drawn by the four of you.
 - Use the formula for permutations to find the number of ways in which the straws can be drawn.
 - What is the likelihood that you will draw the shortest straw? *Explain* your answer by using probability.

6.3

Find Probabilities Using Combinations



Georgia Performance Standard(s)

MM1D1b

Your Notes

Goal • Use combinations to count possibilities.

VOCABULARY

Combination

Example 1 Count combinations

Count the combinations of two letters from the list A, B, C, D, E.

Solution

List all of the permutations of two letters in the list A, B, C, D, E. Because order is not important in a combination, cross out any duplicate pairs.

AB AC AD AE ~~BA~~ BC BD BE ~~CA~~ ~~CB~~
CD CE ~~DA~~ ~~DB~~ ~~DC~~ DE ~~EA~~ ~~EB~~ ~~EC~~ ~~ED~~

There are possible combinations of 2 letters from the list A, B, C, D, E.

COMBINATIONS

Formula

The number of combinations of n objects taken r at a time, where $r \leq n$, is given by:

$${}_n C_r = \frac{\boxed{}}{\boxed{}}$$

Example

The number of combinations of 5 objects taken 2 at a time is:

$${}_5 C_2 = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} = \underline{\hspace{2cm}}$$

Your Notes

Example 2 Use the combinations formula

Toppings You order a pizza at a restaurant. You can choose 3 toppings from a list of 12. How many combinations of toppings are possible?

Solution

The order in which you choose the toppings is not important. So, to find the number of combinations of 12 toppings taken 3 at a time, find ${}_{12}C_3$.

$${}_{12}C_3 = \frac{\boxed{}}{\boxed{}} \quad \text{Combinations formula}$$

$$= \frac{\boxed{}}{\boxed{}} \quad \text{Subtract.}$$

$$= \frac{\boxed{}}{\boxed{}} \quad \text{Expand factorials. Divide out common factor.}$$

$$= \underline{} \quad \text{Simplify.}$$

There are different combinations of toppings.

✓ **Checkpoint** Complete the following exercises.

1. Count the combinations of two letters from the list A, B, C, D, E, F.

2. In Example 2, suppose you can choose only 2 toppings out of the 12 topping choices. How many combinations are possible?

Your Notes

Example 3 Find a probability using combinations

Scholarships A committee must award three students with scholarships. Fifteen students are candidates for the scholarship including you and your two best friends. If the awardees are selected randomly, what is the probability that you and your two best friends are awarded the scholarships?

Solution

Step 1 Write the number of possible outcomes as the number of combinations of 15 candidates taken 3 at a time, ${}_{15}C_3$.

$${}_{15}C_3 = \frac{\boxed{}}{\boxed{}}$$

$$= \frac{\boxed{}}{\boxed{}}$$

$$= \frac{\boxed{}}{\boxed{}}$$

$$= \underline{\hspace{2cm}}$$

Step 2 Find the number of favorable outcomes. Only _____ of the possible combinations includes scholarships for you and your two best friends.

Step 3 Calculate the probability.

$$P(\text{scholarships awarded to you and your friends}) = \underline{\hspace{2cm}}$$

Homework

✔ **Checkpoint** Complete the following exercise.

3. In Example 3, suppose there are 20 candidates for the scholarships. Find the probability that you and your two best friends are awarded the 3 scholarships.

LESSON
6.3**Practice**

Count the number of combinations of the given number of letters from the list K, L, M, N, O, P.

1. two letters 2. three letters 3. five letters

Write the notation for the given description.

4. The number of combinations of 12 objects taken 2 at a time
5. The number of combinations of 18 objects taken 7 at a time

Write the meaning of the notation in words.

6. ${}_{14}C_3$ 7. ${}_{22}C_9$

Match the notation with the correct expression.

8. ${}_{12}C_4$ 9. ${}_{12}C_3$ 10. ${}_{12}C_2$

- A. $\frac{12!}{9!3!}$ B. $\frac{12!}{8!4!}$ C. $\frac{12!}{10!2!}$

Evaluate the expression.

11. ${}_8C_1$ 12. ${}_{10}C_3$ 13. ${}_9C_6$

LESSON
6.3**Practice** *continued*

In Exercises 14 and 15, tell whether the question can be answered using combinations or permutations. Explain your choice.

- 14.** Five students from your class of 100 students will be selected to be sent on to a leadership conference. How many groups of 5 students are possible?
- 15.** Ten students try out to be in one of the 4 different positions of your track's relay team. In how many ways can the 4 positions be filled?
- 16. Art Fair** On the last day of an art fair, an artist offers a special on her photographs. You can buy 2 small photos and 1 large photo for \$75. She has 14 different small photos and 10 different large photos left to choose from.
- a.** How many different choices of 2 small photos are possible?
- b.** How many different choices of 2 small photos and 1 large photo are possible?
- 17. Movies** The programming committee at a college is selecting the movies that will be shown throughout the semester. They have a list of 30 possible movies and they must select 5 movies from the list. How many different combinations of 5 movies are possible?
- 18. Group Project** You have been working on a group project with 4 other people. Your group has decided to have two people present the project together. How many combinations of 2 people from the group are possible? What is the probability that you are one of the two people?

6.4

Find Probabilities of Compound Events



Georgia
Performance
Standard(s)

MM1D2a,
MM1D2b,
MM1D2c

Your Notes

Goal • Find the probability of compound events.

VOCABULARY

Compound event

Mutually exclusive events

Overlapping events

Independent events

Dependent events

Conditional probability

Your Notes

Example 1 Find the probability of A or B

You randomly choose a card from a standard deck of 52 playing cards.

- Find the probability that you choose a 9 or a King.
- Find the probability that you choose an Ace or a spade.

Solution

- Choosing a 9 or a King are mutually exclusive events.

$$P(9 \text{ or King}) = P(9) + P(\text{King})$$

$$= \quad + \quad$$

$$= \quad$$

$$= \quad$$

- Because there is an Ace of spades, choosing an Ace or a spade are _____. There are 4 Aces, 13 spades, and 1 Ace of spades.

$$P(\text{Ace or spade}) = P(\text{Ace}) + P(\text{spade}) - P(\text{Ace and spade})$$

$$= \quad + \quad - \quad$$

$$= \quad$$

$$= \quad$$

Example 2 Find the probability of A and B

You roll two number cubes. What is the probability that you roll a 1 first and a 2 second?

Solution

The events are _____. The number on one number cube does not affect the other.

$$P(1 \text{ and } 2) = \quad \cdot \quad = \quad \cdot \quad = \quad$$

Your Notes

Example 3 Find the probability of A and B

Markers A box contains 8 red markers and 3 blue markers. You choose one marker at random, do not replace it, then choose a second marker at random. What is the probability that both markers are blue?

Solution

Because you do not replace the first marker, the events are dependent. Before you choose a marker, there are **11** markers, 3 of them are blue. After you choose a blue marker, there are **10** markers left and two of them are blue. So, the probability that the second marker is blue given that the first marker is blue, is

$$\frac{\square}{\square} \cdot \frac{\square}{\square}$$

$P(\text{blue and then blue})$

$$= \frac{\quad}{\quad} \cdot \frac{\quad}{\quad}$$

$$= \frac{\quad}{\quad} \cdot \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

✔ **Checkpoint** Complete the following exercises.

1. In a standard deck of cards, find the probability you randomly select the King of diamonds or any spade.

2. In Example 3, suppose there are also 4 orange markers in the box. Calculate the probability of selecting a blue marker and then an orange marker, without replacement.

Homework

LESSON
6.4**Practice**

In Exercises 1–6, you draw a card from a bag that contains 4 yellow cards numbered 1–4 and 4 blue cards numbered 1–4. Tell whether the events *A* and *B* are mutually exclusive or overlapping. Then find $P(A \text{ or } B)$.

1. **Event A:** You choose a yellow card.
Event B: You choose a blue card.
2. **Event A:** You choose a blue card.
Event B: You choose a number 3 card.
3. **Event A:** You choose a number 1 card.
Event B: You choose a yellow card.
4. **Event A:** You choose a card with an odd number.
Event B: You choose a number 2 card.
5. **Event A:** You choose a blue number 4 card.
Event B: You choose a blue card.
6. **Event A:** You choose a card with an odd number.
Event B: You choose a yellow card.

LESSON
6.4**Practice** *continued*

In Exercises 7–9, tell whether the events *A* and *B* are dependent or independent. Then find $P(A \text{ and } B)$.

7. A bag contains 3 red balls and 4 green balls. You randomly draw one ball, replace it, and randomly draw a second ball.

Event A: The first ball is red.

Event B: The second ball is red.

8. You write each of the letters of the word LISTED on pieces of paper and place them in a bag. You randomly draw one letter, do not replace it, then randomly draw a second letter.

Event A: The first letter is an L.

Event B: The second letter is a T.

9. You write each of the letters of the word BRIGHTNESS on pieces of paper and place them in a bag. You randomly draw one letter, replace it, then randomly draw a second letter.

Event A: The first letter is a B.

Event B: The second letter is an H.

10. **Project** The students in your English class are broken up into groups of 4. Each group is assigned a project, and the groups will present the results of the project on Wednesday, Thursday, and Friday of the following week. In each group, one person will be chosen to present the results. What is the probability that your group will present your results next Thursday and you will be chosen as the presenter?

11. **Sports** A survey of 500 students in a school found that about 225 households consist of students who participate in some kind of sport, 250 consist of students who do and do not participate in some kind of sport, and 25 consist of students who do not participate in some kind of sport.

a. What is the probability that one of the households surveyed, chosen at random, consists of students who do *or* do not participate in some kind of sport?

b. What is the probability that one of the households surveyed, chosen at random, consists of students who do *and* do not participate in some kind of sport?

6.5

Find Expected Value



Georgia
Performance
Standard(s)

MM1D2d

Your Notes

Goal • Use expected value.

VOCABULARY

Expected value

Example 1 *Find an expected value*

Game Consider a game in which two players each choose an integer from 2 to 4. If the sum of the two integers is odd, then player A scores 3 points and player B loses 1 point. If the sum is even, then player B scores 3 points and player A loses 1 point. Find the expected value for player A.

Solution

The possible outcomes are $2 + 2$, $2 + 3$, $2 + 4$, $3 + 2$, $3 + 3$, $3 + 4$, $4 + 2$, $4 + 3$, and $4 + 4$. The probability of an even sum is _____. The probability of an odd sum is _____.

$$\text{Player A: } E = \text{_____} \cdot \frac{4}{9} + \text{_____} \cdot \frac{5}{9} = \text{_____}$$

Your Notes

- ✓ **Checkpoint** The outcome values and their probabilities are given. Find the expected value.

1.

Outcome value, x	2	3	-1
Probability, p	0.20	0.45	0.35

2.

Outcome value, x	\$8	\$4	-\$2
Probability, p	0.10	0.25	0.65

Example 2 Use expected value

Game Show You participate in a game show in which you respond to questions that have 3 possible answers. You gain \$10 for each correct answer, and lose \$6 for each incorrect answer. Every question must be answered. If you do not know the answer to one of the questions, is it to your advantage to guess the answer?

Solution

Step 1 Find the probability of each outcome. Because each question has 1 right answer and 2 wrong answers the probability of guessing correctly is _____ and the probability of guessing incorrectly

is _____.

Step 2 Find the expected value of guessing an answer. Multiply the money gained or lost by the corresponding probability, then find the sum of these products.

$$E = \underline{\hspace{2cm}} \cdot \left(\frac{1}{3}\right) + \underline{\hspace{2cm}} \cdot \left(\frac{2}{3}\right) = \frac{\square}{\square}$$

Because the expected value is negative, it is not to your advantage to guess.

Example 3 Find expected value

Theater A movie theater is giving away a \$100 prize and a \$50 prize. To enter the drawing, you need to simply buy a movie ticket for \$6. The ticket collectors will take the tickets from the first 1000 guests, and after the movie they will randomly choose one ticket. If the number chosen matches the number on your ticket stub, you win 1st or 2nd prize. What is the expected value of your gain?

Solution

Step 1 Find the gain for each prize by subtracting the cost of the ticket from the prize money.

Step 2 Find the probability of each outcome. There are 1000 tickets sold, and the probability of winning

one of the prizes is $\frac{\boxed{}}{\boxed{}}$. Because there are

2 prizes there are 2 winning tickets and _____ losing tickets. So, the probability you will not win

a prize is $\frac{\boxed{}}{\boxed{}}$.

Step 3 Summarize the information in the table.

Gain, x	_____	_____	_____
Probability, p	$\frac{1}{1000}$	$\frac{1}{1000}$	$\frac{998}{1000}$

Step 4 Find the expected value by finding the sum of each outcome multiplied by its corresponding probability.

$$E = \text{_____} \left(\frac{1}{1000} \right) + \text{_____} \left(\frac{1}{1000} \right) + \text{_____} \left(\frac{998}{1000} \right)$$

$$= \text{_____}$$

The expected value of your gain is _____. This means that you can expect to _____ an average of _____ for each ticket you buy.

Your Notes

✔ **Checkpoint** Complete the following exercises.

3. In Example 2, suppose you gain \$8 for each correct answer and lose \$4 for each incorrect answer. Find the expected value and then determine if it is to your advantage to guess on a particular question.

4. There is a prize drawing for home electronics. Tickets are \$8. There are a total of 5000 tickets sold for the drawing. The three prizes are a new computer worth \$1500, a high-definition TV worth \$800, and a stereo system worth \$300. If you buy one ticket, what is the expected value of your gain?

Homework

LESSON
6.5**Practice**

In Exercises 1 and 2, use the information in the table to find the expected value.

1.

Outcome value, x	-4	5
Probability, p	0.35	0.65

2.

Outcome value, x	12	18
Probability, p	0.45	0.55

3. Given the outcome values 3, 8, and 10 and the corresponding probabilities 0.30, 0.45, and 0.25, find the expected value.
4. **Coins** Consider a game in which you and a friend each flip a coin. If the coin lands heads up, you gain 2 points and your friend loses 1 point. If the coin lands tails up, you lose 1 point and your friend gains 2 points. Find the expected value of the game for each person.
5. **Craftsperson** A craftsperson is able to etch 25 windows per day if they are able to use the laser etching device, and 8 windows per day if they must etch by hand. If they can use the laser etching device 70% of the time, find the expected number of windows they etch per day.

LESSON
6.5**Practice** *continued*

- 6. Football** A football team plays in a city that has a fair chance of rain on any given day. Suppose this upcoming Sunday there is a 62% chance of rain. If it is raining, they can expect to score 3 touchdowns. If the weather is clear, they can expect to score 5 touchdowns. What is the expected number of touchdowns they score?
- 7. Raffle** There is a prize drawing for a new computer and a new stereo system. Suppose there are 1500 raffle tickets sold. The computer is worth \$1000 and the stereo system is worth \$500. If the tickets cost \$5, what is the expected value if you purchase one ticket?
- 8. New Car** A game show gives away 2 new cars. One car is worth \$50,000 and the other car is worth \$40,000. Two members of the audience are randomly selected to participate and receive one of the two new cars. What is the expected value if you are randomly chosen and you had to pay \$4 to be a member of the audience with 1999 other people?

6.6

Analyze Surveys and Samples



Georgia
Performance
Standard(s)

MM1D3c

Your Notes

Goal • Identify populations and sampling methods.

VOCABULARY

Survey

Population

Random sample

Stratified random sample

Systematic sample

Convenience sample

Self-selected sample

Representative sample

Biased sample

Biased question

Example 1 *Classify a sampling method*

Study Time A high school is conducting a survey to determine the average number of hours that their students spend doing homework each week. At the school, only the members of the sophomore class are chosen to complete the survey. Identify the population and classify the sampling method.

Solution

The population is _____. Because a rule (sophomore class only) is used to select members of the population, the sample is a _____ sample.

Example 2 *Identify a potentially biased sample*

Is the sampling method used in Example 1 likely to result in a biased sample?

Solution

Students in other grades may have different study habits, so the method _____ in a biased sample.

Example 3 *Identify potentially biased questions*

Tell whether the question is potentially biased. Explain your answer. If the question is potentially biased, rewrite it so that it is not.

- a. Do you still support the school basketball team, even though the team is having its worst season in 5 years?
- b. Don't you think that dogs are better pets than cats?

Solution

a. This question is biased because _____.
_____. An unbiased question is,
" _____ "

b. This question is biased because _____.
_____. An unbiased question is
" _____ "

Your Notes

✓ **Checkpoint** Complete the following exercises.

1. In Example 1, suppose the school asks students to volunteer to take the survey. Classify the sampling method.

2. An amusement park owner wants to evaluate the customer service given by the park's ride operators. One day, every 10th customer leaving the park was asked, "Don't you think that our friendly, well-trained ride operators provided excellent customer service today?"

a. Is this sampling method likely to result in a biased sample? Explain.

b. Is this question potentially biased? Explain your answer. If the question is potentially biased, rewrite it so that it is not.

Homework

LESSON
6.6**Practice**

In Exercises 1–3, identify the population and classify the sampling method.

1. The manager of a movie rental store wants to evaluate how customers rate the selection of movies that the store has in stock. Customers are given comment cards with their receipts.
2. Your school’s administrators want to know if students are satisfied with food choices on the school menu. In each grade, every eighth student in alphabetical order is surveyed.
3. At a family reunion, the next year’s reunion location is randomly chosen to be at one of the family member’s homes. Each home is put on a slip of paper into a hat. The location is chosen by pulling a name from a hat.

Tell whether the survey method used is likely to result in a biased sample.

4. The owners of a music store chain want to determine whether or not they should open their stores an hour earlier. They survey the customers in one of their stores at random.
5. A group of students living in the dormitories at a college wants to gather information about the need for a shuttle bus to a local shopping center. They survey every fourth room on each floor of each dormitory.

LESSON
6.6**Practice** *continued*

In Exercises 6 and 7, tell whether the question is potentially biased. Explain your answer.

6. Do you think that fresh fruit should be offered on the lunch menu?

7. Don't you agree that replacing a parking lot with a city park would help decrease pollution in the city?

In Exercises 8–10, explain why the question is biased. Then rewrite it so that it is not.

8. Don't you agree that it is better to offer water than soft drinks in the school's vending machine?

9. Don't you agree that a music concert would be more fun to go to than a museum?

10. Would you pay even higher taxes to fund a new civic center?

11. **Street Conditions** A television station does a report on the condition of the streets in a large city. Part of the report includes a survey of people that live in the area. The survey is done by asking people at a bus stop what they think of local street conditions. Is this sample necessarily representative of local residents? *Explain.*

6.7

Use Measures of Central Tendency and Dispersion



Georgia
Performance
Standard(s)

MM1D4

- Goal** • Compare measures of central tendency and dispersion.

Your Notes

VOCABULARY

Mean

Median

Mode

Measure of dispersion

Range

Deviation from the mean

Mean absolute deviation

Your Notes

Example 1

Compare measures of central tendency

Temperature The daily February temperatures (in degrees Fahrenheit) for 10 days are listed for a certain city. Find the mean, median, and mode(s) of the data

20, 21, 22, 24, 25, 26, 27, 29, 37, 37

Solution

$$\bar{x} = \frac{20 + 21 + 22 + 24 + 25 + 26 + 27 + 29 + 37 + 37}{\square}$$

$$= \frac{\square}{\square}$$

$$= \underline{\hspace{2cm}}$$

The median is the mean of the two middle values, _____.

The mode is _____.

The _____ and _____ best represent the data.

✔ Checkpoint Complete the following exercise.

1. Find (a) the mean, (b) the median, and (c) the mode(s) of the data set.

67, 70, 73, 73, 78, 80

Example 2 Compare measures of dispersion

Running The top 4 finishing times (in seconds) for two different teams in the 50 meter dash are given. Compare the spread of the data for the two sets using (a) the range and (b) the mean absolute deviation.

Team A: 5.8, 6.0, 6.2, 6.4

Team B: 5.7, 5.9, 6.5, 6.7

Solution

a. Team A: _____ - _____ = _____

Team B: _____ - _____ = _____

The range for Team ____ is greater than the range for Team ____, so the data for Team ____ covers a wider interval than the data for Team ____.

b. The mean for Team A is _____, so the mean absolute deviation is:

$$\frac{|5.8 - 6.1| + |6.0 - 6.1| + |6.2 - 6.1| + |6.4 - 6.1|}{4}$$

= _____.

The mean for Team B is _____, so the mean absolute deviation is:

$$\frac{|5.7 - 6.2| + |5.9 - 6.2| + |6.5 - 6.2| + |6.7 - 6.2|}{4}$$

= _____.

The mean absolute deviation of Team ____ is greater, so the average variation from the mean is greater for the data for Team ____ than for the data for Team ____.

✓ Checkpoint Complete the following exercise.

Homework

2. In Example 2, suppose the slowest time for Team B was 6.6 seconds. Recalculate the range and mean absolute deviation.

**LESSON
6.7****Practice****Evaluate the expression.**

1. $\frac{4 + 14 + 8 + 3 + 6}{5}$

2. $\frac{|6 - 4| + |5 - 4| + |9 - 4| + |2 - 4|}{4}$

Complete the statement.

3. The ? of a numerical set of data is the difference of the greatest value and the least value.

4. The ? of a numerical set of data is the middle number when the numbers are written in numerical order.

5. The ? of a numerical set of data is the value that occurs most frequently.

Find the mean, median, and mode(s) of the data.

6. 5, 3, 2, 6, 5, 2, 5

7. 24, 12, 10, 15, 10, 22, 12

8. 14, 9, 20, 5, 17, 13

9. 21, 15, 16, 25, 13, 18

10. 20, 17, 10, 31, 25, 18, 12

11. 48, 40, 53, 43, 52, 46

LESSON
6.7**Practice** *continued*

Find the range and mean absolute deviation of the data. Round to the nearest hundredth, if necessary.

12. 9, 15, 28, 10, 8

13. 32, 33, 22, 85, 58

14. 24, 35, 18, 20, 17, 30

15. 116, 130, 120, 125, 140, 125

16. 105, 98, 95, 100, 95, 107

17. 36, 39, 58, 42, 106, 39, 48, 45

18. Tomato Plants The heights (in inches) of eight tomato plants are 36, 45, 52, 40, 38, 41, 50, and 48.

- What is the range of the tomato plant heights?
- Find the mean, median, and mode(s) of the tomato plant heights.
- Which measure of central tendency best represents the data? *Explain.*

19. World Population The populations (in millions) in 2000 on each of the six inhabited continents were 803, 487, 348, 3686, 730, and 31.

- What is the range of the populations?
- Find the mean, median, and mode(s) of the populations. Round your answers to the nearest tenth.
- Which measure of central tendency best represents the data? *Explain.*

20. Quiz Scores You and your friend have a friendly competition going on about the scores on your math quizzes. Both of your scores for the first five quizzes are given below. *Compare* the spreads of the data sets by using (a) the range and (b) the mean absolute deviation.

Your quiz scores: 18, 16, 19, 15, 17

Friend's quiz scores: 20, 20, 13, 12, 17

6.8

Compare Statistics from Samples



Georgia
Performance
Standard(s)

MM1D3a,
MM1D3b

Your Notes

Goal • Compare statistics from different samples.

VOCABULARY

Quartile

Upper quartile

Lower quartile

Interquartile range

Example 1 Compare statistics from different samples

Production The data sets below give the number of computers assembled by each employee during one month in the East production division and the West production division. Compare the data using the mean, median, range, and interquartile range.

East: 19, 18, 17, 16, 15, 13, 12, 10

West: 21, 20, 19, 18, 17, 16, 14, 9

East	West
$\bar{x} = \frac{19 + \cdots + 10}{8} = \underline{\hspace{2cm}}$	$\bar{x} = \frac{21 + \cdots + 9}{8} = \underline{\hspace{2cm}}$
Median: $\frac{\boxed{\hspace{1cm}}}{2} = \underline{\hspace{2cm}}$	Median: $\frac{\boxed{\hspace{1cm}}}{2} = \underline{\hspace{2cm}}$
Range: $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$	Range: $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
Lower Quartile: $\frac{\boxed{\hspace{1cm}}}{2} = \underline{\hspace{2cm}}$	Lower Quartile: $\frac{\boxed{\hspace{1cm}}}{2} = \underline{\hspace{2cm}}$
Upper Quartile: $\frac{\boxed{\hspace{1cm}}}{2} = \underline{\hspace{2cm}}$	Upper Quartile: $\frac{\boxed{\hspace{1cm}}}{2} = \underline{\hspace{2cm}}$
Interquartile Range: $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$	Interquartile Range: $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

The _____ mean is greater than the _____ mean, so the _____ averaged more computers assembled per person.

The East's _____ is less than the West's, so their data is less spread out.

The West's _____ is _____ than the East's, so the West's middle 50% of the data showed _____ variation than the middle 50% of the East's data.

Example 2 Compare sample and population statistics

Production At the end of the month, summary production statistics are calculated for the entire company. The table below shows the summary data for the North and South divisions. Find the averages of the summary statistics from the four samples: the East and West divisions from Example 1, the North division and the South division.

	Mean	Median	Range	Interquartile Range
North	17.5	15.5	7	3
South	13.3	12	11	6
All	14.9	13	10	5.5

Solution

Average Mean:

$$\bar{x} = \frac{\boxed{}}{\boxed{}} = \underline{\hspace{2cm}}$$

Average Median:

$$\frac{\boxed{}}{\boxed{}} = \underline{\hspace{2cm}}$$

Average Range: $\frac{\boxed{}}{\boxed{}} = \underline{\hspace{2cm}}$

Average Interquartile Range:

$$\frac{\boxed{}}{\boxed{}} = \underline{\hspace{2cm}}$$

The average mean and median are _____ than the corresponding population measures. The average range and interquartile range are _____ than the corresponding population measures.

Your Notes

Checkpoint Complete the following exercises.

1. Compare the data using the mean, median, range, and the interquartile range.

Data set 1: 102, 105, 108, 110, 112, 113

Data set 2: 100, 103, 105, 106, 109, 113

-
2. In Example 2, suppose the Northeast division had the following statistics. How does the inclusion of the Northeast division's statistics change the averages in Example 2?

	Mean	Median	Range	Interquartile Range
Northeast	15.7	18	12	7.2

Homework

LESSON
6.8**Practice**

In Exercises 1–5 find the mean, median, range, lower quartile, upper quartile, and interquartile range of the data set.

1. 4, 7, 14, 8, 9

2. 85, 90, 76, 65, 92, 88

3. 1.1, 2.7, 3.6, 2.8, 1.5, 4.7, 3.4

4. 110, 145, 115, 124, 118, 129, 119

5. 50, 55, 60, 58, 62, 57, 68, 51, 63

LESSON
6.8
Practice *continued*

6. Compare the two samples using mean, median, range, and interquartile range.

Sample A: 8, 9, 10, 7, 11, 6, 15, 12

Sample B: 11, 16, 4, 5, 2, 15, 12, 10

7. **Sales** A sales department is trying to analyze and improve their sales techniques by reviewing the sales data from past years. The table shows summary statistics for the number of sales for each quarter, Q1 (January–March), Q2 (April–June), Q3 (July–September), Q4 (October–December), and the entire year. Compare the average of the four quarters with the entire year using mean, median, range, and interquartile range.

	Mean	Median	Range	Interquartile Range
Q1	35	28	6	5
Q2	44	34	8	4
Q3	43	32	5	8
Q4	42	30	8	5
Year	38	31	7	6

Words to Review

Give an example of the vocabulary word.

Multiplication Counting Principle	Addition Counting Principle
n factorial	Permutation
Combination	Compound event
Mutually exclusive events	Overlapping events
Independent events	Dependent events

Conditional probability	Expected value
Survey	Population
Random sample	Stratified random sample
Systematic sample	Convenience sample
Self-selected sample	Representative sample
Biased sample	Biased question

Mean	Median
Mode	Measure of dispersion
Range	Deviation from the mean
Mean absolute deviation	Quartile
Upper quartile	Lower quartile
Interquartile range	