LESSON 3.12

Exercise Set A

MM1A1g Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

MM1A1i Understand that any equation in x can be interpreted as the equation $f(x) = g(x)$, and interpret the solutions of the equation as the x-value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

Find the average rate of change of the function from x_1 to x_2.

1. $f(x) = -3x + 15; x_1 = 0, x_2 = 2$

2. $f(x) = 2x + 8; x_1 = -1, x_2 = 4$

3. $f(x) = -\sqrt{x - 2} + 4; x_1 = 2, x_2 = 11$

4. $f(x) = -\sqrt{x + 1} + 2; x_1 = 0, x_2 = 8$

Compare **the average rates of change of the functions from x_1 to x_2.**

5. $f(x) = x^2 + 12x - 3, g(x) = 7\sqrt{16x}; x_1 = 0, x_2 = 4$

6. $f(x) = x^3 - 2x^2 - x, g(x) = 12x + 5; x_1 = -1, x_2 = 5$

7. **Error Analysis** *Describe* and correct the student's error in finding the average rate of change of $f(x) = x^3 + 2x^2$ from $x_1 = -1$ to $x_2 = 0$.

$$\frac{f(0) - f(-1)}{-1 - 0} = \frac{0 - 1}{-1} = 1$$

The average rate of change is 1. ✗

Solve the equation by graphing. If necessary, use a graphing calculator and round your answer to the nearest hundredth.

8. $x^2 + 1 = -x + 3$

9. $\frac{1}{3}x^3 + x = -\frac{1}{8}x^2$

10. $\sqrt{x - 1} = x^3 - 7$

11. $2\sqrt{x} + 1 = \frac{1}{4}(x^2 + x)$

12. $x^3 - \sqrt{2} = 2x$

13. $-\sqrt{x + 10} = x^2 - 7$

14. **Multiple Choice** The graphs of $f(x)$ and $g(x)$ intersect at the points $(-1, 3)$ and $(4, -2)$. What are the solutions of the equation $f(x) = g(x)$?

A. -2 and -1 **B.** -2 and 3 **C.** -1 and 4 **D.** -1 and 3

15. **Multiple Representations** Two objects are thrown upward at the same time. One is thrown from a height of 6 feet with an initial vertical velocity of 64 feet per second. The other is thrown from a height of 20 feet with an initial vertical velocity of 48 feet per second.

a. **Writing Functions** Write functions $f(t)$ and $g(t)$ for the height (in feet) t seconds after each was thrown.

b. **Drawing a Graph** Graph the functions on the same graph.

c. **Interpreting a Solution** Solve $f(t) = g(t)$. Interpret the solution in the context of the problem.

LESSON 3.12

Exercise Set B

MM1A1g Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

MM1A1i Understand that any equation in x can be interpreted as the equation $f(x) = g(x)$, and interpret the solutions of the equation as the x-value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

Find the average rate of change of the function from x_1 to x_2.

1. $f(x) = -\frac{2}{3}x + 5; x_1 = -5, x_2 = 3$

2. $f(x) = -x^2 + 8x - 4; x_1 = 0, x_2 = 4$

3. $f(x) = 1 - \sqrt{x + 3}; x_1 = 1, x_2 = 6$

4. $f(x) = 2 - \sqrt{x + 1}; x_1 = 3, x_2 = 7$

Compare **the average rates of change of the functions from x_1 to x_2.**

5. $f(x) = 5x + 3, g(x) = 6\sqrt{9x + 9}; x_1 = 0, x_2 = 3$

6. $f(x) = x^2 + 8x, g(x) = x^3 - x^2 + 4; x_1 = -1, x_2 = 4$

7. **Open-Ended** For the function $f(x) = -x^3 + 5x^2$, state values of x_1 and x_2 so that the average rate of change from x_1 to x_2 is (a) positive and (b) negative.

Solve the equation by graphing. If necessary, use a graphing calculator and round your answer to the nearest hundredth.

8. $14 - x^2 = -\sqrt{x}$

9. $-x^3 + x = \frac{3}{2}x^2$

10. $-\sqrt{x + 4} - 2 = x^3$

11. $\sqrt{x - 2} = x^2 + 2$

In Exercises 12–14, complete the statement.

12. The equation $f(x) = g(x)$ has __?__ solution(s) when the graphs of $f(x)$ and $g(x)$ do not intersect.

13. If a cubic equation has a maximum in the second quadrant and a minimum in the fourth quadrant, then the sign of the average rate of change from the maximum to the minimum is __?__.

14. If the graphs of two equations intersect at (a, b) and (c, d), then the average rates of change for both equations are the same from $x_1 =$ __?__ to $x_2 =$ __?__.

15. **Profit** The profit P (in thousands of dollars) of a company from 1997 to 2007 can be approximated by $P = 0.4t^3 - 4t^2 + 55.6$ where t represents the year, with $t = 0$ corresponding to 1997.

a. Use a graphing calculator to graph the function.

b. Find the average rate of change of the function from 2004 to 2007. Interpret your answer in the context of the problem.

c. Find the three-year time periods when the average rate of change was the most positive and the most negative.