



MM1G2b Understand and use the relationships among a statement and its converse, inverse, and contrapositive.

Rewrite the conditional statement in if-then form.

- **1.** It is time for dinner if it is 6 P.M.
- **2.** There are 12 eggs if the carton is full.
- **3.** An obtuse angle is an angle that measures more than 90° and less than 180° .
- **4.** The car runs when there is gas in the tank.

Write the converse, inverse, and contrapositive of each statement.

- 5. If you like hockey, then you go to the hockey game.
- **6.** If x is odd, then 3x is odd.

Decide whether the statement is *true* or *false*. If false, provide a counterexample.

- 7. The equation 4x 3 = 12 + 2x has exactly one solution.
- **8.** If $x^2 = 36$, then *x* must equal 18 or -18.
- **9.** If $m \angle A = 122^\circ$, then the measure of the supplement of $\angle A$ is 58°.
- **10.** Two lines intersect in at most one point.

Write the converse of each true statement. If the converse is also true, combine the statements to write a true biconditional statement.

- **11.** If an angle measures 30° , then it is acute.
- **12.** If two angles are supplementary, then their sum is 180° .
- **13.** If two circles have the same diameter, then they have the same circumference.
- **14.** If an animal is a panther, then it lives in the forest.

Rewrite the biconditional statement as a conditional statement and its converse.

- **15.** Two lines are perpendicular if and only if they intersect to form right angles.
- **16.** A point is a midpoint of a segment if and only if it divides the segment into two congruent segments.

Decide whether the statement is a valid definition.

- **17.** If a number is divisible by 2 and 3, then it is divisible by 6.
- **18.** If two angles have the same measure, then they are congruent.
- **19.** If an angle is a right angle, then its measure is greater than that of an acute angle.

Exercise Set A (continued)

20. **Error Analysis** *Describe* and correct the error made in writing the if-then statement.

Given statement: All high school students take four Mathematics courses.

If-then statement: If a high school student takes four courses, then all four are Mathematics courses.

In Exercises 21–23, use the information in the table to write a definition for each type of saxophone.

	Frequency (cycles per second)	
Instrument	Lower limit (Hz)	Upper limit (Hz)
E-flat baritone saxophone	69	415
B-flat tenor saxophone	103	622
E-flat alto saxophone	138	830

- **21.** E-flat baritone saxophone
- **22.** B-flat tenor saxophone
- **23.** E-flat alto saxophone

In Exercises 24 and 25, use the information in the table above and the answers to Exercises 21–23.

- 24. If the frequency of a saxophone was 95 Hz, what could you conclude?
- 25. If the frequency of a saxophone was 210 Hz, what could you conclude?
- **26. Profits** The statements below were made during an executive meeting at a department store. Use these statements in parts (a)–(c).

The department store's profits will decrease if wages are increased.

Offering more sales will increase the department store's profits.

Profits will increase when inventory is increased.

- **a.** Write each statement in if-then form.
- **b.** Write the converse of each of the statements in part (a). Is the converse of each statement true? *Explain* your reasoning.
- **c.** Write a true if-then statement about the department store's profit. Is the converse of your statement *true* or *false*? *Explain*.





b Understand and use the relationships among a statement and its converse, inverse, and contrapositive.

Rewrite the conditional statement in if-then form.

- **1.** A car with leaking antifreeze has a problem.
- 2. Don't say anything at all when you don't have something nice to say.
- **3.** You can't teach an old dog new tricks.
- **4.** A vein is a blood vessel that carries blood toward the heart.

The statement is either the converse, the inverse, or the contrapositive of the *meaning* of a well-known saying. What is the saying?

- **5.** If you learn things, then use them to make mistakes.
- **6.** If it isn't lost easily, then it isn't gained easily.
- **7.** If dogs are not lying down, then don't let them sleep.
- 8. If you don't see something, then you won't get it.

For the given statement, write the if-then form, the converse, the inverse, and the contrapositive and indicate whether each statement is *true* or *false*.

- **9.** A circle with a radius of *r* has a circumference of $2\pi r$.
- **10.** A regular pentagon has five sides.

In a plane, point *F* lies between points *C* and *D* and *EF* intersects *CD* so that $\angle CFE \cong \angle DFE$. Decide whether the given statement is true. Explain your answer using definitions and properties that you have learned.

- **11.** FC and \overline{FE} are opposite rays.
- **12.** $\angle CFE$ and $\angle DFE$ are supplementary angles.
- **13.** $m \angle CFD = 180^{\circ}$
- **14.** Points *C*, *F*, and *E* are collinear points.
- **15.** $\angle CFE$ is an obtuse angle.
- **16.** $\overrightarrow{CD} \perp \overrightarrow{EF}$

Rewrite the definition as a biconditional statement.

- **17.** A conditional statement is a logical statement that has two parts, a hypothesis and a conclusion.
- **18.** A conjecture is an unproven statement that is based on observations.
- **19.** A counterexample is a specific case for which a given conjecture is false.

Exercise Set B (continued)

20. Rewrite the following definition as a biconditional statement: A polygon is a closed plane figure that is formed by three or more sides, with each side intersecting exactly two other sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

Decide whether the statement is a valid definition.

- **21.** If a triangle has one right angle, then it is a right triangle.
- **22.** If a solid is a cylinder, then it has two circular bases.
- **23.** If a number is divisible by 9, then it is divisible by 3.

Write the converse of each true statement. Tell whether the converse is true. If false, explain why.

- **24.** If x > 5, then x > 0. **25.** If x < 8, then -x > -8. **26.** If $-x \ge x$, then $x \le 0$.
- **27.** Can the statement, "If $x^2 14 = x + 6$, then x = 5," be combined with its converse to form a true biconditional statement?

In Exercises 28–32, use the following information.

Cyclones Cyclones are areas of rotating air that can be associated with many types of severe weather. Tornadoes are sometimes considered to be cyclones that occur over land. Tropical cyclones are cyclones that form over tropical ocean waters. A weak tropical cyclone with winds of less than 38 miles per hour is called a tropical depression. A tropical depression turns into a tropical storm if its winds increase to 39 miles per hour or faster. The most severe type of tropical cyclone occurs if the wind speeds increase to greater than 74 miles per hour. This type of storm has many different names, depending on where it forms. Some of the names used in different locations are: *typhoon* over much of the Pacific Ocean, *willy-nilly* near Australia, and *hurricane* over the Atlantic Ocean.

Tell whether the statement is a valid definition of a hurricane. If not, explain why.

- **28.** If a tropical cyclone develops in the Atlantic Ocean, then the storm is a called a hurricane.
- **29.** If a tropical cyclone has winds in excess of 74 miles per hour, then the storm is called a hurricane.
- **30.** If a storm occurs in the tropics over the Atlantic Ocean and the storm has winds of over 74 miles per hour, then the storm is a hurricane.
- **31.** If a cyclone with winds of over 74 miles per hour forms in the Atlantic Ocean, then the storm is a hurricane.
- **32.** If a tropical cyclone is formed over the Atlantic Ocean and develops wind speeds of over 74 miles per hour, then the storm is classified as a hurricane.